COMMENTS ON QUESTIONS

MATHEMATICAL TRIPOS PART II, 2021

Course: ALGEBRAIC GEOMETRY

Paper no. 1 Question no. 25I
Comments: The most popular of the questions on this course. Several candidates lost marks giving an incorrect proof that every variety is a finite union of irreducibles, or the last part, for giving a hand-waving argument (or no argument at all) that the tangent space to V at the origin was 3-dimensional.

Paper no. 2 Question no. 25
Comments: One of those questions where some candidates struggled with things they clearly thought were obvious but needed to prove. Unsuprisingly the last part (no nonconstant morphisms from $\mathbf{P}^n$ to $\mathbf{P}^1$) was found hard.

Paper no. 3 Question no. 24I
Comments: By far the least popular of the algebraic geometry questions, possibly because of the non-standard (although not difficult) example at the end.

Paper no. 4 Question no. 24I
Comments: A routine question on the Riemann–Roch theorem and applications. Some candidates lost marks in the last part by simply quoting a stronger result for plane curves.

Course: ALGEBRAIC TOPOLOGY

Paper no. 1 **Question no. 21F**
Comments: This turned out to be the most challenging question, with the last part serving to distinguish the strongest students.

Paper no. 2 **Question no. 21F**
Comments: The average mark here was quite high. There was a variety of correct forms for the answer to the last part.

Paper no. 3 **Question no. 20F**
Comments: A quite high average with less attempts however than the other problems from this course. In general students answered this well.

Paper no. 4 **Question no. 21F**
Comments: Students were asked to describe carefully how to triangulate and few answers were completely satisfactory from this point of view, though some did succeed in obtaining full marks.

Course: ANALYSIS OF FUNCTIONS

Paper no. 1 Question no. 23H
Comments: Lebesgue theorem was knows, but differentiation had to be proved in R^n (not R). Abssolute continuity was not proved correctly by most attempts.

Paper no. 2 Question no. 23H
Comments: Question (a) using a contradiction argument was mostly done correctly. Question (b) required extending the positivity property from $\mathcal{S}$ to $X_{N,k}$ which required a smoothing argument which was not done correctly.

Paper no. 3 Question no. 22H
Comments: Computation of B in Fourier was mostly done correctly. Estimates of Sobolev norms was too often done with mistakes: students do not master the computation of basic three dimensional integrals.

Paper no. 4 Question no. 23H
Comments: Mostly bookwork and done correctly (either directly or using Hahn Banach for the lower semi continuity of norms).

Course: APPLICATIONS OF QUANTUM MECHANICS
Questions were in general standard and in retrospect should have been made a bit more difficult.

Paper no. 1 Question no. 35B
Comments: This question was found particularly easy.

Paper no. 2 Question no. 36B
Comments: Straightforward question. Well answered.

Paper no. 3 Question no. 34B
Comments: This question was found very easy for students.

Paper no. 4 Question no. 34B
Comments: The trickiest of the four questions, but still answered well by most students.

Course: ASYMPTOTIC METHODS

Paper no. 2 **Question no. 32A**
Comments: There were many excellent attempts at this question, particularly on the stationary-phase calculation. One mark was lost for not commenting that the error term (from the end-point) was $O(1/x)$ by Riemann–Lebesgue. Rather more marks were lost on the Laplace integral for not realising that the maximum of $|\phi|$ was at the end-point, which thus gives the dominant contribution. A good question for revealing the differences between the two methods.

Paper no. 3 **Question no. 30A**
Comments: There were many excellent attempts at this question, showing clear understanding of both Watson's lemma and the method of steepest descent. This is particularly impressive as the question provided no routemap or intermediate results, yet candidates knew where they were going.

Paper no. 4 **Question no. 31A**
Comments: Aside from the odd arithmetical error, most candidates did the standard bookwork to find q and derive $S'' = (S')^2 = -q$. Unfortunately, many then did not take note of the *three* terms requested in the question, and posed only $S \sim S_0 + S_1$, while expanding expressions like $q^{1/4}$ binomially. This is inconsistent, and it is necessary to include the next term S_2 to calculate the $O(x^{-1/2})$ term correctly. A good test of understanding of the ideas behind the recipe.

Course: APPLIED PROBABILITY

Paper no. 1 Question no. 28
Comments: The first part of the question was bookwork, the second was a variation of the standard proof of the central limit theorem using characteristic functions, and the third involved some computations. Apparently students found it a little bit on the difficult side, probably more because it involved methods that, although simple and standard, were not central to the course material.

Paper no. 2 Question no. 28
Comments: Reasonable question of appropriate difficulty, except perhaps for the last part (which carried 10 out of the 20 marks of the question) which most students made almost no progress on.

Paper no. 3 Question no. 27
Comments: Although no parts of this question were straight bookwork, they were all small variations on results the students had seen in lectures. As expected, students generally did quite well.

Paper no. 4 Question no. 27
Comments: This question appears to have been a little bit on the difficult side. None of the three parts were bookwork, and the students struggled with all of them. Fewer than half attempted part (b), and only a handful attempted part (c). The fact that there were a lot of explicit computations involved in the last two parts may have been off putting.

Course: AUTOMATA AND FORMAL LANGUAGES

Paper no. 1 **Question no. 4F**
Comments: The answers, even those essentially correct, were quite sloppy.

Paper no. 1 **Question no. 12F**
Comments: Many attempts. I found most answers quite sloppily written, even if essentially correct. See above remark.

Paper no. 2 **Question no. 4F**
Comments: This question was probably on the easy side and many students earned full marks.

Paper no. 3 **Question no. 4F**
Comments: Less attempts than the other short questions. This was probably closer to a reasonable level of difficulty. Again, even essentially "correct" answers were often quite sloppy.

Paper no. 3 **Question no. 12F**
Comments: Many attempts and high marks. Perhaps this turned out to be on the easy side.

Paper no. 4 **Question no. 4F**
Comments: This question turned out to be too short and too easy. It's not surprising that it attracted many attempts.

Course: CLASSICAL DYNAMICS

Paper no. 1 **Question no. 8**
Comments: This question was mostly done well. The algebra at the end was sometimes rather messy, but did not cause too many problems. Candidates who struggled mostly did so because they did not set the problem up correctly initially.

Paper no. 2 **Question no. 8**
Comments: This question was, surprisingly, found to be rather difficult despite being largely bookwork. Demonstrations that the net torque vanishes were often unconvincing. Very few candidates obtained the correct conditions on (I_1, I_3) for the precession to be prograde or retrograde.

Paper no. 2 **Question no. 14**
Comments: Most candidates who attempted this question could manage the early, standard parts well, but then struggled with applying the theory of adiabatics to the given problem. The algebra required in the middle of the problem, though messy, did not cause too many problems. However, few candidates understood the consequences of ω varying on a much longer timescale than the original oscillation period.

Paper no. 3 **Question no. 8**
Comments: This question was mostly found easy by the candidates. Disappointingly few candidates knew the correct expression for an electric field, with most dropping the $\dot{\mathbf{A}}$ term, but this did not cause problems in the rest of the question.

Paper no. 4 **Question no. 8**
Comments: Candidates gave various descriptions of a Lagrange top. Most were able to derive the correct EL equations from the given Lagrangian, and identify the constants of motion. They typically struggled to find the equation for nutation.

Paper no. 4 **Question no. 15**
Comments:
This question was largely done well. Most candidates knew what it meant for a coordinate transfromation to be canonical, though in the infinitesimal case many forgot to look for the implications of $\{Q^i, Q^j\} = 0 = \{P_i, P_j\}$ and instead just checked $\{Q^i, P_j\} = \delta^i{}_j$. The fact that conserved quantities correspond to symmetries of H was known by most candidates.

Course: CODING AND CRYPTOGRAPHY

Paper no. 1 Question no. 3
Comments: Reasonable and appropriate short question.

Paper no. 1 Question no. 11
Comments: Reasonable and appropriate question, mostly on bookwork or small variations of arguments students had seen in class. The material tested here is central to the course, most students did quite well.

Paper no. 2 Question no. 3
Comments: Reasonable and appropriate short question on what is perhaps the most basic result in the course, the channel coding theorem on capacity. Perhaps slightly surprising that there were only 19 attempts.

Paper no. 2 Question no. 12
Comments: Reasonable question of appropriate difficulty.

Paper no. 3 Question no. 3
Comments: Reasonable short question of appropriate difficulty.

Paper no. 4 Question no. 3
Comments: Reasonable short question on standard cryptographic algorithms. The main source of difficulty was the fact that the algorithms described consist of numerous steps, all of which must be connected/justified through appropriate number-theoretic properties.

Course: COSMOLOGY
Questions were in general standard but some were more challenging than others. Very few attempts on the short cosmology questions. One of them turned out to be more difficult than expected.

Paper no. 1 **Question no. 9B**
Comments: Straightforward. Few attempts.

Paper no. 1 **Question no. 15B**
Comments: A good and fairly popular question that differentiated between better and weaker students.

Paper no. 2 **Question no. 9B**
Comments: Part (a) (Bose and Fermi statistics) was surprisingly found too challenging for the students.

Paper no. 3 **Question no. 9B**
Comments: Part (a) was found surprisingly difficult by the students even though it is straightforward calculation.

Paper no. 3 **Question no. 14B**
Comments: Straightforward question. Well answered with a good take-up.

Paper no. 4 **Question no. 9B**
Comments: Part (b) was found particularly difficult by the students.

Course: DIFFERENTIAL GEOMETRY

Paper no. 1 Question no. 26F
Comments: Many attempts. The last part served well to distinguish the best answers.

Paper no. 2 Question no. 26F
Comments: This was well answered on the whole. Answering the last part completely correctly proved challenging.

Paper no. 3 Question no. 25F
Comments: I was impressed how many answered well the unseen part at the end.

Paper no. 4 Question no. 25F
Comments: Slightly fewer attempts than the previous; again, I was impressed by the general quality of the answers to the unseen parts.

Course: DYNAMICAL SYSTEMS

Paper no. 1   Question no. 32A
Comments: This was a well-done question overall, showing a sound grasp of the use of Lyapunov functions and La Salle. In part (b), one should use a contour of V around 0 to justify forward invariance before applying La Salle. In part (c) the domain of stability is defined by the stable manifolds of the saddles, which needed to be drawn, and $V < 1/4$ is only a subset of this domain (see Q11 example sheet 2).

Paper no. 2   Question no. 33A
Comments: This question was found challenging for various reasons. The main stumbling blocks were showing that the sign of $\dot{y} - \dot{x}$ is such that trajectories cannot leave the parallelogram via its top or bottom (a simple, but ad hoc, argument), calculating the energy-balance integral around the unperturbed circular orbits (much easier using the polar representation $\mathbf{x} = (F + R\cos\theta, -R\sin\theta)$), and sketching the case $F > 1$ (a fixed point on the slow manifold).

Paper no. 3   Question no. 31A
Comments: Not particularly straightforward, but well-done overall. The bifurcation diagrams in part (b) were very well done by many candidates. By contrast, the phase-plane sketches near the bifurcation in (d) were more wayward, missing the idea of collapse onto the centre manifold and evolution on the manifold between the saddle and the two nodes.

Paper no. 4   Question no. 32A
Comments: This question on period 3 implying all periods was generally done very well, and I was impressed by the way most candidates were able to use the directed graph to find specific orbits of periods 2, 4 and 5. A few candidates did not appreciate that the wording and allowed assumptions of part (a) required a more formal proof based on the chained trimming lemma. A few thought period 3 implies chaos implies all periods, without realising that it is F^2 that has a horseshoe and so this argument only implies all even periods.

Course: ELECTRODYNAMICS

Paper no. 1 **Question no. 37**
Comments: Although there were not that many attempts at this electrodynamics question, the proportion who did well was high. The invariant scalars were well known and candidates could manipulate the field strength tensor comfortably. There were two routes to finding particle trajectories in the EM background, with the most efficient keeping the equations of motion in matrix form, but most found the solution by one means or another.

Paper no. 3 **Question no. 36**
Comments: Most candidates could reproduce the bookwork derivation of the Larmor radiation formula, though in rather skeletal form. In the second part, most also recognised that if the particle failed to overcome the potential barrier it would radiate again on the return journey.

Paper no. 4 **Question no. 36**
Comments: There were a dwindling number of attempts at this question on dielectrics and boundary conditions between different media. Most recognised the correct matching conditions for the perpendicular and tangential components of the electric field. However, a proportion did not apply Maxwell's equations seamlessly or got lost somewhat in the algebra, so few found the correct solution. More should have recognised (and/or derived) the dipole field for the final comparison.

Course: FLUID DYNAMICS

Paper no. 1 Question no. 39A
Comments: This question concerned Stokes flow between concentric rotating spheres, slightly more complex algebraically, but exactly the same in principle as the rotating-sphere calculation in Q5 on example sheet 2. (See also Q2 for the symmetry arguments.) It was thus surprising to see the number of attempts foundering on not knowing that the couple is $\int \mathbf{x} \wedge \sigma \cdot \mathbf{n} \, dS$.

Paper no. 2 Question no. 39A
Comments: The novel first part on 2D lubrication flow in a variable-width channel was done very well. The solution in the second part of $\nabla^2\psi = 0$ with $\psi = f(r)\sin\theta$, which is essentially IB Methods, caused a surprising amount of mathematical difficulty. The fluid-mechanical aspects of this Hele-Shaw problem seemed fairly well understood.

Paper no. 3 Question no. 38A
Comments: A fairly standard and easy boundary-layer question, which should have attracted more attempts. The scaling arguments were generally done well, leading to the correct similarity ansatz. The subsequent routine calculations to obtain the similarity equation went adrift more often than I would have expected. A few candidates thought $U \propto R^{-3}$ instead of R^{-2}.

Paper no. 4 Question no. 38A
Comments: A fair number of candidates were able to reproduce the bookwork from lectures on the strained vortex. Few understood that advection of angular momentum, $\rho v r$, into the cylinder involved calculating the flux $\int (\rho v r)\mathbf{u} \cdot \mathbf{n} dS$ on its boundaries. Had they done so, the integrals were easy. A bit disappointing as advection of mass, momentum and energy should be familiar, and the angular momentum integral equation is a natural variation on the momentum integral equation. In retrospect, the wording might have helped candidates make the connection.

Course: FURTHER COMPLEX METHODS
No comments received

Course: GALOIS THEORY

Paper no. 1 Question no. 18I
Comments: Most of this question was answered well, with a good number of alphas. The proofs of the existence of splitting fields given were for the most part quite shoddy.

Paper no. 2 Question no. 18I
Comments: The first half of the question, on finite fields, was generally done well, although many said simply that "$\mathbf{F}_q$ is unique" and drew unjustified conclusions. The computation of the Galois group in the second part was well done — most candidates chose to use reduction mod p rather than the resolvent cubic.

Paper no. 3 Question no. 18I
Comments: A novel question on symmetric functions and discriminants. The last part was a bit challenging, but most people realised that the discriminant could only contain two monomials even if they were unable to compute when the discriminant vanished.

Paper no. 4 Question no. 18I
Comments: The candidates who failed to get an alpha mostly did so by quoting results they were asked to prove. The last part was generally well done.

Course: GENERAL RELATIVITY

Paper no. 1 Question no. 38
Comments: This proved to be a challenging question because many candidates lost their way in the calculation of the (vanishing) Weyl tensor in part (b) and then most were unable to make an estimate of its "leading-order" r^{-3} contribution to C_{trtr} in part (c). Given that the connections vanish, the expression for the Riemann tensor is simplified, but it still proved difficult for most to systematically follow through the algebra, let alone perform the contractions to find the Ricci tensor and scalar. Good exam technique should have had more candidates cutting their losses and moving on when their expressions exploded. This also led to straightforward marks being missed at the end about tides and the equivalence principle.

Paper no. 2 Question no. 38
Comments: This question was attempted well by most candidates who were able to determine the constants of the motion and use these effectively to determine the properties of a spacelike geodesic. One deficiency was the understanding of redshift which should be found by considering the effect on a wavecrest emitted a small time later at $t_1 + \Delta t_1$ and detected at $t_2 + \Delta t_2$.

Paper no. 3 Question no. 37
Comments: The first part of this question was a simple diagonalisation to identify that the metric was Lorentzian, but this was just picked off by a modest number of candidates for easy marks with few going on to attempt the whole question. Despite Schwarzschild and Finkelstein coordinates for black holes being on the schedules, only one candidate could correctly reproduce the transformation between them. With the benefit of hindsight, more guidance should have been offered for this bookwork element. The relatively straightforward but unseen last parts of the question could have been attempted by more candidates using their knowledge of the course, notably Birkhoff's theorem and Kruskal coordinates describing infalling trajectories.

Paper no. 4 Question no. 37
Comments: Most candidates started this question well, but then their calculations of the few non-trivial components of the Ricci tensor became undisciplined. A surprising number were unable to correctly contract this to find the Ricci scalar, which was a pre-requisite for correctly obtaining the well-known Friedmann equation. Those who did not waste further time on unproductive algebra, but ventured on to the latter parts of the question, found them quite straightforward.

Course: GRAPH THEORY

Paper no. 1 Question no. 17G
Comments: The bookwork for this question is often found difficult, but here it was on the whole well done. Students came up with a variety of clever ways of doing the problem part.

Paper no. 2 Question no. 17G
Comments: The non-bookwork parts of this question were found very hard. Many students thought that they could find a cycle of logarithmic length without using the fact that the graph was cubic!

Paper no. 3 Question no. 17G
Comments: An extremely well answered question. Students could reproduce the bookwork accurately and had no trouble with the last part of the question.

Paper no. 4 Question no. 17G
Comments: Students seem to have Hall's theorem, and all the associated concepts, very well understood.

Course: INTEGRABLE SYSTEMS

Paper no. 1 Question no. 33

Comments: The first few parts of question was done well by most candidates who attempted them. Things got tougher in part (c), and many candidates could not apply to formalism to actually find a first integral of the Sinh-Gordon equation. (I do not recall any candidate realising that one could actually ignore the formalism, multiply the Sh-G eqn by ϕ' and integrate directly.)

Paper no. 2 Question no. 34

Comments:

This question was found to be difficult. Many candidates could not give a good definition of the scattering data, not obtain the time-dependence of (κ_n, c_n, R). Almost no-one was able to correctly manipulate the GLM kernel to the point of obtaining a solution to the KdV equation, and despite some very generous marking candidates scored poorly on this question.

Paper no. 3 Question no. 32

Comments:

Candidates lost marks at various points in this question, which I thought would be found standard. Many forgot to show that the given transformations did indeed form a group. Others struggled to combine real vectors acting on (ψ_1, ψ_2) to form a complex vector rotating the phase of $\psi = \psi_1 + i\psi_2$. While the notion of a prolongation of a vector field seemed to be understood, few students could apply this to the nonlinear Schrodinger eqn, and very few correctly wrote down a solution corresponding to a travelling soliton.

Course: LINEAR ANALYSIS

Paper no. 1 **Question no. 22H**
Comments: Basis question of Hilbert spaces. Few students did this right, most students do not master neither the diagonal extraction nor the basic meaning of Parseval identity.

Paper no. 2 **Question no. 22H**
Comments: Very close to bookwork supervision, was mostly done correctly. The Baire category theorem is well understood.

Paper no. 3 **Question no. 21H**
Comments: Abstract question close to supervisions, students master the finite covering of compact sets argument.

Paper no. 4 **Question no. 22H**
Comments: Very basic question of Hilbert spaces. Even the definition of the adjoint operator was not obvious for most students. Many formal proofs of the computation of the adjoint, but very few realized that the very definition of U^* requires proving that the map $\ell^2 \to H$ which to $(c_i)_{i\geq 1}$ associates $\sum_{i\geq 1} c_i e_i$ is well defined and continuous which since e_i is a frame only (and not an orthogonal basis) requires an argument.

Course: LOGIC & SET THEORY

Paper no. 1 Question no. 16G
Comments: This question on propositional logic gave students no trouble. With hindsight, perhaps the problem parts were too easy.

Paper no. 2 Question no. 16G
Comments: This question produced many very good answers. Students needed to have a real understanding of ordinals to make progress, and the fact that so many of them did, even solving the non-trivial final part of the question, is really impressive.

Paper no. 3 Question no. 16G
Comments: The section on cardinals is always found difficult. For the last part, students needed to have really understood what makes the proof of Hartogs' Lemma work, and clearly they had not.

Paper no. 4 Question no. 16G
Comments: Very impressive mastery shown of one of the hardest parts of this course: the use of ZF axioms. But perhaps the last part was too straightforward, especially given the hint.

Course: MATHEMATICAL BIOLOGY

Paper no. 3 Question no. 6E

Comments: A very well done question. A few candidates overlooked that the death rate $d(a)$ was age-dependent.

Paper no. 4 Question no. 6E

Comments: Many candidates found a nonzero homogeneous steady state for the special case $D = 0$, which does not, of course, satisfy the boundary condition $n(L) = 0$. They then attempted a linear perturbation analysis about this state for $D \neq 0$, not realising that this is invalid because the homogeneous state cannot be a steady state, whether stable or unstable, because of the boundary condition. The correct approach is to linearise about a zero population; if that is unstable to a growing eigenmode then a nonzero population will not die out.

Course: MATHEMATICS OF MACHINE LEARNING

Paper no. 1 **Question no. 31**
Comments: This question was edited on recommendation of the examiners, and a hint was added to make it easier. In retrospect, the hint could have been left out, as the results were very good. Nonetheless, it was very suitable for the exam.

Paper no. 2 **Question no. 31**
Comments: The marks on this question were also rather high, with the average marks falling close to the alpha boundary. There was a typo in the question (the RHS in the definitions of κ_1 and κ_2 should have been divided by n), which thankfully didn't render the statement incorrect; it simply made the condition $|2\kappa_1 - \kappa_2| \leq \gamma$ unnecessarily strict. As some students might have been confused by this, I didn't deduct any points for missing factors of $1/n$ in the derivation.

Paper no. 4 **Question no. 30**
Comments: A correction for this question was issued about half-way through the exam (there was an extra factor of $\sqrt{M}$ in the final display). However, I did not see evidence that this affected students significantly. There were fewer attempts and the results were significantly worse than for other questions in this course, but they are within the targeted range, and a couple of scripts were perfect. There were a surprising number of mistakes in the definition of random forests, which led several candidates to lose easy bookwork marks. Revision of this topic should be encouraged.

Course: NUMBER FIELDS

Paper no. 1 Question no. 20G
Comments: This was generally well done. A depressing number of students (more that half of those who answered the question) believe that if the norm and trace of an algebraic number are integers then the number is itself an algebraic integer.

Paper no. 2 Question no. 20G
Comments: The bookwork for this question was well done, but students struggled with the manipulation of ideals in the later parts.

Paper no. 4 Question no. 20G
Comments: This difficult question was on the whole well done. Students seemed very good at sorting out the complicated concepts in the problem parts.

Course: NUMBER THEORY

Paper no. 1 Question no. 1I
Comments: The section I questions on this course contained perhaps more unseen problem elements than usual, but this didn't seem to disadvantage students. This question in particular was generally well done. A few candidates failed to realise that all they had to do for the last part was to show that 3 was a quadratic non-residue.

Paper no. 2 Question no. 1I
Comments: Reassuringly, everyone who attempted this question gave a correct definition of multiplicative function. Most people spotted that in the last part they had only to find a sequence of consecutive non-squarefree integers.

Paper no. 3 Question no. 1I
Comments: A completely routine question on continued fractions. Those who failed to get a beta mostly did so through incomplete or inaccurate proofs of the bookwork.

Paper no. 3 Question no. 11I
Comments: This question on quadratic forms had quite a tricky last part, which most candidates found challenging.

Paper no. 4 Question no. 1I
Comments: By far the least popular of the short questions on the course. The short unseen problem at the end proved surprisingly hard.

Paper no. 4 Question no. 11I
Comments: Most candidates had a good stab at the first 3 parts (two of which were bookwork). Surprisingly many ignored the hint in part (b), or failed to realise that the criterion of (c) was useful in part (d). Some candidates decided to solve part (b) by showing that there were infinitely many Mersenne primes, unfortunately with no success.

Course: NUMERICAL ANALYSIS
No comments received

Course: PRINCIPLES OF QUANTUM MECHANICS
Questions were in general standard but some were more challenging than others.

Paper no. 1 Question no. 34B
Comments: Part (b) was found particularly difficult by those who attempted the question. Many students simply avoided it.

Paper no. 2 Question no. 35B
Comments: This question was found very easy for the students judging by both the number of attempts and the number of alphas.

Paper no. 3 Question no. 33B
Comments: A good question with slightly fewer attempts, but many good answers.

Paper no. 4 Question no. 33B
Comments: Part (b) was found difficult. Many did not try to find the exact solution but only the approximated one.

Course: PRINCIPLES OF STATISTICS

Paper no. 1 Question no. 29
Comments: This question was of suitable difficulty and length, despite its largely computational nature. It is comparable to questions from previous years.

Paper no. 2 Question no. 29
Comments: The results on this question were quite poor. Even though a full 15 marks on the question were bookwork (a definition from lecture, and an example sheet problem), it received fewer attempts than other questions and only 4 students in this popular course obtained alphas. Nonetheless, several candidates solved the problem element in part (b), on exact hypothesis tests, and got perfect marks. I believe the reason for the low average is that this topic is not frequently represented in past exams, which are skewed toward a narrow selection of the topics from the schedules. This situation should be rectified, or the schedules should be amended.

Paper no. 3 Question no. 28
Comments: The results in this question were within the target range. However, there was a surprising number of mistakes in the definition of nonparametric Bootstrap confidence intervals, which is essentially bookwork. I believe this reflects the narrow selection of topics which the students revise for, based on past year exams. Many students suggested an asymptotically correct confidence interval in part (b) which, unlike the one in model solutions, does not require applying Slutsky's lemma. A nontrivial number of students made the mistake of defining confidence intervals depending on the parameter.

Paper no. 4 Question no. 28
Comments: This question received a large number of attempts, with a good spread of marks. It was comparable in difficulty and subject matter to questions from past years.

Course: PROBABILITY AND MEASURE

Paper no. 1 **Question no. 27H**
Comments: Easy question.

Paper no. 2 **Question no. 27H**
Comments: Essentially bookwork, well mastered.

Paper no. 3 **Question no. 26H**
Comments: First part is well mastered bookwork. Some realized that for the second part, the hint is useless: the claim follows by computing $E(\sum_{n,m} X_i)^2 = \sum_{n,m} \sigma_i^2$ which gives the result using the Cauchy criterion.

Paper no. 4 **Question no. 26H**
Comments: The fact that pointwise convergence implies convergence in probability on a probability space is a one line dominated convergence argument. The rest was mostly well done bookwork.

Course: QUANTUM INFORMATION AND COMPUTATION

Paper no. 1 Question no. 10
Comments: Candidates seemed a little confused how to apply the Helstrom-Holevo theorem to the given circumstance. While the no-cloning theorem was well known, there was mixed success in seeing how it followed from HH here.

Paper no. 2 Question no. 10
Comments: This question was largely done well, with most candidates able to see how the 'warm-up' part of the question contained the key to solving the second part.

Paper no. 2 Question no. 15
Comments: Part a) of this question was largely done well, but many candidates really struggled with part b) and many did not attempt this part at all. In particular, few candidates could describe a unitary operation that Alice could perform on $t|00\rangle_A|0\rangle_B + s|01\rangle_A|1\rangle_B$ so as to remove the state $|10\rangle_A$ after a partial measurement.

Paper no. 3 Question no. 10
Comments:
This question was found to be mostly straightforward by those candidates who attempted it. Several different proofs of the required result were offered, some of which were actually rather inventive!

Paper no. 3 Question no. 15
Comments:
This was a very popular question that candidates obviously enjoyed and also scored very well on. A rough geometric description of the operations I, J and $-JI$ was usually understood, though often wrong in small details. These did not usually interfere with the remainder of the question which tended to be done algebraically rather than geometrically.

Paper no. 4 Question no. 10
Comments: This question was also done well by most candidates who attempted it, and the quantum Fourier transform seems to be well understood. I was pleased that candidates even handled the final part of the question with relative ease.

Course: REPRESENTATION THEORY

Paper no. 1 Question no. 19I
Comments: This question tested understanding of basic concepts in representation theory, and attracted many excellent attempts.

Paper no. 2 Question no. 19I
Comments: This question was found to be the hardest on the course. Although the first part had been set on an example sheet, there were few correct solutions.

Paper no. 3 Question no. 19I
Comments: Unsurprisingly, this question on induced representations had few attempts, although the problem element — computing the characters of the non-abelian group of order 21 — was not difficult, and those who did attempt it were generally successful

Paper no. 4 Question no. 19I
Comments: There were few attempts at this rather easy question on $SO(2)$ and $SU(2)$. Remarkably, many candidates could not give a proof of the character orthogonality relations for charaters of $SO(2)$. The second part of the question on $SU(2)$ was mostly well done.

Course: RIEMANN SURFACES

Paper no. 1 Question no. 24F
Comments: Very few attempts and not well answered; not clear why.

Paper no. 2 Question no. 24F
Comments: The last part was difficult for most students, though there were examples of answers achieving full marks.

Paper no. 3 Question no. 23F
Comments: This question had the most attempts and the highest average marks from questions of the course. The level of rigour and precision in the justification of the last part varied greatly.

Course: STATISTICAL MODELLING

Paper no. 1 Question no. 5
Comments: This question was very straightforward, and a large majority of the candidates obtained betas.

Paper no. 1 Question no. 13
Comments: This question was harder than expected. Part (b.i) of the problem, in particular, stumped many students. This suggests an inadequate level of preparation in the practical elements of the course.

Paper no. 2 Question no. 5
Comments: This question was very straightforward, dealing with a core definition in the course, and without a genuine problem element. However, the solutions were very poor on average.

Paper no. 3 Question no. 5
Comments: This question was very suitable as a short question in the course, with the average mark falling just above the beta threshold.

Paper no. 4 Question no. 5
Comments: This question again demonstrates that the practical elements of the course are not successfully examined. The number of attempts was half that of the short question in Paper 3, which was theoretical. Even though this question was arguably simpler, the results were abysmal. This leads me to believe that either students are not revising adequately for this type of question, or a Part II exam is not a good method to evaluate this material.

Paper no. 4 Question no. 13
Comments: This question was computational and straightforward, very suitable for a C course. The results are within the targeted range.

Course: STATISTICAL PHYSICS

Paper no. 1 Question no. 36
Comments: This question on heat capacities and the Carnot heat engine was attempted well by most candidates. A number of candidates incorrectly assumed that heat capacity C_p at constant pressure p could be defined in terms of the internal energy change $dE/dT|_p$ rather than the heat ΔQ supplied (which works for C_V but not C_p). Most were able to calculate changes in work done W_i and heat energy Q_i at each stage i of the Otto cycle, but needed to be more careful about how they are put together to find the efficiency $\eta = W/Q_{\rm in} = 1 - Q_{\rm out}/Q_{\rm in}$.

Paper no. 2 Question no. 37
Comments: Another statistical physics question with a good uptake and which most candidates tackled successfully. In providing ensemble definitions, a single brief coherent sentence was sought, but some candidates wasted significant time by not being concise. Although unseen, this question mainly involving Gaussian integrals which proved straightforward for most; when the 'large volume limit' is clearly requested, the large distance boundary correction need not be explicitly given.

Paper no. 3 Question no. 35
Comments: Less popular on the Bose-Einstein distribution, though well done by those who attempted this question. Part (a) was straightforward, while (b) required an understanding that the derivation of the average number $\langle n_r \rangle$ depends on $\mu < 0$ in order for the underlying series to converge. For (c), given the starting point it was not sufficient to arrive at familiar results without deriving them, e.g. through integration by parts. Part (d) required candidates to note that the condensate was not included in the distribution.

Paper no. 4 Question no. 35
Comments: This question on phase transitions was well done, with the phase transitions in part (c) given in lectures. Although part (d) was more complex and unseen, the application of the same methodology to this case was performed effectively.

Course: STOCHASTIC FINANCIAL MODELS

Paper no. 1 Question no. 30
Comments: Question on the standard probabilistic material of the course, reasonable mix of bookwork and unseen exercise, perhaps slightly on the long side.

Paper no. 2 Question no. 30
Comments: Reasonable and appropriate question.

Paper no. 3 Question no. 29
Comments: As above.

Paper no. 4 Question no. 29
Comments: As above.

Course: TOPICS IN ANALYSIS

Paper no. 1 Question no. 2H
Comments: Few attempts, though the question was a basic convexity argument which was mostly well done by those who attempted it.

Paper no. 2 Question no. 2H
Comments: Very few attempts, questions were elementary though.

Paper no. 2 Question no. 11H
Comments: Bookwork. The question was long, and required careful estimates which were hardly correctly reproduced. A correct proof in particular required showing $A_i \geq 0$. Most attempts were not rigorous enough in describing sequences of estimates.

Paper no. 3 Question no. 2H
Comments: This question was poorly understood.

Paper no. 4 Question no. 2H
Comments: Brouwer's theorem was well quoted, and mostly correctly applied to obtain the eigenvalue claim.

Paper no. 4 Question no. 12H
Comments: Very popular question which was mostly very well done. However one could not just quote the result from a number theory class like some did, the question was precisely about giving a proof.

Course: WAVES

Paper no. 1 **Question no. 40A**
Comments: Many attempts ran into the sand and were abandoned after writing down the d'Alembert solution for in-going and out-going travelling waves in an unhelpful form with factors $\pm i\omega t$. The question is actually very straightforward if one looks for a standing-wave (separable/normal-mode) solution of the form $\mathrm{Re}\, R(r)\exp(i\omega t)$, as in Q6 on example sheet 1 (the oboe) or Q5 on example sheet 2 (elastic normal modes of a sphere). With more effort, it can be done from d'Alembert if one uses the form with factors $i\omega t \pm ikr$.

Paper no. 2 **Question no. 40A**
Comments: This completely standard question on Love waves was generally done well. The only tricky part was converting the graph of LHS and RHS vs c into one of ω vs k, with tangency to $\omega = c_s k$ at cut-off.

Paper no. 3 **Question no. 39A**
Comments: Fewer attempts than the other waves questions, but a good success rate. It was a conceptually straightforward reflection–transmission problem, and most candidates rose to the challenge of getting the algebra right in calculating the amplitudes. Howvever, not a single candidate correctly stated that the signs were chosen such that the *group* velocities were in the right direction for incoming and outgoing waves (2 marks).

Paper no. 4 **Question no. 39A**
Comments: Almost all candidates knew the Rankine–Hugoniot relations, and most could handle the messy algebra to do part (b). Fewer saw that the key to part (c) was to resolve parallel and perpendicular to the shock, with the perpendicular velocity components U and $U - u_2$ being given by part (b) and the parallel component being U throughout.