

5K Statistical Modelling

Let Y_1, \dots, Y_n be independent with $Y_i \sim \frac{1}{n_i} \text{Bin}(n_i, \mu_i)$, $i = 1, \dots, n$, and

$$\log \left(\frac{\mu_i}{1 - \mu_i} \right) = x_i^\top \beta, \quad (1)$$

where x_i is a $p \times 1$ vector of regressors and β is a $p \times 1$ vector of parameters. Write down the likelihood of the data Y_1, \dots, Y_n as a function of $\mu = (\mu_1, \dots, \mu_n)$. Find the unrestricted maximum likelihood estimator of μ , and the form of the maximum likelihood estimator $\hat{\mu} = (\hat{\mu}_1, \dots, \hat{\mu}_n)$ under the logistic model (1). Finally, obtain an expression for the deviance residual in this generalised linear model.