UNIVERSITY OF CAMBRIDGE
Faculty of Mathematics

SCHEDULES OF LECTURE COURSES
AND FORM OF EXAMINATIONS
FOR THE MATHEMATICAL TRIPOS 2023-24
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**Notes**

1. Variational Principles is normally taken in the third term.
2. Optimisation may be attended in the Easter term of either the first or second year.
3. The Computational Projects may be done at any time after the Projects booklet is made available (in late July or early August of the first year). Students should attend the associated lectures in their third term.
4. Students taking the option Mathematics with Physics take courses from the Natural Sciences Tripos instead of both Numbers and Sets and Dynamics and Relativity.
5. Students may choose to take either Complex Methods or Complex Analysis.
6. Students who have not studied at least three mechanics modules (or the equivalent) should attend all or part of the 10-lecture non-examinable Mechanics course in the Michaelmas term.
THE MATHEMATICAL TRIPPOS 2023-24

INTRODUCTION

This booklet is the formal description of the content and structure of Parts IA, IB and II of the Mathematical Tripos.\(^1\) In particular, it contains the schedules, or syllabus specifications, that define each course in the undergraduate Tripos, and it contains detailed information about the structure and marking of examinations, and the classification criteria. In addition, the booklet contains many useful pieces of advice and information for students regarding the Mathematical Tripos. It is updated every year to reflect changes approved by the Faculty Board.

Lectures and Examinations Post COVID-19

On 5 May 2023 the WHO Director-General declared, with great hope, the end to COVID-19 as a global health emergency. Therefore, it is the working assumption of the Faculty that lectures and examinations in 2023-24 will all be held ‘normally’ and in person. Students are expected to attend lectures in order to take full advantage of the benefits of in-person teaching.

SCHEDULES

Syllabus

The schedule for each lecture course is a list of topics that define the course. The schedule is agreed by the Faculty Board. Some schedules contain topics that are ‘starred’ (listed between asterisks); all the topics must be covered by the lecturer but examiners can only set questions on unstarred topics. The numbers which appear in brackets at the end of subsections or paragraphs in these schedules indicate the approximate number of lectures likely to be devoted to that subsection or paragraph. Lecturers decide upon the amount of time they think appropriate to spend on each topic, and also on the order in which they present topics. There is no requirement for this year’s lectures to match the previous year’s notes. Some topics in Part IA and Part IB courses have to be introduced in a certain order so as to tie in with other courses.

Recommended Books

A list of books is given after each schedule. Books marked with \(\dagger\) are particularly well suited to the course. Some of the books are out of print; these are retained on the list because they should be available in college libraries (as should all the books on the list) and may be found in second-hand bookshops. There may well be many other suitable books not listed; it is usually worth browsing college libraries (and/or the internet). In most cases, the contents of the book will not be exactly the same as the content of the schedule, and different styles suit different people. Hence you are advised to consult library copies in the first instance to decide which, if any, would be of benefit to you. Up-to-date prices, and the availability of hard- and soft-back versions, can most conveniently be checked online.

STUDY SKILLS

The Faculty produces a booklet Study Skills in Mathematics which can be obtained online in PDF format linked from https://www.maths.cam.ac.uk/undergrad/studyskills/.

\(^1\) This booklet, full name Schedules of Lecture Courses and Form of Examinations for the Mathematical Tripos but often referred to simply as ‘The Schedules’, can be found online linked from https://www.maths.cam.ac.uk/undergrad/course/.

There is also a booklet, Supervision in Mathematics, that gives guidance to supervisors, which can be obtained online in PDF format linked from https://www.maths.cam.ac.uk/undergrad/supervisions/ which may also be of interest to students.

Aims and objectives

The aims of the Faculty for Parts IA, IB and II of the Mathematical Tripos are:

- to provide a challenging course in mathematics and its applications for a range of students that includes some of the best in the country;
- to provide a course that is suitable both for students aiming to pursue research and for students going into other careers;
- to provide an integrated system of teaching which can be tailored to the needs of individual students;
- to develop in students the capacity for learning and for clear logical thinking, and the ability to solve unseen problems;
- to continue to attract and select students of outstanding quality;
- to produce the high calibre graduates in mathematics sought by employers in universities, the professions and the public services.
- to provide an intellectually stimulating environment in which students have the opportunity to develop their skills and enthusiasms to their full potential;
- to maintain the position of Cambridge as a leading centre, nationally and internationally, for teaching and research in mathematics.

The objectives of Parts IA, IB and II of the Mathematical Tripos are as follows:

After completing Part IA, students should have:

- made the transition in learning style and pace from school mathematics to university mathematics;
- been introduced to basic concepts in higher mathematics and their applications, including (i) the notions of proof, rigour and axiomatic development, (ii) the generalisation of familiar mathematics to unfamiliar contexts, (iii) the application of mathematics to problems outside mathematics;
- laid the foundations, in terms of knowledge and understanding, of tools, facts and techniques, to proceed to Part IB.

After completing Part IB, students should have:

- covered material from a range of pure mathematics, statistics and operations research, applied mathematics, theoretical physics and computational mathematics, and studied some of this material in depth;
- acquired a sufficiently broad and deep mathematical knowledge and understanding to enable them both to make an informed choice of courses in Part II and also to study these courses.

After completing Part II, students should have:

- developed the capacity for (i) solving both abstract and concrete unseen problems, (ii) presenting a concise and logical argument, and (iii) (in most cases) using standard software to tackle mathematical problems;
- studied advanced material in the mathematical sciences, some of it in depth.
INTRODUCTION

EXAMINATIONS

There are three examinations for the undergraduate Mathematical Tripos: Parts IA, IB and II, normally taken in consecutive years. Candidates are awarded a class in each examination and are required to pass in order to progress from one year to the next and to be eligible to graduate with a BA (honours) degree after completing all three years. In the Mathematical Tripos, the overall class for the BA degree is the class awarded in the Part II examination.

The following sections contain information that is common to the examinations in Parts IA, IB and II. Information that is specific to individual examinations is given later in this booklet in the General Arrangements sections for the appropriate part of the Tripos.

Overview of Responsibilities

The form of each examination (number of papers, numbers of questions on each lecture course, distribution of questions in the papers and in the sections of each paper, number of questions which may be attempted) is determined by the Faculty Board of Mathematics. The main structure has to be agreed by University committees and is published as a Regulation in the Statutes and Ordinances of the University of Cambridge (https://www.admin.cam.ac.uk/univ/reg). Any significant change to the format is announced in the Reporter as a Form and Conduct Notice. The actual questions and marking schemes, and precise borderlines (following general classification criteria agreed by the Faculty Board — see below) are determined by the examiners.

The examiners for each part of the Tripos are appointed by the General Board of the University. The internal examiners are normally teaching staff of the two mathematics departments and they are joined by one or more external examiners from other universities (one for Part IA, two for Part IB and three for Part II).

For all three parts of the Tripos, the examiners are collectively responsible for the examination questions, though for Part II the questions are proposed by the individual lecturers. All questions have to be signed off by the relevant lecturer; no question can be used unless the lecturer agrees that it is fair and appropriate.

Form of the Examination

The form of the examination is guided by the core competence standard for assessment in the Mathematical Tripos as a whole (as approved by the Faculty Board of Mathematics), which is the ability to recall and accurately apply knowledge to solve unseen problems within a time limit.

The examination for each part of the Tripos consists of four written papers and candidates take all four. For Parts IB and II, candidates may in addition submit Computational Projects. Each written paper has two sections: Section I contains questions that are intended to be accessible to any student who has studied the material conscientiously. They should not contain any significant ‘problem’ element. Section II questions are intended to be more challenging.

Calculators are not allowed in any paper of the Mathematical Tripos; questions will be set in such a way as not to require the use of calculators. The rules for the use of calculators in the Physics paper of the Mathematics-with-Physics option of Part I A are set out in the regulations for the Natural Sciences Tripos. Formula booklets are not permitted, but candidates will not be required to quote elaborate formulae from memory.

Marking Conventions

On the written papers of the Mathematical Tripos, Section I questions are marked out of 10 and Section II questions are marked out of 20. In addition to a numerical mark, extra credit in the form of a quality mark may be awarded for each question depending on the completeness and quality of each answer. For a Section I question, an alpha quality mark is awarded for a mark of 7 or more. For a Section II question, a beta quality mark is awarded for a mark of 15 or more, and a gamma quality mark is awarded for a mark between 10 and 14, inclusive.

On each written paper the number of questions for which credit may be obtained is restricted. The relevant restrictions are specified in the introductions to Parts IA, IB and II later in this booklet, and indicated on the examination paper by a rubric such as ‘Candidates may obtain credit for attempts on at most N questions from Section M’. If a candidate submits more attempts than are allowed for credit in the rubric then examiners will mark all attempts and the candidate is given credit only for the best attempts consistent with the rubric. This policy is intended to deal with candidates who accidentally attempt too many questions: it is clearly not in candidates’ best interests to spend time tackling extra questions for which they will receive no credit.

The marks available on the Computational Projects courses are described later in this booklet in the introductions to Parts IB and II, and in more detail in the Computational Projects Manuals, which are available at https://www.maths.cam.ac.uk/undergrad/catam/.

Examinations are ‘single-marked’, but safety checks are made on all scripts to ensure that all work is marked and that all marks are correctly added and transcribed. Faculty policy is that examiners should make every effort to read poor handwriting (and they almost always succeed), but if an answer, or part of an answer, is indecipherable then it will not be awarded the relevant marks. Exceptions will be made, where appropriate, for candidates with disabilities or diagnosed specific learning difficulties.

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Scripts are identified only by candidate number until the final class list has been drawn up. In drawing up the class list, examiners make decisions based only on the work they see: no account is taken of the candidates’ personal situation or of supervision reports.

Following the posting of results on CamSIS, candidates and their Colleges will be sent a more detailed list of the marks gained on each question and each computational project attempted.

Mitigating Circumstances

Candidates who are seriously hindered in preparing for, or sitting, their examinations should contact their College Tutor at the earliest possible opportunity. The Tutor will advise on what further action is needed (e.g. securing medical or other evidence) and, in cases of illness or other grave cause, the Tutor can make an application on the candidate’s behalf to the University for an Examination Allowance.

Queries and Corrections

Examiners are present for the duration of each examination paper and available to answer queries if a candidate suspects there may be an error in one of the questions. The candidate should raise their hand to gain the attention of an invigilator, write the query clearly on a piece of rough paper so that it can be taken to the duty examiners, then continue to work on the exam paper while waiting for a response. If an error in a question is discovered, a correction will be announced to all candidates. The examiner will mark each attempt at the question generously if there is any evidence that the candidate has been affected by the error, and will note any candidate whose script shows evidence that they lost significant time due to the error, for example by making several attempts to reach an answer that is actually incorrect.
INTRODUCTION

Classification Criteria

For each examination, each candidate is placed in one of the following categories: first class (1), upper second class (2.1), lower second class (2.2), third class (3), fail or other. ‘Other’ here includes, for example, candidates who were ill for all or part of the examination. The examiners place candidates into the different classes with particular attention given to all candidates near each borderline. The primary classification criteria for each borderline, which are determined by the Faculty Board, are as follows:

<table>
<thead>
<tr>
<th>Classification</th>
<th>Formula</th>
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<tbody>
<tr>
<td>First / upper second</td>
<td>$30\alpha + 5\beta + m$</td>
</tr>
<tr>
<td>Upper second / lower second</td>
<td>$15\alpha + 5\beta + m$</td>
</tr>
<tr>
<td>Lower second / third</td>
<td>$15\alpha + 5\beta + m$</td>
</tr>
<tr>
<td>Third/ fail</td>
<td>$2\alpha + \beta$ together with $m$</td>
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</table>

Here, $m$ denotes the number of marks and $\alpha$ and $\beta$ denote the numbers of quality marks. Other factors besides marks and quality marks may be taken into account. At the third/fail borderline, examiners may consider if most of the marks have been obtained on only one or two courses.

The Faculty Board recommends that no distinction should be made between marks obtained on the Computational Projects courses in Parts IB and II and marks obtained on the written papers. The Faculty Board recommends approximate percentages of candidates for each class: 30% firsts; 70–75% upper seconds and above; 90–95% lower seconds and above; and 5–10% thirds and below. (These percentages exclude candidates who did not sit all the written papers.) The Faculty Board expects that the classification criteria described above should result in classes that can be broadly characterised as follows (after allowing for the possibility that in Parts IB and II stronger performance on the Computational Projects may compensate for weaker performance on the written papers or vice versa):

**First Class**
Candidates placed in the first class will have demonstrated a good command and secure understanding of examinable material. They will have presented standard arguments accurately, showed skill in applying their knowledge, and generally will have produced substantially correct solutions to a significant number of more challenging questions.

**Upper Second Class**
Candidates placed in the upper second class will have demonstrated good knowledge and understanding of examinable material. They will have presented standard arguments accurately and will have shown some ability to apply their knowledge to solve problems. A fair number of their answers to both straightforward and more challenging questions will have been substantially correct.

**Lower Second Class**
Candidates placed in the lower second class will have demonstrated knowledge but sometimes imperfect understanding of examinable material. They will have been aware of relevant mathematical issues, but their presentation of standard arguments will sometimes have been fragmentary or imperfect. They will have produced substantially correct solutions to some straightforward questions, but will have had limited success at tackling more challenging problems.

Third Class
Candidates placed in the third class will have demonstrated some knowledge of the examinable material. They will have made reasonable attempts at a small number of questions, but will not have shown the skills needed to complete many of them.

Transcripts and Overall Degree Classification

The class that a student is assigned in each Tripos examination is part of their academic record and appears on their University transcript, which can be accessed via CamSIS. Until recently there had been no official overall class assigned to a BA degree at Cambridge. However, in a change to past practice, from 2023 onwards, students who graduate with a BA are awarded an official overall class for their degree (provided they started their course in 2020 or later). Following consideration by the Faculty Board, it has been agreed that in the Mathematical Tripos, the overall class for the BA degree will be the class awarded in Part II.

For each Tripos examination, University guidelines also require the Faculty to produce a UMS percentage mark and a rank for each candidate, to appear on their University transcript. These are calculated from the distribution of ‘merit marks’ as follows.

The merit mark $M$ is defined in terms of the numbers of marks, alphas and betas by:

$$M = \begin{cases} 30\alpha + 5\beta + m - 120 & \text{for candidates in the first class, or in the upper second class with } \alpha \geq 8, \\ 15\alpha + 5\beta + m & \text{otherwise} \end{cases}$$

The UMS percentage mark is obtained by piecewise linear scaling of the merit marks within each class. The 1/2.1, 2.1/2.2, 2.2/3 and 3/fail boundaries are mapped to 69.5%, 59.5%, 49.5% and 39.5% respectively, and the merit mark of the 5th ranked candidate is mapped to 95%. If, after linear mapping of the first class, the percentage mark of any candidate is greater than 100, it is reduced to 100%. The percentage of each candidate is then rounded appropriately to integer values. The rank of the candidate is determined by merit-mark order within each class.

Mark Checks and Examination Reviews

All appeals must be made through official channels, and examiners must not be approached directly, either by the candidate or their Director of Studies. A candidate who thinks that there is an error in their detailed marks should discuss this with their Director of Studies. If there is good reason to believe that an error has occurred, the Director of Studies should contact the Undergraduate Office within 14 days of the detailed marks being released, requesting a mark check and providing details of the reason for the request. A request for a mark check outside the aforementioned time frame will not be accepted unless an evidenced good reason for lateness is included.

The Faculty procedure exists to check for errors in the marking (including the very rare occasions when an error has been made by the assessors or examiners). Matters of academic judgement of the assessors or examiners will not be revisited.

A candidate can also appeal to the University if they believe there is a case for an Examination Review using the procedure outlined at [https://www.studentcomplaints.admin.cam.ac.uk/examination-reviews](https://www.studentcomplaints.admin.cam.ac.uk/examination-reviews). Further information can be obtained from College Tutors, and also from the exams section of the students’ advice service website: see [https://www.cambridgesu.co.uk/advice/student-advice-service/](https://www.cambridgesu.co.uk/advice/student-advice-service/)
Examination Data Retention Policy

To meet the University’s obligations under the data protection legislation, the Faculty deals with data relating to individuals and their examination marks as follows:

- All marks for individual questions and computational projects are released routinely to individual candidates and their Colleges after the examinations. The final examination mark book is kept indefinitely by the Undergraduate Office.
- Scripts and Computational Projects submissions are kept, in line with the University policy, for six months following the examinations (in case of appeals). Scripts are then destroyed; and Computational Projects are anonymised and stored in a form that allows comparison (using anti-plagiarism software) with current projects.
- Neither the Data Protection Act nor the Freedom of Information Act entitle candidates to have access to their scripts. Data appearing on individual examination scripts is technically available on application to the University Information Compliance Officer. However, such data consists only of a copy of the examiner’s ticks, crosses, underlines, etc., and the mark subtotals and totals.

Examiners’ Reports

For each part of the Tripos, the examiners (internal and external) write a joint report. In addition, the external examiners each submit a report addressed to the Vice-Chancellor. The reports of the external examiners are scrutinised by the Education Committee of the University’s General Board. All the reports, the examination statistics (number of attempts per question, etc.), student feedback on the examinations and lecture courses (via the end of year questionnaire and paper questionnaires), and other relevant material are considered by the Faculty Teaching Committee at the start of the Michaelmas term. The Teaching Committee includes two student representatives, and may include other students (for example, previous members of the Teaching Committee and student members of the Faculty Board).

The Teaching Committee compiles a detailed report on examinations including various recommendations for the Faculty Board to consider at its second meeting in the Michaelmas term. This report also forms the basis of the Faculty Board’s response to the reports of the external examiners. Previous Teaching Committee reports and recent examiners’ comments on questions can be found at https://www.maths.cam.ac.uk/facultyboard/teachingcommittee.

MISCELLANEOUS MATTERS

Numbers of Supervisions, Example Sheets and Workload

The primary responsibility for supervisions rests with colleges, and Directors of Studies are expected to make appropriate arrangements for their students. Lecturers provide example sheets for each course, which supervisors are generally recommended to use. According to Faculty Board guidelines, the number of example sheets for 24-lecture, 16-lecture and 12-lecture courses should be 4, 3 and 2, respectively, and the content and length of each example sheet should be suitable for discussion (with a typical pair of students) in an hour-long supervision. For a student studying the equivalent of 4 24-lecture courses in each of Michaelmas and Lent Terms, as in Part IA, the 32 example sheets would then be associated with an average of about two supervisions per week, and with revision supervisions in the Easter Term, a norm of about 40 supervisions over the year. Since supervisions on a given course typically begin sometime after the first two weeks of lectures, the fourth supervision of a 24-lecture course is often given at the start of the next term to spread the workload and allow students to catch up.

Past Papers

Past Tripos papers, since circa 2001, are available for download from the Faculty web site at https://www.maths.cam.ac.uk/undergrad/pastpapers/. Some examples of solutions and mark schemes for the 2011 Part IA examination can be found with an explanatory comment at https://www.maths.cam.ac.uk/examples-solutions-part-ia. Otherwise, solutions and mark schemes are not available except in rough draft form for supervisors.

Student Support: Colleges and the Wider University

An extensive support network is available through colleges and the wider university, to help students get the most from their time in Cambridge and to assist with any issues of a more personal nature that may arise. The first points of contact for any student should be their College Director of Studies (for academic matters) and their College Tutor (for both academic and pastoral concerns). They will be able to offer help and advice directly, or to guide students to others with appropriate expertise, either within their College or elsewhere. While pastoral matters do not usually fall within the remit of the Mathematics Faculty, we strongly encourage our students to seek help and get support if they are experiencing any difficulties, and a summary of some relevant resources can be found by following the Student Support links on the undergraduate course webpages https://www.maths.cam.ac.uk/undergrad/undergrad.

Faculty Committees and Student Representation

The Faculty Board is responsible for setting policies governing arrangements for lecturing and examining in the Mathematical Tripos (https://www.maths.cam.ac.uk/internal/faculty/facultyboard) such as, for example, those described in these Schedules. It meets formally, and also considers other matters. There are two committees that deal exclusively with matters relating to the undergraduate Tripos, namely the Teaching Committee (https://www.maths.cam.ac.uk/facultyboard/teachingcommittee/) and the Curriculum Committee (https://www.maths.cam.ac.uk/facultyboard/curriculumcommittee/). The role of the Teaching Committee is mainly to monitor feedback (questionnaires, examiners’ reports, etc.) and make recommendations to the Faculty Board on the basis of this feedback. It also formulates policy recommendations at the request of the Faculty Board. The Curriculum Committee is responsible for recommending (to the Faculty Board) changes to the undergraduate Tripos and to the schedules for individual lecture courses.

Student representatives have a very important role to play on each of these committees: to advise on the student point of view and to collect opinion and liaise with the wider student body. There are two student representatives on both the Teaching and Curriculum Committees (others may be co-opted). There are also three student members on the Faculty Board, two undergraduate and one graduate, elected each year in November.

Further details regarding the student representatives, their roles and contributions, can be found at https://www.maths.cam.ac.uk/undergrad/student-representation. They can be contacted by email at student.reps@maths.cam.ac.uk.
Feedback

Constructive feedback of all sorts and from all sources is welcomed by everyone concerned in providing courses for the Mathematical Tripos.

There are many different feedback routes.

- Each lecturer distributes a questionnaire towards the end of the course.
- There are brief web-based questionnaires issued about a quarter of the way through each course.
- Students are sent a combined online questionnaire at the end of each year.
- Students (or supervisors) can e-mail feedback@maths.cam.ac.uk at any time. Such e-mails are received by the Director of Undergraduate Education and the Chair of the Teaching Committee, who will either deal with your comment, or pass your e-mail (after stripping out your identity) to the relevant person (a lecturer, for example). Students should receive a prompt response.
- If a student wishes to be entirely anonymous and does not want any response, the web-based comment form that can be found at https://www.maths.cam.ac.uk/undergrad/feedback.html can be used. (A consequence of anonymity is that clarifying information cannot be sought to help us deal with the comment.)
- Feedback on college-provided teaching (supervisions, classes) can be given to Directors of Studies or Tutors at any time.

The questionnaires are particularly important in shaping the future of the Tripos and the Faculty Board urges all students to respond.

Formal Complaints

The formal complaints procedure to be followed within the University can be found at https://www.studentcomplaints.admin.cam.ac.uk/student-complaints. The Responsible Officer in Step 1 of this procedure for the Faculty of Mathematics is the Chair of the Faculty Board — see https://www.maths.cam.ac.uk/facultyboard for the name of the current Chair.
GENERAL ARRANGEMENTS

Structure of Part IA

There are two options:
(a) Pure and Applied Mathematics;
(b) Mathematics with Physics.

Option (a) is intended primarily for students who expect to continue to Part IB of the Mathematical Tripos, while Option (b) is intended primarily for those who are undecided about whether they will continue to Part IB of the Mathematical Tripos or change to Part IB of the Natural Sciences Tripos (Physics option). For Option (b), two of the lecture courses (Numbers and Sets, and Dynamics and Relativity) are replaced by the complete Physics course from Part IA of the Natural Sciences Tripos; Numbers and Sets because it is the least relevant to students taking this option, and Dynamics and Relativity because much of this material is covered in the Natural Sciences Tripos anyway. Students wishing to examine the schedules for the physics courses should consult the documentation supplied by the Physics department, for example on https://www.phy.cam.ac.uk/students/teaching/.

Examinations

Details of arrangements common to all examinations of the undergraduate Mathematical Tripos start on page 2 of this booklet.

All candidates for Part IA of the Mathematical Tripos take four papers, as follows:
(a) Candidates taking Option (a) (Pure and Applied Mathematics) will take Papers 1, 2, 3 and 4 of the Mathematical Tripos (Part IA).
(b) Candidates taking Option (b) (Mathematics with Physics) take Papers 1, 2 and 3 of the Mathematical Tripos (Part IA) and the Physics paper of the Natural Sciences Tripos (Part IA); they must also submit practical notebooks.

For Mathematics-with-Physics candidates, the marks for the Physics paper are scaled to bring them in line with Paper 4. This is done as follows. The Physics scripts of the Mathematics-with-Physics candidates are marked by the Natural Sciences examiners for Part IA Physics, and a mark for each candidate is given to the Mathematics examiners. Class boundaries for the Physics paper are determined such that the percentages in each class (1, 2.1, 2.2, 3) of all candidates on the Physics paper (including those from NST) are 25, 35, 30, 10 (which are the guidelines across Natural Sciences). All candidates for Paper 4 (ranked by merit mark on that paper) are assigned nominally to classes so that the percentages in each class are 30, 40, 20, 10 (which is the Faculty Board rough guideline for the initial proportion in each class prior to the final overall classification). Piecewise linear mapping of the Physics marks in each class to the Mathematics merit marks in each nominal Mathematics class is used to provide a merit mark for each Mathematics-with-Physics candidate. The merit mark is then broken down into marks, alphas and betas by comparison (for each candidate) with the break down for Papers 1, 2 and 3.

Examination Papers

Papers 1, 2, 3 and 4 of Part IA of the Mathematical Tripos are each divided into two Sections. There are four questions in Section I and eight questions in Section II. Candidates may obtain credit for attempts on all the questions in Section I and at most five questions from Section II, of which no more than three may be on the same lecture course.

Each section of each of Papers 1–4 is divided equally between two courses as follows:

| Paper | Examination | Course
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<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Paper 1</td>
<td>Vectors and Matrices, Analysis I</td>
<td>Pure and Applied Mathematics</td>
</tr>
<tr>
<td>Paper 2</td>
<td>Differential Equations, Probability</td>
<td>Pure and Applied Mathematics</td>
</tr>
<tr>
<td>Paper 3</td>
<td>Groups, Vector Calculus</td>
<td>Pure and Applied Mathematics</td>
</tr>
<tr>
<td>Paper 4</td>
<td>Numbers and Sets, Dynamics and Relativity</td>
<td>Pure and Applied Mathematics</td>
</tr>
</tbody>
</table>

Approximate Class Boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines. For convenience, we define $M_1$ and $M_2$ by

\[ M_1 = 30\alpha + 5\beta + m - 120, \quad M_2 = 15\alpha + 5\beta + m. \]

$M_1$ is related to the primary classification criterion for the first class and $M_2$ is related to the primary classification criterion for the upper and lower second classes.

The second column of each table shows a sufficient criterion for each class. The third and fourth columns show $M_1$ (for the first class) or $M_2$ (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline. The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

### Part IA 2023

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 569$</td>
<td>510/275, 11, 5</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 336$</td>
<td>337/222, 5, 8</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 244$</td>
<td>245/185, 2, 6</td>
</tr>
<tr>
<td>3</td>
<td>$2\alpha + \beta &gt; 8$ or $m &gt; 153$</td>
<td>186/136, 1, 7</td>
</tr>
</tbody>
</table>

### Part IA 2022

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 574$</td>
<td>575/382, 11, 8</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 466$</td>
<td>368/227, 5, 9</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 244$</td>
<td>245/185, 2, 6</td>
</tr>
<tr>
<td>3</td>
<td>$2\alpha + \beta &gt; 8$</td>
<td>191/146, 0, 9</td>
</tr>
</tbody>
</table>
GROUPS

24 lectures, Michaelmas Term

Axioms for groups. Examples from geometry: symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group; cross-ratio, preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius maps and iteration.

Lagrange's theorem

Coezts. Lagrange's theorem. Groups of small order (up to order 8). Quaternions. Fermat-Euler theorem from the group-theoretic point of view.

Group actions

Group actions; orbits and stabilizers. Orbit-stabilizer theorem. Cayley's theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes. Cauchy's theorem.

Quotient groups

Normal subgroups, quotient groups and the isomorphism theorem.

Matrix groups

The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups. Proof (in $\mathbb{R}^3$) that every element of the orthogonal group is the product of reflections and every rotation in $\mathbb{R}^3$ has an axis. Basis change as an example of conjugation.

Permutations

Permutations, cycles and transpositions. The sign of a permutation. Conjugacy in $S_n$ and in $A_n$. Simple groups; simplicity of $A_5$.

Appropriate books

M.A. Armstrong Groups and Symmetry. Springer–Verlag 1988
1 Alan F Beardon Algebra and Geometry. CUP 2005
R.P. Burn Groups, a Path to Geometry. Cambridge University Press 1987
W. Lederman Introduction to Group Theory. Longman 1976
Nathan Carter Visual Group Theory. Mathematical Association of America Textbooks

VECTORS AND MATRICES

24 lectures, Michaelmas Term

Complex numbers

Review of complex numbers, including complex conjugate, inverse, modulus, argument and Argand diagram. Informal treatment of complex logarithm, $n$-th roots and complex powers. de Moivre’s theorem.

Vectors

Review of elementary algebra of vectors in $\mathbb{R}^3$, including scalar product. Brief discussion of vectors in $\mathbb{R}^n$; scalar product and the Cauchy-Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension. Suffix notation: including summation convention, $\delta_{ij}$ and $\epsilon_{ijk}$. Vector product and triple product: definition and geometrical interpretation. Solution of linear vector equations. Applications of vectors to geometry, including equations of lines, planes and spheres.

Matrices

Elementary algebra of $3 \times 3$ matrices, including determinants. Extension to $n \times n$ complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears; kernel and image, rank-nullity theorem (statement only). Simultaneous linear equations: matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination. Symmetric, anti-symmetric, orthogonal, hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts.

Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors; geometric significance. Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general change of basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for $2 \times 2$ matrices. Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms.

Rotation matrices and Lorentz transformations as transformation groups.

Appropriate books

Alan F Beardon Algebra and Geometry. CUP 2005
NUMBERS AND SETS 24 lectures, Michaelmas Term

[Note that this course is omitted from Option (b) of Part IA.]

Introduction to number systems and logic
Overview of the natural numbers, integers, real numbers, rational and irrational numbers, algebraic and transcendental numbers. Brief discussion of complex numbers; statement of the Fundamental Theorem of Algebra.

Irrationality of \( \sqrt{2} \) and \( e \). Decimal expansions. Construction of a transcendental number. Brief discussion of complex numbers; statement of the Fundamental Theorem of Algebra.

Sets, relations and functions
Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities. Functions; injections, surjections and bijections. Relations, and equivalence relations. Counting the combinations or permutations of a set. The Inclusion-Exclusion Principle.

The integers
The natural numbers: mathematical induction and the well-ordering principle. Examples, including the Binomial Theorem.

Elementary number theory
Prime numbers: existence and uniqueness of prime factorisation into primes; highest common factors and least common multiples. Euclid’s proof of the infinity of primes. Euclid’s algorithm. Solution in integers of \( ax + by = c \).

Modular arithmetic (congruences). Units modulo \( n \). Chinese Remainder Theorem. Wilson’s Theorem; the least common multiples. Euclid’s proof of the infinity of primes. Euclid’s algorithm. Solution in integers of \( ax + by = c \).

The real numbers
Least upper bounds; simple examples. Least upper bound axiom. Sequences and series; convergence of bounded monotonic sequences. Irrationality of \( \sqrt{2} \) and \( e \). Decimal expansions. Construction of a transcendental number.

Countability and uncountability
Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of \( \mathbb{R} \). Non-existence of a bijection from a set to its power set. Indirect proof of existence of transcendental numbers.

Appropriate books
R.P. Burn Numbers and Functions: steps into analysis. Cambridge University Press 2000
H. Davenport The Higher Arithmetic. Cambridge University Press 1999

DIFFERENTIAL EQUATIONS 24 lectures, Michaelmas Term

Basic calculus
Informal treatment of differentiation as a limit, the chain rule, Leibniz’s rule, Taylor series, informal treatment of \( O \) and \( o \) notation and l’Hôpital’s rule; integration as an area, fundamental theorem of calculus, integration by substitution and parts.

Informal treatment of partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule, implicit differentiation. Informal treatment of differentials, including exact differentials. Differentiation of an integral with respect to a parameter.

First-order linear differential equations
Equations with constant coefficients: exponential growth, comparison with discrete equations, series solution; modelling examples including radioactive decay.

Equations with non-constant coefficients: solution by integrating factor.

Nonlinear first-order equations
Separable equations. Exact equations. Sketching solution trajectories. Equilibrium solutions, stability by perturbation; examples, including logistic equation and chemical kinetics. Discrete equations: equilibrium solutions, stability; examples including the logistic map.

Higher-order linear differential equations
Complementary function and particular integral, linear independence, Wronskian (for second-order equations), Abel’s theorem. Equations with constant coefficients and examples including radioactive sequences, comparison in simple cases with difference equations, reduction of order, resonance, transients, damping. Homogeneous equations. Response to step and impulse function inputs; introduction to the notions of the Heaviside step-function and the Dirac delta-function. Series solutions including statement only of the need for the logarithmic solution.

Multivariate functions: applications
Directional derivatives and the gradient vector. Statement of Taylor series for functions on \( \mathbb{R}^n \). Local extrema of real functions, classification using the Hessian matrix. Coupled first order systems: equivalence to single higher order equations; solution by matrix methods. Non-degenerate phase portraits local to equilibrium points; stability.

Simple examples of first- and second-order partial differential equations, solution of the wave equation in the form \( f(x + ct) + g(x - ct) \).

Appropriate books
J. Robinson An introduction to Differential Equations. Cambridge University Press, 2004
PART IA

ANALYSIS I 24 lectures, Lent Term

Limits and convergence
Sequences and series in $\mathbb{R}$ and $\mathbb{C}$. Sums, products and quotients. Absolute convergence; absolute convergence implies convergence. The Bolzano-Weierstrass theorem and applications (the General Principle of Convergence). Comparison and ratio tests, alternating series test.

Continuity
Continuity of real- and complex-valued functions defined on subsets of $\mathbb{R}$ and $\mathbb{C}$. The intermediate value theorem. A continuous function on a closed bounded interval is bounded and attains its bounds.

Differentiability
Differentiability of functions from $\mathbb{R}$ to $\mathbb{R}$. Derivative of sums and products. The chain rule. Derivative of the inverse function, Rolle’s theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor’s theorem from $\mathbb{R}$ to $\mathbb{R}$; Lagrange’s form of the remainder. Complex differentiation.

Power series
Complex power series and radius of convergence. Exponential, trigonometric and hyperbolic functions, and relations between them. *Direct proof of the differentiability of a power series within its circle of convergence*.

Integration

Appropriate books
T.M. Apostol *Calculus, vol 1*. Wiley 1967-69
J.C. Burkill *A First Course in Mathematical Analysis*. Cambridge University Press 1978
D.J.H. Garling *A Course in Mathematical Analysis (Vol 1)*. Cambridge University Press 2013
J.B. Reade *Introduction to Mathematical Analysis*. Oxford University Press
M. Spivak *Calculus*. Addison–Wesley/Benjamin–Cummings 2006
David M. Bressoud *A Radical Approach to Real Analysis*. Mathematical Association of America Textbooks

PROBABILITY 24 lectures, Lent Term

Basic concepts
Classical probability, equally likely outcomes. Combinatorial analysis, permutations and combinations. Stirling’s formula (asymptotics for $\log n!$ proved).

Axiomatic approach

Discrete random variables

Continuous random variables

Inequalities and limits

Appropriate books
S. Ross *A First Course in Probability*. Prentice Hall 2009
VECTOR CALCULUS

24 lectures, Lent Term

Curves in $\mathbb{R}^3$
Parameterised curves and arc length, tangents and normals to curves in $\mathbb{R}^3$; curvature and torsion. [1]

Integration in $\mathbb{R}^2$ and $\mathbb{R}^3$
Line integrals. Surface and volume integrals: definitions, examples using Cartesian, cylindrical and spherical coordinates; change of variables. [4]

Vector operators
Directional derivatives. The gradient of a real-valued function: definition; interpretation as normal to level surfaces; examples including the use of cylindrical, spherical *and general orthogonal curvilinear* coordinates.

Divergence, curl and $\nabla^2$ in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical *and general orthogonal curvilinear* coordinates. Solenoidal fields, irrotational fields and conservative fields; scalar potentials. Vector derivative identities. [5]

Integration theorems
Divergence theorem, Green’s theorem, Stokes’ theorem, Green’s second theorem: statements; informal proofs; examples; application to fluid dynamics, and to electromagnetism including statement of Maxwell’s equations. [5]

Laplace’s equation
Laplace’s equation in $\mathbb{R}^2$ and $\mathbb{R}^3$; uniqueness theorem and maximum principle. Solution of Poisson’s equation by Gauss’s method (for spherical and cylindrical symmetry) and as an integral. [4]

Cartesian tensors in $\mathbb{R}^3$
Tensor transformation laws, addition, multiplication, contraction, with emphasis on tensors of second rank. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Quotient theorem. Examples including inertia and conductivity. [5]

DYNAMICS AND RELATIVITY

24 lectures, Lent Term

[Note that this course is omitted from Option (b) of Part IA.]

Familiarity with the topics covered in the non-examinable Mechanics course is assumed.

Basic concepts
Space and time, frames of reference, Galilean transformations. Newton’s laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz. [4]

Newtonian dynamics of a single particle
Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping. Angular velocity, angular momentum, torque. Orbits: the $u(\theta)$ equation; escape velocity; Kepler’s laws; stability of orbits; motion in a repulsive potential (Rutherford scattering). Rotating frames: centrifugal and Coriolis forces. *Brief discussion of Foucault pendulum.* [8]

Newtonian dynamics of systems of particles
Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation. [2]

Rigid bodies
Moments of inertia, angular momentum and energy of a rigid body. Parallel axis theorem. Simple examples of motion involving both rotation and translation (e.g. rolling). [3]

Special relativity
The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in (1+1)-dimensional spacetime. Time dilation and length contraction. The Minkowski metric for (1+1)-dimensional spacetime. Lorentz transformations in (3+1) dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4-momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit. [7]

Appropriate books

2. G.F.R. Ellis and R.M. Williams *Flat and Curved Space-times*. Oxford University Press 2000

Appropriate books

2. G.F.R. Ellis and R.M. Williams *Flat and Curved Space-times*. Oxford University Press 2000
COMPUTATIONAL PROJECTS 8 lectures, Easter Term of Part IA

The Computational Projects course is examined in Part IB. However introductory lectures are given in the Easter Full Term of the Part IA year. The lectures cover an introduction to algorithms and aspects of the MATLAB programming language.

The projects that need to be completed for credit are published by the Faculty in a manual usually at the end of July or the beginning of August at the end of the Part IA year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at https://www.maths.cam.ac.uk/undergrad/catam/.

Full credit may obtained from the submission of the two core projects and a further two additional projects. Once the manual is available, these projects may be undertaken at any time up to the submission deadlines, which are near the start of the Full Lent Term in the Part IB year for the two core projects, and near the start of the Full Easter Term in the Part IB year for the two additional projects.

A list of suitable books can be found in the manual.

MECHANICS (non-examinable) 10 lectures, Michaelmas Term

This course is intended for students who have taken only a limited amount of Mechanics at A-level (or the equivalent). The material is prerequisite for Dynamics and Relativity in the Lent Term.

Lecture 1
Brief introduction

Lecture 2: Kinematics of a single particle

Lecture 3: Equilibrium of a single particle
The vector nature of forces, addition of forces, examples including gravity, tension in a string, normal reaction (Newton’s third law), friction. Conditions for equilibrium.

Lecture 4: Equilibrium of a rigid body
Resultant of several forces, couple, moment of a force. Conditions for equilibrium.

Lecture 5: Dynamics of particles
Newton’s second law. Examples of pulleys, motion on an inclined plane.

Lecture 6: Dynamics of particles
Further examples, including motion of a projectile with air-resistance.

Lecture 7: Energy
Definition of energy and work. Kinetic energy, potential energy of a particle in a uniform gravitational field. Conservation of energy.

Lecture 8: Momentum
Definition of momentum (as a vector), conservation of momentum, collisions, coefficient of restitution, impulse.

Lecture 9: Springs, strings and SHM
Force exerted by elastic springs and strings (Hooke’s law). Oscillations of a particle attached to a spring, and of a particle hanging on a string. Simple harmonic motion of a particle for small displacement from equilibrium.

Lecture 10: Motion in a circle
Derivation of the central acceleration of a particle constrained to move on a circle. Simple pendulum; motion of a particle sliding on a cylinder.

Appropriate books
Peter J O’Donnell Essential Dynamics and Relativity. CRC Press, 2014
J. Heiborn and J. Littlewood Mechanics 1, Mechanics 2 and Mechanics 3 (Edexcel). Heinemann, 2000
Anything similar to the above, for the other A-level examination boards
PART IB

GENERAL ARRANGEMENTS

Structure of Part IB

Sixteen courses, including Computational Projects, are examined in Part IB. The schedules for Complex Analysis and Complex Methods cover much of the same material, but from different points of view: students may attend either (or both) sets of lectures. One course, Optimization, can be taken in the Easter term of either the first year or the second year. Another course, Variational Principles, can also be taken in either Easter term, but is normally taken in the first year as the material forms a good background for a number of courses in Part IB.

Students are not expected to take all the courses in Part IB, and the structure of the Part IB examination papers allows for considerable flexibility. The Faculty Board guidance regarding choice of courses in Part IB is as follows:

Part IB of the Mathematical Tripos provides a wide range of courses from which students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred workload, bearing in mind that it is better to do a smaller number of courses thoroughly than to do many courses scrappily.

Students might sensibly assess their workload by comparing the numbers of example sheets and lectures they are taking on per term with their previous experience in Part IA (see page 6 of this booklet). The table of dependencies on the next page may also help them with their choice.

Computational Projects

The lectures for Computational Projects should be attended in the Easter term of the first year. The Computational Projects themselves are best started in the summer vacation between Part IA and Part IB, and then completed in the Michaelmas and Lent terms, and the Christmas and Easter vacations, of the second year.

No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of reports. The maximum credit obtainable is 160 marks and there are no alpha or beta quality marks. Credit obtained is added directly to the credit gained on the written papers. The maximum contribution to the final merit mark is thus 160, which is roughly the same (averaging over the alpha weightings) as for a 16-lecture course. The Computational Projects are considered to be a single piece of work within the Mathematical Tripos.

Examinations

Details of arrangements common to all examinations of the undergraduate Mathematical Tripos start on page 2 of this booklet.

Each of the four papers is divided into two sections. Candidates may obtain credit for attempts on at most four questions from Section I and at most six questions from Section II.

The number of questions set on each course varies according to the number of lectures given, as shown:

<table>
<thead>
<tr>
<th>Number of lectures</th>
<th>Section I</th>
<th>Section II</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: The number of Section I questions on a 24-lecture course was changed after 2018/19, and the distribution of questions among the papers also changed.

Examination Papers

Questions on the different courses are distributed among the papers as specified in the following table. The letters S and L appearing in the table denote a question in Section I and a question in Section II, respectively.

<table>
<thead>
<tr>
<th>Paper 1</th>
<th>Paper 2</th>
<th>Paper 3</th>
<th>Paper 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Algebra</td>
<td>L+S</td>
<td>L</td>
<td>L+S</td>
</tr>
<tr>
<td>Groups, Rings and Modules</td>
<td>L</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Analysis and Topology</td>
<td>L</td>
<td>L+S</td>
<td>L</td>
</tr>
<tr>
<td>Geometry</td>
<td>L+S</td>
<td>L</td>
<td>L+S</td>
</tr>
<tr>
<td>Complex Analysis</td>
<td>L+S</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Complex Methods</td>
<td>L</td>
<td>L</td>
<td>S</td>
</tr>
<tr>
<td>Variational Principles</td>
<td>S</td>
<td>L</td>
<td>S</td>
</tr>
<tr>
<td>Methods</td>
<td>L</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Quantum Mechanics</td>
<td>L</td>
<td>L</td>
<td>S</td>
</tr>
<tr>
<td>Electromagnetism</td>
<td>L</td>
<td>L+S</td>
<td>L</td>
</tr>
<tr>
<td>Fluid Dynamics</td>
<td>L</td>
<td>S</td>
<td>L+S</td>
</tr>
<tr>
<td>Numerical Analysis</td>
<td>L+S</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Statistics</td>
<td>L+S</td>
<td>S</td>
<td>L</td>
</tr>
<tr>
<td>Markov Chains</td>
<td>L</td>
<td>L</td>
<td>S</td>
</tr>
<tr>
<td>Optimisation</td>
<td>S</td>
<td>S</td>
<td>L</td>
</tr>
</tbody>
</table>

† On Paper 1 and Paper 2, Complex Analysis and Complex Methods are examined by means of common questions, each of which may contain two sub-questions, one on each course, of which candidates may attempt only one (‘either/or’).
Approximate Class Boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines in recent years.

For convenience, we define $M_1$ and $M_2$ by

$$M_1 = 30\alpha + 5\beta + m - 120, \quad M_2 = 15\alpha + 5\beta + m.$$  

$M_1$ is related to the primary classification criterion for the first class and $M_2$ is related to the primary classification criterion for the upper and lower second and third classes.

The second column of each table shows a sufficient criterion for each class (in terms of $M_1$ for the first class and $M_2$ for the other classes). The third and fourth columns show $M_1$ (for the first class) or $M_2$ (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline.

The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

### Part IB 2023

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 666$</td>
<td>667/407, 12, 4</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 431$</td>
<td>432/412, 5, 9</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 308$</td>
<td>309/224, 3, 8</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 208$</td>
<td>209/194, 0, 3</td>
</tr>
</tbody>
</table>

### Part IB 2022

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 753$</td>
<td>754/464, 12, 10</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 507$</td>
<td>508/373, 6, 9</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 321$</td>
<td>322/247, 2, 9</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 205$</td>
<td>206/141, 3, 4</td>
</tr>
</tbody>
</table>

### Part II Dependencies

The relationships between Part IB courses and Part II courses are shown in the following tables. A blank in the table means that the material in the Part IB course is not directly relevant to the Part II course.

The terminology is as follows:

- **Essential**: (E) a good understanding of the methods and results of the Part IB course is essential;
- **Desirable**: (D) knowledge of some of the results of the Part IB course is required;
- **Background**: (B) some knowledge of the Part IB course would provide a useful background.
LINEAR ALGEBRA 24 lectures, Michaelmas Term


GROUPS, RINGS AND MODULES 24 lectures, Lent Term

Groups

Rings

Modules
Definitions, examples of vector spaces, abelian groups and vector spaces with an endomorphism. Submodules, homomorphisms, quotient modules and direct sums. Equivalence of matrices, canonical form. Structure of finitely generated modules over Euclidean domains, applications to abelian groups and Jordan normal form.

Appropriate books

LINEAR ALGEBRA
P.R. Halmos Finite-dimensional vector spaces. Springer 1974

GROUPS, RINGS AND MODULES
P.M. Cohn Classic Algebra. Wiley, 2000
P.J. Cameron Introduction to Algebra. OUP
I. Herstein Topics in Algebra. John Wiley and Sons, 1975
P.M. Neumann, G.A. Stoy and E.C. Thomson Groups and Geometry. OUP 1994
ANALYSIS AND TOPOLOGY 24 lectures, Michaelmas Term

Uniform convergence and uniform continuity
The general principle of uniform convergence. A uniform limit of continuous functions is continuous. Uniform convergence and term-wise integration and differentiation of series of real-valued functions. Local uniform convergence of power series. Uniform continuity. Continuous functions on closed bounded intervals are uniformly continuous; Riemann integrability of continuous functions from Analysis I revisited. [3]

Metric spaces

Topological spaces

Connectedness
Definition using open sets and integer-valued functions. Examples. Components. The continuous image of a connected space is connected. Path-connectedness and its relation to connectedness. Connected open sets in Euclidean space are path-connected. Connectedness of products. [3]

Compactness
Definition using open covers. Examples including [0, 1]. Closed subsets of compact spaces are compact. Compact subsets of Hausdorff spaces are closed. The compact subsets of the real line. Continuous images of compact sets are compact. Continuous real-valued functions on a compact space are bounded and attain their bounds. Compactness of products. Compactness of quotient spaces. Sequential compactness. [3]

Differentiation from \( \mathbb{R}^m \) to \( \mathbb{R}^n \)

GEOMETRY 24 lectures, Lent Term

Surfaces
Topological surfaces via charts and atlases. Examples including the sphere via stereographic projection, the real projective plane and polygons with side identifications. Smooth surfaces and smooth parametrizations. Orientability. The implicit-function theorem. Surfaces in \( \mathbb{R}^3 \). [3]

Informal discussion of triangulations; Euler characteristic and genus. [1]

Surfaces in 3-space
The first fundamental form of an embedded surface in \( \mathbb{R}^3 \). Length and area. Examples including surfaces of revolution. Change of parametrisation. [3]


Geodesics

Hyperbolic surfaces
Abstract Riemannian metrics on a disc; isometries. Möbius group of the sphere, disc and half-plane. The hyperbolic metric on the disc and half-plane; geodesics and isometries. Gauss–Bonnet theorem for hyperbolic triangles. Hyperbolic hexagons; hyperbolic structures on closed surfaces. [5]

Further topics

Appropriate books

1. P.M.H. Wilson Curved Spaces. CUP 2008
3. M. Reid and B. Szendroi Geometry and Topology. CUP 2005
4. J.C. Burkill and H. Burkill A Second Course in Mathematical Analysis. CUP 2002
5. E. Bloch A first course in geometric topology and differential geometry. Birkauser 1997

1. J.C. Burkill and H. Burkill A Second Course in Mathematical Analysis. Cambridge University Press 2002
3. T.W. Körner A Companion to Analysis. AMS, 2004
COMPLEX ANALYSIS
16 lectures, Lent Term

Analytic functions
Complex differentiation and the Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of log $z$ and $z^a$. [3]

Contour integration and Cauchy’s theorem

Expansions and singularities

The residue theorem

Appropriate books
A.F. Beardon Complex Analysis. Wiley
D.J.H. Garling A Course in Mathematical Analysis (Vol 3). Cambridge University Press 2014
I. Stewart and D. Tall Complex Analysis. Cambridge University Press 1983

COMPLEX METHODS
16 lectures, Lent Term

Analytic functions

Contour integration and Cauchy’s Theorem
[Proofs of theorems in this section will not be examined in this course.]

Residue calculus

Fourier and Laplace transforms
Laplace transform: definition and basic properties; inversion theorem (proof not required); convolution theorem. Examples of inversion of Fourier and Laplace transforms by contour integration. Applications to differential equations. [3]

Appropriate books
PART IB

VARIATIONAL PRINCIPLES 12 lectures, Easter Term

Stationary points for functions on \( \mathbb{R}^n \). Necessary and sufficient conditions for minima and maxima. Importance of convexity. Variational problems with constraints; method of Lagrange multipliers. The Legendre Transform; need for convexity to ensure invertibility; illustrations from thermodynamics. The idea of a functional and a functional derivative. First variation for functionals. Euler-Lagrange equations, for both ordinary and partial differential equations. Use of Lagrange multipliers and multiplier functions. Fermat’s principle; geodesics; least action principles, Lagrange’s and Hamilton’s equations for particles and fields. Noether theorems and first integrals, including two forms of Noether’s theorem for ordinary differential equations (energy and momentum, for example). Interpretation in terms of conservation laws. Second variation for functionals; associated eigenvalue problem.

Appropriate books
D.S. Lemons Perfect Form. Princeton University Press 1997
C. Lanczos The Variational Principles of Mechanics. Dover 1986
R. Weinstock Calculus of Variations with applications to physics and engineering. Dover 1974
I.M. Gelfand and S.V. Fomin Calculus of Variations. Dover 2000
S. Hildebrandt and A. Tromba Mathematics and Optimal Form. Scientific American Library 1985

METHODS 24 lectures, Michaelmas Term

Self-adjoint ODEs
Periodic functions. Fourier series: definition and simple properties; Parseval’s theorem. Equations of second order. Self-adjoint differential operators. The Sturm–Liouville equation; eigenfunctions and eigenvalues; reality of eigenvalues and orthogonality of eigenfunctions; eigenfunction expansions (Fourier series as prototype), approximations in mean square, statement of completeness.

PDEs on bounded domains: separation of variables
Physical basis of Laplace’s equation, the wave equation and the diffusion equation. General method of separation of variables in Cartesian, cylindrical and spherical coordinates. Legendre’s equation: derivation, solutions including explicit forms of \( P_0 \), \( P_1 \) and \( P_2 \), orthogonality. Bessel’s equation of integer order as an example of a self-adjoint eigenvalue problem with non-trivial weight. Examples including potentials on rectangular and circular domains and on a spherical domain (axisymmetric case only), waves on a finite string and heat flow down a semi-infinite rod.

Inhomogeneous ODEs: Green’s functions
Properties of the Dirac delta function. Initial value problems and forced problems with two fixed end points; solution using Green’s functions. Eigenfunction expansions of the delta function and Green’s functions.

Fourier transforms
Fourier transforms: definition and simple properties; inversion and convolution theorems. The discrete Fourier transform. Examples of application to linear systems. Relationship of transfer function to Green’s function for initial value problems.

PDEs on unbounded domains
Classification of PDEs in two independent variables. Well posedness. Solution by the method of characteristics. Green’s functions for PDEs in 1, 2 and 3 independent variables; fundamental solutions of the wave equation, Laplace’s equation and the diffusion equation. The method of images. Application to the forced wave equation, Poisson’s equation and forced diffusion equation. Transient solutions of diffusion problems: the error function.

Appropriate books
Erwin Kreyszig Advanced Engineering Mathematics. Wiley
QUANTUM MECHANICS

16 lectures, Michaelmas Term

Physical background
Photoelectric effect. Electrons in atoms and line spectra. Particle diffraction.

Schrödinger equation and solutions

Observables and expectation values
Position and momentum operators and expectation values. Canonical commutation relations. Uncertainty principle.
Observables and Hermitian operators. Eigenvalues and eigenfunctions. Formula for expectation value.

Hydrogen atom
Spherically symmetric wave functions for spherical well and hydrogen atom.
Orbital angular momentum operators. General solution of hydrogen atom.

Appropriate books
Feynman, Leighton and Sands vol. 3 Ch 1-3 of the Feynman lectures on Physics. Addison-Wesley 1970
S. Gasiorowicz Quantum Physics. Wiley 2003
A.I.M. Rae Quantum Mechanics. Institute of Physics Publishing 2002

ELECTROMAGNETISM

16 lectures, Lent Term

Electrostatics

Magnetostatics

Electrodynamics

Electromagnetism and relativity
Review of special relativity; tensors and index notation. Charge conservation. 4-vector potential, gauge transformations. Electromagnetic tensor. Lorentz transformations of electric and magnetic fields. Maxwell’s equations in relativistic form. Lorentz force law.

Appropriate books
D.J. Griffiths Introduction to Electrodynamics. Cambridge University Press 2017
E.M. Purcell and D.J. Morin Electricity and Magnetism. Cambridge University Press 2013
A. Zangwill Modern Electromagnetism. Cambridge University Press 2013
J.D. Jackson Classical Electrodynamics. Wiley 1999
P. Lorrain and D. Corson Electromagnetism, Principles and Applications. Freeman 1990
**FLUID DYNAMICS** 16 lectures, Lent Term

**Parallel viscous flow**
Plane Couette flow, dynamic viscosity. Momentum equation and boundary conditions. Steady flows including Poiseuille flow in a channel. Unsteady flows, kinematic viscosity, brief description of viscous boundary layers (skin depth).

**Kinematics**
Material time derivative. Conservation of mass and the kinematic boundary condition. Incompressibility; streamfunction for two-dimensional flow. Streamlines and path lines.

**Dynamics**
Statement of Navier-Stokes momentum equation. Reynolds number. Stagnation-point flow; discussion of viscous boundary layer and pressure field. Conservation of momentum; Euler momentum equation. Bernoulli’s equation. Vorticity, vorticity equation, vortex line stretching, irrotational flow remains irrotational.

**Potential flows**
Velocity potential; Laplace’s equation, examples of solutions in spherical and cylindrical geometry by separation of variables. Translating sphere. Lift on a cylinder with circulation. Expression for pressure in time-dependent potential flows with potential forces. Oscillations in a manometer and of a bubble.

**Geophysical flows**
Linear water waves: dispersion relation, deep and shallow water, standing waves in a container, Rayleigh-Taylor instability. Euler equations in a rotating frame. Steady geostrophic flow, pressure as streamfunction. Motion in a shallow layer, hydrostatic assumption, modified continuity equation. Conservation of potential vorticity, Rossby radius of deformation.

**Appropriate books**
2. G.K. Batchelor *An Introduction to Fluid Dynamics*. Cambridge University Press 2000
4. M. van Dyke *An Album of Fluid Motion*. Parabolic Press

**NUMERICAL ANALYSIS** 16 lectures, Lent Term

**Polynomial approximation**

**Computation of ordinary differential equations**
Euler’s method and proof of convergence. Multistep methods, including order, the root condition and the concept of convergence. Runge-Kutta schemes. Stiff equations and A-stability.

**Systems of equations and least squares calculations**

**Appropriate books**
4. E. Suli and D.F. Meyers *An introduction to numerical analysis*. CUP 2003
5. A. Ralston and P. Rabinowitz *A first course in numerical analysis*. Dover 2001
7. P.J. Davis *Interpolation and Approximation*. Dover 1975
STATISTICS 16 lectures, Lent Term

Estimation

Hypothesis testing
Simple examples of hypothesis testing, null and alternative hypothesis, critical region, size, power, type I and type II errors, Neyman–Pearson lemma. Significance level of outcome. Uniformly most powerful tests. Likelihood ratio, and use of generalised likelihood ratio to construct test statistics for composite hypotheses. Examples, including $t$-tests and $F$-tests. Relationship with confidence intervals. Goodness-of-fit tests and contingency tables. [4]

Linear models

Appropriate books

MARKOV CHAINS 12 lectures, Michaelmas Term

Discrete-time chains
Definition and basic properties, the transition matrix. Calculation of $n$-step transition probabilities. Communicating classes, closed classes, absorption, irreducibility. Calculation of hitting probabilities and mean hitting times; survival probability for birth and death chains. Stopping times and statement of the strong Markov property.

Recurrence and transience; equivalence of transience and summability of $n$-step transition probabilities; equivalence of recurrence and certainty of return. Recurrence as a class property, relation with closed classes. Simple random walks in dimensions one, two and three. [3]

Invariant distributions, statement of existence and uniqueness. Mean return time, positive recurrence; equivalence of positive recurrence and the existence of an invariant distribution. Convergence to equilibrium for irreducible, positive recurrent, aperiodic chains and proof by coupling. *Long-run proportion of time spent in given state*. [3]

Time reversal, detailed balance, reversibility; random walk on a graph. [1]

Appropriate books
J.R. Norris Markov Chains. CUP 1997
**OPTIMISATION**  
12 lectures, Easter Term

**Elements of convex optimisation**
Convex sets and functions in $\mathbb{R}^n$, global and constrained optimality. Algorithms for unconstrained convex optimisation: gradient descent, Newton’s algorithm. Introduction to convex optimisation on a convex set, the barrier method. Examples. [3]

**Lagrangian methods & duality**

**Linear programming in the nondegenerate case**
Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm and the tableau. Examples. [3]

**Applications of linear programming**
Two person zero-sum games. Network flows: the max-flow min-cut theorem; the Ford–Fulkerson algorithm, the rational case. Network flows with costs, the transportation algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks. The formulation of simple practical and combinatorial problems as linear programming or network problems. [3]

**Appropriate books**
2. D. Luenberger *Linear and Nonlinear Programming*. Addison-Wesley 1984

**COMPUTATIONAL PROJECTS**  
8 lectures, Easter Term of Part IA

Lectures are given in the Part IA year.

The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July or the beginning of August preceding the Part IB year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at [https://www.maths.cam.ac.uk/undergrad/catam/](https://www.maths.cam.ac.uk/undergrad/catam/).

Full credit may be obtained from the submission of the two core projects and a further two additional projects. Once the manual is available, these projects may be undertaken at any time up to the submission deadlines, which are near the start of the Lent Full Term in the IB year for the two core projects, and near the start of the Easter Full Term in the IB year for the two additional projects. A list of suitable books can be found in the CATAM manual.
GENERAL ARRANGEMENTS

Structure of Part II

There are two types of lecture courses in Part II: C courses and D courses. C courses are intended to be straightforward and accessible, and of general interest, whereas D courses are intended to be more demanding. The Faculty Board recommend that students who have not obtained at least a good second class in Part IB should include a significant number of C courses amongst those they choose.

There are 10 C courses and 27 D courses. All C courses are 24 lectures; of the D courses, 21 are 24 lectures and 6 are 16 lectures. The complete list of courses is as follows (an asterisk denotes a 16-lecture course):

<table>
<thead>
<tr>
<th>C courses</th>
<th>D courses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Theory</td>
<td>Logic and Set Theory</td>
</tr>
<tr>
<td>Topics in Analysis</td>
<td>Stochastic Financial Models</td>
</tr>
<tr>
<td>Coding and Cryptography</td>
<td>*Mathematics of Machine Learning</td>
</tr>
<tr>
<td>Automata and Formal Lang.</td>
<td>*Asymptotic Methods</td>
</tr>
<tr>
<td>Statistical Modelling</td>
<td>*Number Fields</td>
</tr>
<tr>
<td>Mathematical Biology</td>
<td>*Integrable Systems</td>
</tr>
<tr>
<td>Further Complex Methods</td>
<td>Algebraic Topology</td>
</tr>
<tr>
<td>Classical Dynamics</td>
<td>Principles of Quantum Mechanics</td>
</tr>
<tr>
<td>Cosmology</td>
<td>*Riemann Surfaces</td>
</tr>
<tr>
<td>Quantum Inf. and Comp.</td>
<td>*Electrodynamics</td>
</tr>
<tr>
<td>Differential Geometry</td>
<td>General Relativity</td>
</tr>
<tr>
<td>Probability and Measure</td>
<td>Fluid Dynamics</td>
</tr>
<tr>
<td>Applied Probability</td>
<td>Waves</td>
</tr>
<tr>
<td>Principles of Statistics</td>
<td>Numerical Analysis</td>
</tr>
</tbody>
</table>

As in Part IB, students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred workload, bearing in mind that it is better to do a smaller number of courses thoroughly than to do many courses scrappily. Students might sensibly assess their proposed workload by comparing the numbers of example sheets and lectures they are taking on per term with their previous experiences in Parts IA and IB.

Computational Projects

In addition to the lectured courses, there is a Computational Projects course.

No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of reports. The maximum credit obtainable is 150 marks and there are no alpha or beta quality marks. Credit obtained is added directly to the credit gained on the written papers. The maximum contribution to the final merit mark is thus 150, which is the same as the maximum for a 16-lecture course. The Computational Projects are considered to be a single piece of work within the Mathematical Tripos.

Examinations

Details of arrangements common to all examinations of the undergraduate Mathematical Tripos start on page 2 of this booklet.

There are no restrictions on the number or type of courses that may be presented for examination, but examiners may consider if most of the marks are obtained on only one or two courses. The Faculty Board has recommended to the examiners that no distinction be made, for classification purposes, between quality marks obtained on the C-course questions in Section II and quality marks obtained on D course questions.

On each of the four papers, candidates may obtain credit for attempts on at most six questions in Section I; there is no restriction on the number of questions in Section II that may be attempted for credit.

The number of questions set on each course is determined by the type and length of the course, as shown in the following table:

<table>
<thead>
<tr>
<th>Section I</th>
<th>Section II</th>
</tr>
</thead>
<tbody>
<tr>
<td>C course, 24 lectures</td>
<td>2</td>
</tr>
<tr>
<td>D course, 24 lectures</td>
<td>4</td>
</tr>
<tr>
<td>D course, 16 lectures</td>
<td>3</td>
</tr>
</tbody>
</table>

In Section I of each paper, there are 10 questions, one on each C course.

In Section II of each paper, there are 5 questions on C courses, one question on each of the 20 24-lecture D courses and either one question or no questions on each of the 7 16-lecture D courses, giving a total of 30 or 31 questions on each paper.

The distribution in Section II of the C course questions and the 16-lecture D course questions is shown in the following table.

<table>
<thead>
<tr>
<th>C courses</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>16-lecture D courses</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Theory</td>
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<td>*</td>
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<td>*</td>
<td>*</td>
<td>Electrodynamics</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Further Complex Methods</td>
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<td>*</td>
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<td>*</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Classical Dynamics</td>
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</tr>
<tr>
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</tr>
</tbody>
</table>
Approximate Class Boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines in recent years.

For convenience, we define $M_1$ and $M_2$ by

$$M_1 = 30\alpha + 5\beta + m - 120, \quad M_2 = 15\alpha + 5\beta + m.$$

$M_1$ is related to the primary classification criterion for the first class and $M_2$ is related to the primary classification criterion for the upper and lower second and third classes.

The second column of each table shows a sufficient criterion for each class (in terms of $M_1$ for the first class and $M_2$ for the other classes). The third and fourth columns show $M_1$ (for the first class) or $M_2$ (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline.

The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

### Part II 2023

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 693$</td>
<td>694/499, 13, 3 694/399, 13, 5</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 481$</td>
<td>482/357, 5, 10 483/343, 6, 10</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 313$</td>
<td>314/279, 1, 4 317/262, 6, 5</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 185$</td>
<td>186/146, 1, 5 218/158, 1, 9</td>
</tr>
</tbody>
</table>

### Part II 2022

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 658$</td>
<td>659/494, 12, 5 664/429, 10, 11</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 361$</td>
<td>462/312, 8, 6 465/340, 4, 13</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 314$</td>
<td>315/250, 2, 7 325/260, 1, 10</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 200$</td>
<td>215/155, 1, 9 221/166, 4, 2</td>
</tr>
</tbody>
</table>
PART II

NUMBER THEORY (C) 24 lectures, Michaelmas Term

Review from Part IA Numbers and Sets: Euclid’s Algorithm, prime numbers, fundamental theorem of arithmetic. Congruences. The theorems of Fermat and Euler. [2]

Chinese remainder theorem. Lagrange’s theorem. Primitive roots to an odd prime power modulus. [3]

The mod-$p$ field, quadratic residues and non-residues, Legendre’s symbol. Euler’s criterion. Gauss’ lemma, quadratic reciprocity. [2]

Proof of the law of quadratic reciprocity. The Jacobi symbol. [1]


Distribution of the primes. Divergence of $\sum_{p \leq x} p^{-1}$. The Riemann zeta-function and Dirichlet series. Statement of the prime number theorem and of Dirichlet’s theorem on primes in an arithmetic progression. Legendre’s formula. Chebyshev’s theorem. [4]

Continued fractions. Pell’s equation. [3]

Primality testing. Fermat, Euler and strong pseudo-primes. [2]

Factorization. Fermat factorization, factor bases, the continued-fraction method. Pollard’s method. [2]

Appropriate books
A. Baker  A Concise Introduction to the Theory of Numbers. Cambridge University Press 1984
G.H. Hardy and E.M. Wright  An Introduction to the Theory of Numbers. Oxford University Press
T. Nagell  Introduction to Number Theory. AMS 1964
H. Davenport  The Higher Arithmetic. Cambridge University Press
A. Granville  Number Theory Revealed: an Introduction. AMS 2019

TOPICS IN ANALYSIS (C) 24 lectures, Lent Term

Analysis courses from IB will be helpful, but it is intended to introduce and develop concepts of analysis as required.

Discussion of metric spaces; compactness and completeness. Brouwer’s fixed point theorem. Proof(s) in two dimensions. Equivalent formulations, and applications. The degree of a map. The fundamental theorem of algebra, the Argument Principle for continuous functions, and a topological version of Rouché’s theorem. [6]


Liouville’s proof of the existence of transcendentals. The irrationality of $e$ and $\pi$. The continued fraction expansion of real numbers; the continued fraction expansion of $e$. [4]


Appropriate books
E.W. Cheney  Introduction to Approximation Theory. AMS, 1999
CODING AND CRYPTOGRAPHY (C) 24 lectures, Lent Term

Part IB Linear Algebra is useful and Part IB Groups, Rings and Modules is very useful.

Introduction to communication channels, coding and channel capacity. [1]
Applications to gambling and the stock market. [1]
Information rate of a Bernoulli source. Capacity of a memoryless binary symmetric channel; Shannon’s noisy coding theorem for such channels. [3]

Appropriate books
2 D. Welsh Codes and Cryptography. OUP 1988
5 John Talbot and Dominic Welsh Complexity and cryptography, an introduction. CUP 2006
6 Richard W. Hamming Coding and information theory. Prentice-Hall 1980
7 Thomas M Thompson From error correcting codes through sphere packings to simple groups. Carus Mathematical Monographs 1983

AUTOMATA AND FORMAL LANGUAGES (C) 24 lectures, Michaelmas Term

Part IA Numbers and Sets is essential.

Recursively enumerable languages

Regular languages

Context-free languages
Context-free grammars. Context-free languages. Chomsky normal form. Regular languages are context-free. Limitations of context-free grammars: the pumping lemma for context-free languages; examples of non-context-free languages. [5]

Appropriate books
P.T. Johnstone Notes on logic and set theory (Chapter 4). CUP 1987
D.C. Kozen Automata and computability. Springer 1997
STATISTICAL MODELLING (C) 24 lectures, Michaelmas Term

Part IB Statistics is essential. About two thirds of this course will be lectures, with the remaining hours as practical classes, using R. R may be downloaded at no cost via https://cran.r-project.org

Introduction to the statistical programming language R

Graphical summaries of data, e.g. histograms. Matrix computations. Writing simple functions. Simulation.

Linear models


Exponential families


Generalised linear models


Examples in R

Linear and generalised linear models. Interpretation of models, inference and model selection.

Appropriate books

A.C. Davison Statistical Models. CUP 2008
J. Albert and M. Rizzo R by Example. Springer 2012
S. Chaterjee and J.S. Simonoff Handbook of Regression Analysis. Wiley 2013
A. Agresti Foundations of Linear and Generalized Linear Models. Wiley 2015

MATHMATICAL BIOLOGY (C) 24 lectures, Lent Term

Part II Dynamical Systems is useful.

Introduction to the role of mathematics in biology

Systems without spatial structure: deterministic systems


Stochastic systems

Discrete stochastic models of birth and death processes. Master equations and Fokker-Planck equations. The continuum limit and the importance of fluctuations. Comparison of deterministic and stochastic models, including implications for extinction/invasion. Simple random walk and derivation of the diffusion equation.

Systems with spatial structure: diffusion and reaction-diffusion systems


Appropriate books

L. Edelstein-Keshet Mathematical Models in Biology. SIAM classics in applied mathematics reprint, 2005
FURTHER COMPLEX METHODS (C) 24 lectures, Lent Term

Complex Methods (or Complex Analysis) is essential.

Complex variable
Revision of complex variable. Analyticity of a function defined by an integral (statement and discussion only). Analytic and meromorphic continuation.
Multivalued functions: definitions, branch points and cuts, integration; examples, including inverse trigonometric functions as integrals and elliptic integrals.

Analytic and meromorphic continuation.

Special functions
Gamma function: Euler integral definition; brief discussion of product formulae; Hankel representation; reflection formula; discussion of uniqueness (e.g. Wielandt’s theorem). Beta function: Euler integral definition; relation to the gamma function. Riemann zeta function: definition as a sum; integral representations; functional equation; *discussion of zeros and relation to \( \pi(x) \) and the distribution of prime numbers*.

Differential equations by transform methods
Solution of differential equations by integral representation; Airy equation as an example. Solution of partial differential equations by transforms; the wave equation as an example. Causality. Nyquist stability criterion.

Second order ordinary differential equations in the complex plane
Classification of singularities, exponents at a regular singular point. Nature of the solution near an isolated singularity by analytic continuation. Fuchsian differential equations. The Riemann P-function, hypergeometric functions and the hypergeometric equation, including brief discussion of monodromy.

Appropriate books
4. B. Spain and M.G. Smith, *Functions of Mathematical Physics*. Van Nostrand 1970

CLASSICAL DYNAMICS (C) 24 lectures, Michaelmas Term

Review of Newtonian mechanics

Lagrange’s equations

Motion of a rigid body
Kinematics of a rigid body. Angular momentum, kinetic energy, diagonalization of inertia tensor. Euler top, conserved angular momentum, Euler equations and their solution in terms of elliptic integrals. Lagrange top, steady precession, nutation.

Hamilton’s equations

Hamiltonian systems in nonlinear phase spaces, e.g. classical spin in a magnetic field. 2-D motion of ideal point vortices. *Connections between Lagrangian/Hamiltonian dynamics and quantum mechanics.*

Appropriate books
COSMOLOGY (C) 24 lectures, Michaelmas Term

The expanding universe


The hot universe

Structure formation

Appropriate books
B. Ryden Introduction to cosmology. Addison-Wesley 2003
A. Liddle An introduction to modern cosmology. Wiley 2003
E.R. Harrison Cosmology: The science of the universe. CUP 2000
S. Weinberg Cosmology. OUP 2008 (A more advanced text)

QUANTUM INFORMATION AND COMPUTATION (C) 24 lectures, Lent Term

Part III Quantum Mechanics is desirable.

Introduction
Why quantum information and computation. The basic idea of polynomial vs exponential computational complexity.

Quantum mechanics and quantum information
Basic principles of quantum mechanics and Dirac notation in a finite-dimensional setting. Composite systems and tensor products, projective measurements. Two-dimensional systems: qubits and Pauli operations. Definition of an entangled state.

Quantum states as information carriers
The no-cloning theorem. Optimal discrimination of non-orthogonal pure states; the Helstrom bound. Local quantum operations. The no-signalling theorem.

Quantum teleportation and dense coding
Bell states and basic properties; quantum dense coding. Exposition of quantum teleportation. Consistency of quantum teleportation with the no-signalling and no-cloning theorems.

Quantum cryptography
Cryptographic key distribution and the one-time pad. Quantum key distribution: the BB84 protocol. Sketch of the security of the BB84 protocol against individual attacks. *Brief discussion of implementations of quantum key.*

Basic principles of quantum computing

Basic quantum algorithms

Grover’s quantum searching algorithm
Introduction to search problems and the complexity class NP. Exposition of Grover’s quantum searching algorithm.

Shor’s quantum factoring algorithm
Exposition of Shor’s quantum factoring algorithm (proofs of classical number-theory ingredients not examinable).

Appropriate books
M. Nielsen and I. Chuang Quantum Computation and Quantum Information. CUP 2000
B. Schumacher and M. Westmoreland Quantum Processes, Systems, and Information. CUP 2010
LOGIC AND SET THEORY (D)  
24 lectures, Lent Term  
No specific prerequisites.

Ordinals and cardinals  

Posets and Zorn’s lemma  
Partially ordered sets; Hasse diagrams, chains, maximal elements. Lattices and Boolean algebras. Complete and chain-complete posets; fixed-point theorems. The axiom of choice and Zorn’s lemma. Applications of Zorn’s lemma in mathematics. The well-ordering principle.  

Propositional logic  

Predicate logic  
The predicate calculus with equality. Examples of first-order languages and theories. Statement of the completeness theorem; *sketch of proof*. The compactness theorem and the Löwenheim-Skolem theorems. Limitations of first-order logic. Model theory.  

Set theory  
Set theory as a first-order theory; the axioms of ZF set theory. Transitive closures, epsilon-induction and epsilon-recursion. Well-founded relations. Mostowski’s collapsing theorem. The rank function and the von Neumann hierarchy.  

Consistency  
«Problems of consistency and independence».  

Appropriate books  
A. Hajnal and P. Hamburger *Set Theory*. LMS Student Texts number 48, CUP 1999  
1 P.T. Johnstone *Notes on Logic and Set Theory*. Cambridge University Press 1987  
D. van Dalen *Logic and Structure*. Springer-Verlag 1994

GRAPH THEORY (D)  
24 lectures, Michaelmas Term  
No specific prerequisites.

Introduction  

Connectivity and matchings  
Matchings in bipartite graphs; Hall’s theorem and its variants. Connectivity and Menger’s theorem.  

Extremal graph theory  
Long paths, long cycles and Hamilton cycles. Complete subgraphs and Turán’s theorem. Bipartite subgraphs and the problem of Zarankiewicz. The Erdős-Stone theorem; *sketch of proof*.  

Eigenvalue methods  
The adjacency matrix and the Laplacian. Strongly regular graphs.  

Graph colouring  
Vertex and edge colourings; simple bounds. The chromatic polynomial. The theorems of Brooks and Vizing. Equivalent forms of the four colour theorem; the five colour theorem. Heawood’s theorem for surfaces; the torus and the Klein bottle.  

Ramsey theory  
Ramsey’s theorem (finite and infinite forms). Upper bounds for Ramsey numbers.  

Probabilistic methods  
Basic notions; lower bounds for Ramsey numbers. The model $G(n, p)$; graphs of large girth and large chromatic number. The clique number.  

Appropriate books  
1 B.Bollobás *Modern Graph Theory*. Springer 1998  
R.Diestel *Graph Theory*. Springer 2000  
D.West *Introduction to Graph Theory*. Prentice Hall 1999
GALOIS THEORY (D) 24 lectures, Michaelmas Term

Groups, Rings and Modules is essential.

Field extensions, tower law, algebraic extensions; irreducible polynomials and relation with simple algebraic extensions. Finite multiplicative subgroups of a field are cyclic. Existence and uniqueness of splitting fields.

Existence and uniqueness of algebraic closure.

Separability. Theorem of primitive element. Trace and norm.

Normal and Galois extensions, automorphic groups. Fundamental theorem of Galois theory.


Solubility by radicals. Insolubility of general quintic equations and other classical problems.

Artin’s theorem on the subfield fixed by a finite group of automorphisms. Polynomial invariants of a finite group; examples.

Appropriate books

E. Artin Galois Theory. Dover Publications
B. L. van der Waerden Modern Algebra. Unger Pub 1949
S. Lang Algebra (Graduate Texts in Mathematics). Springer-Verlag New York Inc
I. Kaplansky Fields and Rings. The University of Chicago Press

REPRESENTATION THEORY (D) 24 lectures, Michaelmas Term

Linear Algebra, and Groups, Rings and Modules are essential.

Representations of finite groups


Character theory

Determination of a representation by its character. The group algebra, conjugacy classes, and orthogonality relations. Regular representation. Permutation representations and their characters. Induced representations and the Frobenius reciprocity theorem. Mackey’s theorem. Frobenius’s Theorem.

Arithmetic properties of characters

Divisibility of the order of the group by the degrees of its irreducible characters. Burnside’s $p^aq^b$ theorem.

Tensor products

Tensor products of representations and products of characters. The character ring. Tensor, symmetric and exterior algebras.

Representations of $S_3$ and $SU_2$

The groups $S_3$, $SU_2$ and $SO(3)$, their irreducible representations, complete reducibility. The Clebsch-Gordan formula. *Compact groups.*

Further worked examples

The characters of one of $GL_2(F_q)$, $S_n$ or the Heisenberg group.

Appropriate books

J.L. Alperin and R.B. Bell Groups and representations. Springer 1995
J-P. Serre Linear representations of finite groups. Springer-Verlag 1977
**NUMBER FIELDS (D)**

16 lectures, Lent Term

*Part IB Groups, Rings and Modules is essential and Part II Galois Theory is desirable.*

Definition of algebraic number fields, their integers and units. Norms, bases and discriminants. [3]

Ideals, principal and prime ideals, unique factorisation. Norms of ideals. [3]

Dedekind’s theorem on the factorisation of primes. Application to quadratic fields. [2]

Minkowski’s theorem on convex bodies. Statement of Dirichlet’s unit theorem. Determination of units in quadratic fields. [2]

Ideal classes, finiteness of the class group. Calculation of class numbers using statement of the Minkowski bound. [2]

Discussion of the cyclotomic field and the Fermat equation or some other topic chosen by the lecturer. [3]

**Appropriate books**

- D.A. Marcus, *Number Fields*. Springer 1977

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**ALGEBRAIC TOPOLOGY (D)**

24 lectures, Lent Term

*Part IB Analysis and Topology is essential.*

The fundamental group

Homotopy of continuous functions and homotopy equivalence between topological spaces. The fundamental group of a space, homomorphisms induced by maps of spaces, change of base point, invariance under homotopy equivalence. [3]

Covering spaces

Covering spaces and covering maps. Path-lifting and homotopy-lifting properties, and their application to the calculation of fundamental groups. The fundamental group of the circle; topological proof of the fundamental theorem of algebra. *Construction of the universal covering of a path-connected, locally simply connected space*. The correspondence between connected coverings of $X$ and conjugacy classes of subgroups of the fundamental group of $X$. [5]

The Seifert–Van Kampen theorem

Free groups, generators and relations for groups, free products with amalgamation. Statement *and proof* of the Seifert–Van Kampen theorem. Applications to the calculation of fundamental groups. [4]

Simplicial complexes

Finite simplicial complexes and subdivisions; the simplicial approximation theorem. [3]

Homology

Simplicial homology, the homology groups of a simplex and its boundary. Functorial properties for simplicial maps. *Proof of functoriality for continuous maps, and of homotopy invariance*. [4]

Homology calculations

The homology groups of $S^n$, applications including Brouwer’s fixed-point theorem. The Mayer-Vietoris theorem. *Sketch of the classification of closed combinatorial surfaces*; determination of their homology groups. Rational homology groups; the Euler–Poincaré characteristic and the Lefschetz fixed-point theorem. [5]

**Appropriate books**

PART II

LINEAR ANALYSIS (D) 24 lectures, Michaelmas Term
Part IB Linear Algebra, and Analysis and Topology are essential.

Normed and Banach spaces. Linear mappings, continuity, boundedness, and norms. Finite-dimensional normed spaces.
The Baire category theorem. The principle of uniform boundedness, the closed graph theorem and the inversion theorem; other applications.
Inner product spaces and Hilbert spaces; examples and elementary properties. Orthonormal systems, and the orthogonalization process. Bessel’s inequality, the Parseval equation, and the Riesz–Fischer theorem.
Bounded linear operations, invariant subspaces, eigenvectors; the spectrum and resolvent set. Compact operators on Hilbert space; discreteness of spectrum. Spectral theorem for compact Hermitian operators.

Appropriate books

ANALYSIS OF FUNCTIONS (D) 24 lectures, Lent Term
Part II Linear Analysis and Part II Probability and Measures are essential.

Lebesgue integration theory
Review of integration: simple functions, monotone and dominated convergence; existence of Lebesgue measure; definition of $L^p$ spaces and their completeness. The Lebesgue differentiation theorem. Egorov’s theorem, Lusin’s theorem. Mollification by convolution, continuity of translation and separability of $L^p$ when $p \neq \infty$.

Banach and Hilbert space analysis

Fourier analysis
Definition of Fourier transform in $L^1$; the Riemann–Lebesgue lemma. Fourier inversion theorem. Extension to $L^2$ by density and Plancherel’s isometry. Duality between regularity in real variable and decay in Fourier variable.

Generalized derivatives and function spaces

Applications

Appropriate books
RIEMANN SURFACES (D)  16 lectures, Michaelmas Term

Part IB Complex Analysis is essential, and Analysis and Topology is desirable.

The complex logarithm. Analytic continuation in the plane; natural boundaries of power series. Informal examples of Riemann surfaces of simple functions (via analytic continuation). Examples of Riemann surfaces, including the Riemann sphere, and the torus as a quotient surface. [4]
Analytic, meromorphic and harmonic functions on a Riemann surface; analytic maps between two Riemann surfaces. The open mapping theorem, the local representation of an analytic function as $z \mapsto z^k$. Complex-valued analytic and harmonic functions on a compact surface are constant. [2]
Germs of an analytic map between two Riemann surfaces; the space of germs as a covering surface (in the sense of complex analysis). The monodromy theorem (statement only). The analytic continuation of a germ over a simply connected domain is single-valued. [3]
The degree of a map between compact Riemann surfaces; Branched covering maps and the Riemann-Hurwitz relation (assuming the existence of a triangulation). The fundamental theorem of algebra. Rational functions as meromorphic functions from the sphere to the sphere. [3]

Statement of the Uniformization Theorem; applications to conformal structure on the sphere, *to tori, and the hyperbolic geometry of Riemann surfaces*. [1]

Appropriate books

A.F. Beardon A Primer on Riemann Surfaces. Cambridge University Press, 2004
G.A. Jones and D. Singerman Complex functions: an algebraic and geometric viewpoint. Cambridge University Press, 1987

ALGEBRAIC GEOMETRY (D)  24 lectures, Lent Term

Groups Rings and Modules is essential.

Affine varieties and coordinate rings. Projective space, projective varieties and homogenous coordinates. Rational and regular maps. [4]
Discussion of basic commutative algebra. Dimension, singularities and smoothness. [4]
Conics and plane cubics. Quadric surfaces and their lines. Segre and Veronese embeddings. [4]
Curves, differentials, genus. Divisors, linear systems and maps to projective space. The canonical class. [8]

Statement of the Riemann-Roch theorem, with applications. [4]

Appropriate books

F. Kirwan Complex Algebraic Curves. Cambridge University Press, 1992
M. Reid Undergraduate Algebraic Geometry. Cambridge University Press 1989
B. Hassett Introduction to Algebraic Geometry. Cambridge University Press, 2007
R. Hartshorne Algebraic Geometry, chapters 1 and 4. Springer 1997
DIFFERENTIAL GEOMETRY (D) 24 lectures, Lent Term

Part IB Analysis and Topology, and Geometry are very useful.

Smooth manifolds in $\mathbb{R}^n$, tangent spaces, smooth maps and the inverse function theorem. Examples, regular values, Sard’s theorem (statement only). Transverse intersection of submanifolds. [4]

Manifolds with boundary, degree mod 2 of smooth maps, applications. [3]

Curves in 2-space and 3-space, arc-length, curvature, torsion. The isoperimetric inequality. [2]

Smooth surfaces in 3-space, first fundamental form, area. [1]

The Gauss map, second fundamental form, principal curvatures and Gaussian curvature. Theorema Egregium. [3]


Parallel transport and geodesics for surfaces in 3-space. Geodesic curvature. [2]


Global theorems on curves: Fenchel’s theorem (the total curvature of a simple closed curve is greater than or equal to $2\pi$); the Fary-Milnor theorem (the total curvature of a simple knotted closed curve is greater than $4\pi$). [2]

Appropriate books

P.M.H. Wilson Curved Spaces. CUP, January 2008
A. Pressley Elementary Differential Geometry,. Springer-Verlag, 2010

PROBABILITY AND MEASURE (D) 24 lectures, Michaelmas Term

Part IB Analysis and Topology is essential.


Chebyshev’s inequality, tail estimates. Jensen’s inequality. Completeness of $L^p$ for $1 \leq p \leq \infty$. The Hölder and Minkowski inequalities, uniform integrability. [4]

$L^2$ as a Hilbert space. Orthogonal projection, relation with elementary conditional probability. Variance and covariance. Gaussian random variables, the multivariate normal distribution. [2]

The strong law of large numbers, proof for independent random variables with bounded fourth moments. Measure preserving transformations, Bernoulli shifts. Statements and proofs of maximal ergodic theorem and Birkhoff’s almost everywhere ergodic theorem, proof of the strong law. [4]

The Fourier transform of a finite measure, characteristic functions, uniqueness and inversion. Weak convergence, statement of Lévy’s convergence theorem for characteristic functions. The central limit theorem. [2]

Appropriate books

P. Billingsley Probability and Measure. Wiley 1995
D. Williams Probability with Martingales. Cambridge University Press 1991
APPLIED PROBABILITY (D)  
24 lectures, Lent Term

Part IB Markov Chains is essential and Part II Probability and Measure is desirable

Finite-state continuous-time Markov chains: basic properties. Q-matrix (or generator), backward and forward equations. The homogeneous Poisson process and its properties (thinning, superposition). Birth and death processes.


Applications: the $M/M/1$ and $M/M/\infty$ queues. Burke's theorem. Jackson's theorem for queueing networks. The $M/G/1$ queue.

Spatial Poisson processes in $d$ dimensions. The superposition, mapping, and colouring theorems. Rényi's theorem. Applications including Olbers' paradox.

Appropriate books

J.R. Norris Markov Chains. CUP 1997
J.F.C. Kingman Poisson processes. OUP 1992

PRINCIPLES OF STATISTICS (D)  
24 lectures, Michaelmas Term

Part IB Statistics is essential

The Likelihood Principle

Bayesian Inference
Prior and posterior distributions. Conjugate families, improper priors, predictive distributions. Asymptotic theory for posterior distributions. Point estimation, credible regions, hypothesis testing and Bayes factors.

Decision Theory
Basic elements of a decision problem, including loss and risk functions. Decision rules, admissibility, minimax and Bayes rules. Finite decision problems. Stein estimator.

Multivariate Analysis

Nonparametric Inference and Monte Carlo Techniques

Appropriate books

STOCHASTIC FINANCIAL MODELS (D) 24 lectures, Michaelmas Term

Utility and mean-variance analysis
Utility functions: risk aversion and risk neutrality. Portfolio selection with the mean-variance criterion; the efficient frontier when all assets are risky and when there is one riskless asset. The capital-asset pricing model. Reservation bid and ask prices, marginal utility pricing. Simplest ideas of equilibrium and market clearing. State-price density. [5]

Martingales
Conditional expectation, definition and basic properties. Stopping times. Martingales, supermartingales, submartingales. Use of the optional sampling theorem. [3]

Dynamic models
Introduction to dynamic programming; optimal stopping and exercising American puts; optimal portfolio selection. [3]

Pricing contingent claims
Lack of arbitrage in one-period models; hedging portfolios; martingale probabilities and pricing claims in the binomial model. Extension to the multi-period binomial model. Axiomatic derivation. [4]

Brownian motion
Introduction to Brownian motion; Brownian motion as a limit of random walks. Hitting-time distributions; changes of probability. [3]

Black–Scholes model
The Black–Scholes formula for the price of a European call; sensitivity of price with respect to the parameters; implied volatility; pricing other claims. Binomial approximation to Black–Scholes. Use of finite-difference schemes to compute prices. [6]

Appropriate books
J. Hull Options, Futures and Other Derivative Securities. Prentice-Hall 2003

MATHEMATICS OF MACHINE LEARNING (D) 16 lectures, Lent Term

Part IB Statistics is essential. Part IB Optimisation is helpful, but relevant material will be reviewed in the course.

Introduction to statistical learning

Statistical learning theory

Computation for empirical risk minimisation

Popular machine learning methods

Appropriate books
C. Giraud Introduction to High-Dimensional Statistics. CRC press, 2015
T. Hastie, R. Tibshirani and J. Friedman The Elements of Statistical Learning, 2nd ed. Springer, 2001
M. Wainwright High-Dimensional Statistics: A Non-Asymptotic Viewpoint. CUP, 2019
S. Shalev-Shwartz and S. Ben-David Understanding Machine Learning: From Theory to Algorithms. CUP, 2014
ASYMPTOTIC METHODS (D) 16 lectures, Michaelmas Term

Either Complex Methods or Complex Analysis is essential, Part II Further Complex Methods is useful.

Asymptotic expansions
Definition (Poincare) of \( \phi(z) \sim \sum a_n z^{-n} \); examples; elementary properties; uniqueness; Stokes’ phenomenon. \[4\]

Asymptotics behaviour of functions defined by integrals
Integration by parts. Watson’s lemma and Laplace’s method. Riemann–Lebesgue lemma and method of stationary phase. The method of steepest descent (including derivation of higher order terms). Any function, *and application to wave theory of a rainbow*. \[7\]

Asymptotic behaviour of solutions of differential equations
Asymptotic solution of second-order linear differential equations, including Liouville–Green functions (proof that they are asymptotic not required) and WKBJ with the quantum harmonic oscillator as an example. \[4\]

Recent developments
Further discussion of Stokes’ phenomenon. *Asymptotics ‘beyond all orders’*. \[1\]

Appropriate books
A. Erdelyi Asymptotic Expansions. Dover 1956
E.J. Hinch Perturbation Methods. CUP 1991
J.D. Murray Asymptotic Analysis. Springer 1984

DYNAMICAL SYSTEMS (D) 24 lectures, Michaelmas Term

General introduction
The notion of a dynamical system and examples of simple phase portraits. Relationship between continuous and discrete systems. Reduction to autonomous systems. Initial value problems, uniqueness, finite-time blowup, examples. Flows, orbits, invariant sets, limit sets and topological equivalence. \[3\]

Fixed points of flows
Linearization. Classification of fixed points in \( \mathbb{R}^2 \), Hamiltonian case. Effects of nonlinearity; hyperbolic and non-hyperbolic cases; Stable-manifold theorem (statement only), stable and unstable manifolds in \( \mathbb{R}^2 \). Phase-plane sketching. \[3\]

Stability
Lyapunov, quasi-asymptotic and asymptotic stability of invariant sets. Lyapunov and bounding functions. Lyapunov’s 1st theorem; La Salle’s invariance principle. Local and global stability. \[2\]

Periodic orbits in \( \mathbb{R}^2 \)
The Poincaré index; Dulac’s criterion; the Poincaré–Bendixson theorem (*and proof*). Nearly Hamiltonian flows. Stability of periodic orbits; Floquet multipliers. Examples; van der Pol oscillator. \[5\]

Bifurcations in flows and maps

Chaos
Sensitive dependence on initial conditions, topological transitivity. Maps of the interval, the sawtooth map, horseshoes, symbolic dynamics. Period three implies chaos, the occurrence of \( N \)-cycles, Sharkovsky’s theorem (statement only). The tent map. Unimodal maps and Feigenbaum’s constant. \[6\]

Appropriate books
D.K. Arrowsmith and C.M. Place Introduction to Dynamical Systems. CUP 1990
P.G. Drazin Nonlinear Systems. CUP 1992
P.A. Glendinning Stability, Instability and Chaos. CUP 1994
D.W. Jordan and P. Smith Nonlinear Ordinary Differential Equations. OUP 1999
J. Guckenheimer and P. Holmes Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer, second edition 1986

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INTEGRABLE SYSTEMS (D)  16 lectures, Lent Term
Part IB Methods, and Complex Methods or Complex Analysis are essential


Hamiltonian formulation of soliton equations. [2]

Painlevé equations and Lie symmetries: Symmetries of differential equations, the ODE reductions of certain integrable nonlinear PDEs, Painlevé equations. [3]

Appropriate books
† Dunajski, M Solitons, Instantons and Twistor. (Ch 1–4) Oxford Graduate Texts in Mathematics, ISBN 9780198570639, OUP, Oxford 2009
P.J. Olver Applications of Lie groups to differential equations. Springer 2000
MJ Ablowitz and P Clarkson Solitons, Nonlinear Evolution Equations and Inverse Scattering. CUP 1991

PRINCIPLES OF QUANTUM MECHANICS (D) 24 lectures, Michaelmas Term
IB Quantum Mechanics is essential.

Dirac formalism
Bra and ket notation, operators and observables, probability amplitudes, expectation values, complete commuting sets of operators, unitary operators. Schrödinger equation, wave functions in position and momentum space. [3]

Time evolution operator, Schrödinger and Heisenberg pictures, Heisenberg equations of motion. [2]

Harmonic oscillator
Analysis using annihilation, creation and number operators. Significance for normal modes in physical examples. [2]

Multiparticle systems
Composite systems and tensor products, wave functions for multiparticle systems. Symmetry or antisymmetry of states for identical particles, Bose and Fermi statistics, Pauli exclusion principle. [3]

Perturbation theory
Time-independent theory; second order without degeneracy, first order with degeneracy. [2]

Angular momentum

Translations and rotations
Unitary operators corresponding to spatial translations, momenta as generators, conservation of momentum and translational invariance. Corresponding discussion for rotations. Reflections, parity, intrinsic parity. [4]

Time-dependent perturbation theory
Interaction picture. First-order transition probability, the golden rule for transition rates. Application to atomic transitions, selection rules based on angular momentum and parity, "absorption, stimulated and spontaneous emission of photons". [3]

Quantum basics
Quantum data, qubits, no cloning theorem. Entanglement, pure and mixed states, density matrix. Classical determinism versus quantum probability, Bell inequality for singlet two-electron state, GHZ state. [2]

Appropriate books
† E. Merzbacher Quantum Mechanics, 3rd edition. Wiley 1998
S. Weinberg Lectures on Quantum Mechanics. CUP, 2nd ed., 2015
J.J. Sakurai and J.J. Napolitano Modern Quantum Mechanics. CUP 2017
PART II

APPLICATIONS OF QUANTUM MECHANICS (D)  
24 lectures, Lent Term

Principles of Quantum Mechanics is essential.

Variational Principle
Variational principle, examples. [2]

Bound states and scattering states in one dimension
Bound states, reflection and transmission amplitudes. Examples. Relation between bound states and transmission amplitude by analytic continuation. [3]

Scattering theory in three dimensions

Electrons in a magnetic field

Particle in a one-dimensional periodic potential
Discrete translation group, lattice and reciprocal lattice, periodic functions. Bloch’s theorem, Brillouin zone, energy bands and gaps. Floquet matrix, eigenvalues. Band gap in nearly-free electron model, tight-binding approximation. [3]

Crystalline solids

Appropriate books
D.J. Griffiths Introduction to Quantum Mechanics, 2nd edition, Pearson Education 2005
R. Shankar Principles of Quantum Mechanics, Springer 2008

STATISTICAL PHYSICS (D)  
24 lectures, Lent Term

Part III Quantum Mechanics and “Multiparticle Systems” from Part II Principles of Quantum Mechanics are essential.

Fundamentals of statistical mechanics

Classical gases

Quantum gases

Thermodynamics

Phase transitions

Appropriate books
F. Mandl Statistical Physics. Wiley 1988
M. Kardar Statistical Physics of Particles. CUP 2007. (See also course 8.333, MIT OpenCourseWare https://ocw.mit.edu)
F. Reif Fundamentals of Thermal and Statistical Physics. McGraw–Hill 1965
A.B. Pippard Elements of Classical Thermodynamics. Cambridge University Press, 1957
K. Huang Introduction to Statistical Physics. Taylor and Francis 2001
ELECTRODYNAMICS (D) 16 lectures, Michaelmas Term

IB Electromagnetism and IA Dynamics and Relativity are essential. IB Methods is desirable.

Classical Field Theory

Electromagnetic Radiation

Electromagnetism in Media

Appropriate books
A. Zangwill Modern Electrodynamics. CUP 2012
J.D. Jackson Electrodynamics, Wiley 1999

GENERAL RELATIVITY (D) 24 lectures, Lent Term

Part IB Methods and Variational Principles are very useful.

Brief review of Special Relativity
Notion of proper time. Equation of motion for free point particle derivable from a variational principle. Noether’s theorem.

Introduction and motivation for General Relativity

Tensor calculus

Vacuum field equations
Spherically symmetric spacetimes, the Schwarzschild solution. Birkhoff’s theorem "with proof". Rays and orbits, gravitational red-shift, light deflection, perihelion advance. Shapiro time delay.

Einstein Equations coupled to matter

Linearized theory
Linearized form of the vacuum equations. De-Donder gauge and reduction to wave equation. Comparison of linearized point mass solution with exact Schwarzschild solution and identification of the mass parameter. Gravitational waves in linearized theory. "The quadrupole formula for energy radiated." Comparison of linearized gravitational waves with the exact pp-wave metric.

Gravitational collapse and black holes
Non-singular nature of the surface $r = 2M$ in the Schwarzschild solution using Finkelstein and Kruskal coordinates. The idea of an event horizon and the one-way passage of timelike geodesics through it. Qualitative account of idealized spherically symmetric collapse. The final state: statement of Israel’s Theorem. "Qualitative description of Hawking radiation."

Appropriate books
S.M. Carroll Spacetime and Geometry. Addison-Wesley 2004
FLUID DYNAMICS II (D) 24 lectures, Michaelmas Term

Methods and Fluid Dynamics are essential. It is recommended that students attend the associated Laboratory Demonstrations in Fluid Dynamics, which take place in the Michaelmas term.

Governing equations for an incompressible Newtonian fluid
Stress and rate-of-strain tensors and hypothesis of linear relation between them for an isotropic fluid; equation of motion; conditions at a material boundary; dissipation; flux of mass, momentum and energy; the Navier-Stokes equations. Dynamical similarity; steady and unsteady Reynolds numbers. [4]

Unidirectional flows
Couette and Poiseuille flows; the Stokes layer; the Rayleigh problem. [2]

Stokes flows
Flow at low Reynolds number; linearity and reversibility; uniqueness and minimum dissipation theorems. Flow in a corner; force and torque relations for a rigid particle in arbitrary motion; case of a rigid sphere and a spherical bubble. [4]

Flow in a thin layer
Lubrication theory; simple examples; the Héle-Shaw cell; gravitational spreading on a horizontal surface. [3]

Generation and confinement of vorticity
Vorticity equation; vortex stretching; flow along a plane wall with suction; flow toward a stagnation point on a wall; flow in a stretched line vortex. [3]

Boundary layers at high Reynolds number
The Euler limit and the Prandtl limit; the boundary layer equation for two-dimensional flow. Similarity solutions for flow past a flat plate and a wedge. Discussion of the effect of acceleration of the external stream, separation. Boundary layer at a free surface; rise velocity of a spherical bubble. [6]

Stability of unidirectional inviscid flow
Instability of a vortex sheet and of simple jets (e.g. vortex sheet jets). [2]

Appropriate books
G.K. Batchelor An Introduction to Fluid Dynamics. Cambridge University Press 2000

WAVES (D) 24 lectures, Lent Term

Part IB Methods is essential and Part IB Fluid Dynamics is very helpful.

Sound waves
Equations of motion of an inviscid compressible fluid (without discussion of thermodynamics). Mach number. Linear acoustic waves; wave equation; wave-energy equation; plane waves; spherically symmetric waves. [4]

Elastic waves
Momentum balance: stress and infinitesimal strain tensors and hypothesis of a linear relation between them for an isotropic solid. Wave equations for dilatation and rotation. Compressional and shear plane waves; simple problems of reflection and transmission; Rayleigh waves. [5]

Dispersive waves
Rectangular acoustic wave guide; Love waves; cut-off frequency. Representation of a localised initial disturbance by a Fourier integral (one-dimensional case only); modulated wave trains; stationary phase. Group velocity as energy propagation velocity; dispersing wave trains. Water waves; internal gravity waves. [6]

Ray theory
Group velocity from wave-crest kinematics; ray tracing equations. Doppler effect; ship wave pattern. Cases where Fermat’s principle and Snell’s law apply. [4]

Non-linear waves

Appropriate books
J. Billingham and A.C. King Wave Motion: Theory and application. Cambridge University Press 2000
M.J. Lighthill Waves in Fluids. Cambridge University Press 1978
G.B. Whitham Linear and Nonlinear Waves. Wiley 1999
NUMERICAL ANALYSIS (D) 24 lectures, Michaelmas Term

Part IB Numerical Analysis is essential and Analysis and Topology, Linear Algebra and Complex Methods or Complex Analysis are all desirable.

Finite difference methods for the Poisson’s equation
Approximation of $\nabla^2$ by finite differences. The accuracy of the five-point method in a square. Higher order methods. Solution of the difference equations by iterative methods, including multigrid. Fast Fourier transform (FFT) techniques. [5]

Finite difference methods for initial value partial differential equations

Spectral methods

Iterative methods for linear algebraic systems

Computation of eigenvalues and eigenvectors
The power method and inverse iteration. Transformations to tridiagonal and upper Hessenberg forms. The QR algorithm for symmetric and general matrices, including shifts. [3]

Appropriate books

COMPUTATIONAL PROJECTS

The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July or the beginning of August preceding the Part II year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at https://www.maths.cam.ac.uk/undergrad/catam/. Each project is allocated a number of units of credit. Full credit may obtained from the submission of projects with credit totalling 30 units. Credit for submissions totalling less than 30 units is awarded proportionately. There is no restriction on the choice of projects. Once the manual is available, the projects may be done at any time up to the submission deadline, which is near the beginning of the Easter Full Term.

A list of suitable books can be found in the manual