SCHEDULES OF LECTURE COURSES
AND FORM OF EXAMINATIONS
FOR THE MATHEMATICAL TRIPOS 2020/2021
INTRODUCTION

THE MATHEMATICAL TRIPOS 2020–2021

CONTENTS

This booklet is the formal description of the content and structure of Parts IA, IB and II of the Mathematical Tripos. In particular, it contains the schedules, or syllabus specifications, that define each course in the undergraduate Tripos, and it contains detailed information about the structure and marking of examinations, and the classification criteria. In addition, the booklet contains many useful pieces of advice and information for students regarding the Mathematical Tripos. It is updated every year to reflect changes approved by the Faculty Board.

Adjustment for Covid-19

The arrival of the pandemic required all teaching and assessment to be moved online at short notice in Easter Term 2020, and entailed significant changes to the arrangements for the 2020 examination session compared to those described in the Schedules booklet for 2019/20 (for further details see www.maths.cam.ac.uk/undergrad/files/scheduleupdate.pdf). Though it is currently very unclear how the public health situation will evolve through 2020/21, this edition of the Schedules reflects the Faculty’s working assumption that it should again be possible to hold ‘normal’ examinations by June 2021.

The content of the lecture courses for 2020/21 is described by the schedules below. However, the delivery and location of lectures will certainly be affected by health considerations in Michaelmas and probably subsequent terms. Detailed arrangements, and updates on any further changes, will be announced by email to both students and Directors of Studies, and posted under Covid-19 Announcements on the undergraduate course page www.maths.cam.ac.uk/undergrad/undergrad as soon as they are available.

SCHEDULES

Syllabus

The schedule for each lecture course is a list of topics that define the course. The schedule is agreed by the Faculty Board. Some schedules contain topics that are ‘starred’ (listed between asterisks); all the topics must be covered by the lecturer but examiners can only set questions on unstarred topics. The numbers which appear in brackets at the end of subsections or paragraphs in these schedules reflect the Faculty Board. Some schedules contain topics that are ‘starred’ (listed between asterisks); all the topics must be covered by the lecturer but examiners can only set questions on unstarred topics. Lecturers decide upon the amount of time they think appropriate to spend on each topic, and also on the order in which they present topics. There is no requirement for this year’s lectures to match the previous year’s notes. Some topics in Part IA and Part IB courses have to be introduced in a certain order so as to tie in with other courses.

Recommended books

A list of books is given after each schedule. Books marked with † are particularly well suited to the course. Some of the books are out of print; these are retained on the list because they should be available in college libraries (as should all the books on the list) and may be found in second-hand bookshops. There may well be many other suitable books not listed; it is usually worth browsing college libraries. In most cases, the contents of the book will not be exactly the same as the content of the schedule, and different styles suit different people. Hence you are advised to consult library copies in the first instance to decide which, if any, would be of benefit to you. Up-to-date prices, and the availability of hard- and soft-back versions, can most conveniently be checked online.

STUDY SKILLS

The Faculty produces a booklet Study Skills in Mathematics which is distributed to all first year students and can be obtained in pdf format from www.maths.cam.ac.uk/undergrad/studyskills. There is also a booklet, Supervision in Mathematics, that gives guidance to supervisors obtainable from www.maths.cam.ac.uk/facultyoffice/supervisorsguide/ which may also be of interest to students.

Aims and objectives

The aims of the Faculty for Parts IA, IB and II of the Mathematical Tripos are:

- to provide a challenging course in mathematics and its applications for a range of students that includes some of the best in the country;
- to provide a course that is suitable both for students aiming to pursue research and for students going into other careers;
- to provide an integrated system of teaching which can be tailored to the needs of individual students;
- to develop the students’ capacity for learning and for clear logical thinking, and the ability to solve unseen problems;
- to continue to attract and select students of outstanding quality;
- to produce the high calibre graduates in mathematics sought by employers in universities, the professions and the public services;
- to provide an intellectually stimulating environment in which students have the opportunity to develop their skills and enthusiasm to their full potential;
- to maintain the position of Cambridge as a leading centre, nationally and internationally, for teaching and research in mathematics.

The objectives of Parts IA, IB and II of the Mathematical Tripos are as follows:

After completing Part IA, students should have:

- made the transition in learning style and pace from school mathematics to university mathematics;
- been introduced to basic concepts in higher mathematics and their applications, including (i) the notions of proof, rigour and axiomatic development, (ii) the generalisation of familiar mathematics to unfamiliar contexts, (iii) the application of mathematics to problems outside mathematics;
- laid the foundations, in terms of knowledge and understanding, of tools, facts and techniques, to proceed to Part IB.

After completing Part IB, students should have:

- covered material from a range of pure mathematics, statistics and operations research, applied mathematics, theoretical physics and computational mathematics, and studied some of this material in depth;
- acquired a sufficiently broad and deep mathematical knowledge and understanding to enable them both to make an informed choice of courses in Part II and also to study these courses.

After completing Part II, students should have:

- developed the capacity for (i) solving both abstract and concrete unseen problems, (ii) presenting a concise and logical argument, and (iii) (in most cases) using standard software to tackle mathematical problems;
- studied advanced material in the mathematical sciences, some of it in depth.

*This booklet, full name Schedules of Lecture Courses and Form of Examinations for the Mathematical Tripos but often referred to simply as ‘The Schedules’, can be found online at www.maths.cam.ac.uk/undergrad/course/schedules.pdf
EXAMINATIONS

Overview

There are three examinations for the undergraduate Mathematical Tripos: Parts IA, IB and II. They are normally taken in consecutive years. This page contains information that is common to all three examinations. Information that is specific to individual examinations is given later in this booklet in the General Arrangements sections for the appropriate part of the Tripos.

The form of each examination (number of papers, numbers of questions on each lecture course, distribution of questions in the papers and in the sections of each paper, number of questions which may be attempted) is determined by the Faculty Board. The main structure has to be agreed by University committees and is published as a Regulation in the Statutes and Ordinances of the University of Cambridge (http://www.admin.cam.ac.uk/univ/so). (Any significant change to the format is announced in the Reporter as a Form and Conduct notice.) The actual questions and marking schemes, and precise borderlines (following general classification criteria agreed by the Faculty Board — see below) are determined by the examiners.

The examiners for each part of the Tripos are appointed by the General Board of the University. The internal examiners are normally teaching staff of the two mathematics departments and they are joined by one or more external examiners from other universities (one for Part IA, two for Part IB and three for Part II).

For all three parts of the Tripos, the examiners are collectively responsible for the examination questions, though for Part II the questions are proposed by the individual lecturers. All questions have to be signed off by the relevant lecturer; no question can be used unless the lecturer agrees that it is fair and appropriate to the course he or she lectured.

Form of the examination

The examination for each part of the Tripos consists of four written papers and candidates take all four. For Parts IB and II, candidates may in addition submit Computational Projects. Each written paper has two sections: Section I contains questions that are intended to be accessible to any student who has studied the material conscientiously. They should not contain any significant 'problem' element. Section II questions are intended to be more challenging.

Calculators are not allowed in any paper of the Mathematical Tripos; questions will be set in such a way as not to require the use of calculators. The rules for the use of calculators in the Physics paper of the Mathematics-with-Physics option of Part IA are set out in the regulations for the Natural Sciences Tripos.

Formula booklets are not permitted, but candidates will not be required to quote elaborate formulae from memory.

Past papers

Past Tripos papers for the last 8 or more years can be found on the Faculty web site http://www.maths.cam.ac.uk/undergrad/pastpapers/. Some examples of solutions and mark schemes for the 2011 Part IA examination can be found with an explanatory comment at http://www.maths.cam.ac.uk/examples-solutions-part-ia. Otherwise, solutions and mark schemes are not available except in rough draft form for supervisors.

Marking conventions

On the written papers of the Mathematical Tripos, Section I questions are marked out of 10 and Section II questions are marked out of 20. In addition to a numerical mark, extra credit in the form of a quality mark may be awarded for each question depending on the completeness and quality of each answer. For a Section I question, a beta quality mark is awarded for a mark of 7 or more. For a Section II question, an alpha quality mark is awarded for a mark of 15 or more, and a beta quality mark is awarded for a mark between 10 and 14, inclusive.

On each written paper the number of questions for which credit may be obtained is restricted. The relevant restrictions are specified in the introductions to Parts IA, IB and II later in this booklet, and indicated on the examination paper by a rubric such as ‘Candidates may obtain credit from attempts on at most N questions from Section M’.2 If a candidate submits more attempts than are allowed for credit in the rubric then examiners will mark all attempts and the candidate is given credit only for the best attempts consistent with the rubric. This policy is intended to deal with candidates who accidentally attempt too many questions: it is clearly not in candidates’ best interests to spend time tackling extra questions for which they will receive no credit.

The marks available on the Computational Projects courses are described later in this booklet in the introductions to Parts IB and II, and in more detail in the Computational Projects Manuals, which are available at http://www.maths.cam.ac.uk/undergrad/catam/

Examinations are ‘single-marked’, but safety checks are made on all scripts to ensure that all work is marked and that all marks are correctly added and transcribed. Faculty policy is that examiners should make every effort to read poor handwriting (and they almost always succeed), but if an answer, or part of an answer, is indecipherable then it will not be awarded the relevant marks. Exceptions will be made, where appropriate, for candidates with disabilities or diagnosed specific learning difficulties.

Scripts are identified only by candidate number until the final class list has been drawn up. In drawing up the class list, examiners make decisions based only on the work they see: no account is taken of the candidates’ personal situation or of supervision reports.

Following the posting of individual results on CamSIS and the publication of the class list, candidates and their Colleges will be sent a detailed list of the marks gained on each question and each computational project attempted.

Mitigating circumstances

Candidates who are seriously hindered in preparing for, or sitting, their examinations should contact their College Tutor at the earliest possible opportunity. The Tutor will advise on what further action is needed (e.g. securing medical or other evidence) and in cases of illness or other grave cause, the Tutor can make an application on the candidate’s behalf to the University for an Examination Allowance.

Mark Checks and Examination Reviews

All appeals must be made through official channels, and examiners must not be approached directly, either by the candidate or their Director of Studies.

A candidate who thinks that there is an error in their detailed marks should discuss this with their Director of Studies. If there is good reason to believe that an error has occurred, the Director of Studies can contact the Undergraduate Office within 14 days of the detailed marks being released, requesting a mark check and providing details of the reason for the request.

A candidate can also appeal to the University if they believe there is a case for an Examination Review, see https://www.studentcomplaints.admin.cam.ac.uk/examination-reviews. Further information can be obtained from College Tutors and from the exams section http://www.studentadvice.cam.ac.uk/academic/exams of the students’ advice service website.

2 Slightly different wording was used prior to 2017, but the meaning and the policies were the same.
Classification Criteria

As a result of each examination, each candidate is placed in one of the following categories: first class, upper second class (2.1), lower second class (2.2), third class, fail or ‘other’. ‘Other’ here includes, for example, candidates who were ill for all or part of the examination.

The Faculty Board recommends approximate percentages of candidates for each class: 30% firsts; 70–75% upper seconds and above; 90–95% lower seconds and above; and 5–10% thirds and below. These percentages exclude candidates who did not sit the examination.

The Faculty Board expects that the classification criteria described above should result in classes that can be broadly characterized as follows (after allowing for the possibility that in Parts IB and II stronger performance on the Computational Projects may compensate for weaker performance on the written papers or vice versa):

First Class
Candidates placed in the first class will have demonstrated a good command and secure understanding of examinable material. They will have presented standard arguments accurately, showed skill in applying their knowledge, and generally will have produced substantially correct solutions to a significant number of more challenging questions.

Upper Second Class
Candidates placed in the upper second class will have demonstrated good knowledge and understanding of examinable material. They will have presented standard arguments accurately and will have shown some ability to apply their knowledge to solve problems. A fair number of their answers to both examinable material. They will have presented standard arguments accurately, showed skill in applying their knowledge, and generally will have produced substantially correct solutions to a significant number of more challenging questions.

Lower Second Class
Candidates placed in the lower second class will have demonstrated knowledge but sometimes imperfect understanding of examinable material. They will have been aware of relevant mathematical issues. Their presentation of standard arguments will sometimes have been fragmentary or imperfect. They will have produced substantially correct solutions to some straightforward questions, but will have had limited success at tackling more challenging problems.

Third Class
Candidates placed in the third class will have demonstrated some knowledge of the examinable material. They will have made reasonable attempts at a small number of questions, but will not have shown the skills needed to complete many of them.

Examination Data Retention Policy

To meet the University’s obligations under the data protection legislation, the Faculty deals with data relating to individuals and their examination marks as follows:

- All marks for individual questions and computational projects are released routinely to individual candidates and their Colleges after the examinations. The final examination mark book is kept indefinitely by the Undergraduate Office.
- Scripts and Computational Projects submissions are kept, in line with the University policy, for six months following the examinations (in case of appeals). Scripts are then destroyed; and Computational Projects are anonymised and stored in a form that allows comparison (using anti-plagiarism software) with current projects.
- Neither the GDPR nor the Freedom of Information Act entitle candidates to have access to their scripts. Data appearing on individual examination scripts is technically available on application to the University Information Compliance Officer. However, such data consists only of a copy of the examiner’s ticks, crosses, underlines, etc., and the mark subtotals and totals.

Examiners’ reports

For each part of the Tripos, the examiners (internal and external) write a joint report. In addition, the external examiners each submit a report addressed to the Vice-Chancellor. The reports of the external examiners are scrutinised by the General Board of the University’s Education Committee.

All the reports, the examination statistics (number of attempts per question, etc), student feedback on the examinations and lecture courses (via the end of year questionnaire and paper questionnaires), and other relevant material are considered by the Faculty Teaching Committee at the start of the Michaelmas term. The Teaching Committee includes two student representatives, and may include other students (for example, previous members of the Teaching Committee and student representatives of the Faculty Board).

The Teaching Committee compiles a lengthy report on examinations including various recommendations for the Faculty Board to consider at its second meeting in the Michaelmas term. This report also forms the basis of the Faculty Board’s response to the reports of the external examiners. Previous Teaching Committee reports and recent examiners’ comments on questions can be found at http://www.maths.cam.ac.uk/facultyboard/teachingcommittee.
MISCELLANEOUS MATTERS

Numbers of supervisions, example sheets and workload

Directors of Studies will arrange supervisions for each course as they think appropriate, and the primary responsibility for supervisions rests with the colleges.

Lecturers hand out example sheets, which supervisors are generally recommended to use. According to Faculty Board guidelines, the number of example sheets for 24-lecture, 16-lecture and 12-lecture courses should be 4, 3 and 2, respectively, and the content and length of each example sheet should be suitable for discussion (with a typical pair of students) in an hour-long supervision. For a student studying the equivalent of 4 24-lecture courses in each of Michaelmas and Lent Terms, as in Part IA, the 32 example sheets would then be associated with an average of about two supervisions per week, and with revision supervisions in the Easter Term, a norm of about 40 supervisions over the year. Since supervisions on a given course typically begin sometime after the first two weeks of lectures, the fourth supervision of a 24-lecture course is often given at the start of the next term to spread the workload and allow students to catch up.

As described later in this booklet, the structure of Parts IB and II allows considerable flexibility over the selection and number of courses to be studied, which students can use, in consultation with their Directors of Studies, to adjust their workload as appropriate to their interests and to their previous experience in Part IA. Dependent on their course selection, and the corresponding number of example sheets, most students have 35–45 supervisions in Part IB and Part II, with the average across all students being close to 40 supervisions per year.

It is impossible to say how long an example sheet ‘should’ take. If a student is concerned that they are regularly studying for significantly more than 48 hours per week in total then they should seek advice from their Director of Studies.

Transcripts

In order to conform to University guidelines on examinations, the Faculty is obliged to produce, for use in official transcripts, a UMS percentage mark and a rank for each candidate. These are issued via CamSIS, and are calculated from the merit marks as follows.

The merit mark, denoted by \( M \), is defined in terms of raw mark \( m \), number of alphas, \( \alpha \), and number of betas, \( \beta \), by:

\[
M = \begin{cases} 
30a + 5.3 + m - 120 & \text{for candidates in the first class, or in the upper second class with } \alpha \geq 8, \\
15a + 5.3 + m & \text{otherwise}
\end{cases}
\]

The UMS percentage mark is obtained by piecewise linear scaling of the merit marks within each class. The 1/2.1, 2.1/2.2, 2.2/3 and 3/fail boundaries are mapped to 69.5%, 59.5%, 49.5% and 39.5% respectively and the merit mark of the 5th ranked candidate is mapped to 95%. If, after linear mapping of the first class, the percentage mark of any candidate is greater than 100, it is reduced to 100%. The percentage of each candidate is then rounded appropriately to integer values. The rank of the candidate is determined by merit-mark order within each class and the cohort size is the number of students who sat the examination.

Faculty Committees

The Faculty has two committees which deal with matters relating to the undergraduate Tripos: the Teaching Committee (http://www.maths.cam.ac.uk/facultyboard/teachingcommittee/) and the Curriculum Committee (http://www.maths.cam.ac.uk/facultyboard/curriculumcommittee/). Both have student representatives.

The role of the Teaching Committee is mainly to monitor feedback (questionnaires, examiners’ reports, etc.) and make recommendations to the Faculty Board on the basis of this feedback. It also formulates policy recommendations at the request of the Faculty Board.

The Curriculum Committee is responsible for recommending (to the Faculty Board) changes to the undergraduate Triposes and to the schedules for individual lecture courses.

Student representatives

There are three student representatives, two undergraduate and one graduate, on the Faculty Board, and two representatives on each of the Teaching Committee and the Curriculum Committee. They are normally elected (in the case of the Faculty Board representatives) or appointed in November of each year. Their role is to advise the committees on the student point of view, to collect opinion from and liaise with the student body. They operate a website: http://www.maths.cam.ac.uk/studentreps and their email address is student.reps@maths.cam.ac.uk.

Feedback

Constructive feedback of all sorts and from all sources is welcomed by everyone concerned in providing courses for the Mathematical Tripos.

There are many different feedback routes.

- Each lecturer hands out a paper questionnaire towards the end of the course.
- There are brief web-based questionnaires after roughly six lectures of each course.
- Students are sent a combined online questionnaire at the end of each year.
- Students (or supervisors) can e-mail feedback@maths.cam.ac.uk at any time. Such e-mails are received by the Director of Undergraduate Education and the Chair of the Teaching Committee, who will either deal with your comment, or pass your e-mail (after stripping out any clue to your identity) to the relevant person (a lecturer, for example). Students will receive a rapid response.
- If a student wishes to be entirely anonymous and does not want any response, the web-based comment form at www.maths.cam.ac.uk/feedback.html can be used. (Anonymity also means we can’t ask you for clarifying information to help us deal with the comment.)
- Feedback on college-provided teaching (supervisions, classes) can be given to Directors of Studies or Tutors at any time.

The questionnaires are particularly important in shaping the future of the Tripos and the Faculty Board urges all students to respond.

Formal complaints

The formal complaints procedure to be followed within the University can be found at http://www.studentcomplaints.admin.cam.ac.uk/student-complaints. The Responsible Officer in Step 1 of this procedure for the Faculty of Mathematics is the Chair of the Faculty Board — see http://www.maths.cam.ac.uk/facultyboard for the name of the current Chair.
PART IA

GENERAL ARRANGEMENTS

Structure of Part IA

There are two options:

(a) Pure and Applied Mathematics;
(b) Mathematics with Physics.

Option (a) is intended primarily for students who expect to continue to Part IB of the Mathematical Tripos, while Option (b) is intended primarily for those who are undecided about whether they will continue to Part IB of the Mathematical Tripos or change to Part IB of the Natural Sciences Tripos (Physics option).

For Option (b), two of the lecture courses (Numbers and Sets, and Dynamics and Relativity) are replaced by the complete Physics course from Part IA of the Natural Sciences Tripos; Numbers and Sets because it is the least relevant to students taking this option, and Dynamics and Relativity because much of this material is covered in the Natural Sciences Tripos anyway. Students wishing to examine the schedules for the physics courses should consult the documentation supplied by the Physics department, for example on http://www.phy.cam.ac.uk/teaching/.

Examinations

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

All candidates for Part IA of the Mathematical Tripos take four papers, as follows:

Candidates taking Option (a) (Pure and Applied Mathematics) will take Papers 1, 2, 3 and 4 of the Mathematical Tripos (Part IA).

Candidates taking Option (b) (Mathematics with Physics) take Papers 1, 2 and 3 of the Mathematical Tripos (Part IA) and the Physics paper of the Natural Sciences Tripos (Part IA); they must also submit practical notebooks.

For Mathematics-with-Physics candidates, the marks for the Physics paper are scaled to bring them in line with Paper 4. This is done as follows. The Physics scripts of the Mathematics-with-Physics candidates are marked by the Natural Sciences examiners for Part IA Physics, and a mark for each candidate is given to the Mathematics examiners. Class boundaries for the Physics paper are determined such that the percentages in each class (1, 2.1, 2.2, 3) of all candidates on the Physics paper (including those from NST and CST) are 25, 35, 30, 10 (which are the guidelines across Natural Sciences). All candidates for Paper 4 (ranked by merit mark on that paper) are assigned nominally to classes so that the percentages in each class are 30, 40, 20, 10 (which is the Faculty Board rough guideline for the initial proportion in each class prior to the final overall classification). Piecewise linear mapping of the percentages in each class are 30, 40, 20, 10 (which is the Faculty Board rough guideline for the initial proportion in each class prior to the final overall classification). Piecewise linear mapping of the percentages in each class are 30, 40, 20, 10 (which is the Faculty Board rough guideline for the initial proportion in each class prior to the final overall classification). Piecewise linear mapping of the percentages in each class are 30, 40, 20, 10 (which is the Faculty Board rough guideline for the initial proportion in each class prior to the final overall classification).

The following tables, based on information supplied by the examiners, show approximate borderlines. For convenience, we define $M_1$ and $M_2$ by

$$M_1 = 30\alpha + 5\beta + m - 120, \quad M_2 = 15\alpha + 5\beta + m.$$ 

$M_1$ is related to the primary classification criterion for the first class and $M_2$ is related to the primary classification criterion for the upper and lower second classes.

The second column of each table shows a sufficient criterion for each class. The third and fourth columns show $M_1$ (for the first class) or $M_2$ (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline. The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

### Examination Papers

Papers 1, 2, 3 and 4 of Part IA of the Mathematical Tripos are each divided into two Sections. There are four questions in Section I and eight questions in Section II. Candidates may obtain credit for attempts on all the questions in Section I and at most five questions from Section II, of which no more than three may be on the same lecture course.

Each section of each Papers 1–4 is divided equally between two courses as follows:

- **Paper 1**: Vectors and Matrices, Analysis I
- **Paper 2**: Differential Equations, Probability
- **Paper 3**: Groups, Vector Calculus
- **Paper 4**: Numbers and Sets, Dynamics and Relativity.

### Approximate class boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines.

#### Part IA 2018

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 474$</td>
<td>475/275, 10, 4</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 316$</td>
<td>317/217, 2.14</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 248$</td>
<td>249/189, 1, 9</td>
</tr>
<tr>
<td>3</td>
<td>$2\alpha + \beta &gt; 9$ or $m &gt; 166$</td>
<td>250/180, 1, 9</td>
</tr>
</tbody>
</table>

#### Part IA 2019

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 635$</td>
<td>637/432, 12, 9</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 410$</td>
<td>411/251, 8, 8</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 275$</td>
<td>279/179, 4, 8</td>
</tr>
<tr>
<td>3</td>
<td>$2\alpha + \beta &gt; 9$ or $m &gt; 180$</td>
<td>286/211, 2, 9</td>
</tr>
</tbody>
</table>
GROUPS

24 lectures, Michaelmas Term

Examples of groups
Axions for groups. Examples from geometry: symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group, cross-ratios, preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius maps and iteration. [4]

Lagrange’s theorem
Coeets. Lagrange’s theorem. Groups of small order (up to order 8). Quaternions. Fermat-Euler theorem from the group-theoretic point of view. [5]

Group actions
Group actions; orbits and stabilizers. Orbit-stabilizer theorem. Cayley’s theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes. Cauchy’s theorem. [4]

Quotient groups
Normal subgroups, quotient groups and the isomorphism theorem. [4]

Matrix groups
The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups. Proof (in \( \mathbb{R}^3 \)) that every element of the orthogonal group is the product of reflections and every rotation in \( \mathbb{R}^3 \) has an axis. Basis change as an example of conjugation. [5]

Permutations
Permutations, cycles and transpositions. The sign of a permutation. Conjugacy in \( S_n \) and in \( A_n \). Simple groups; simplicity of \( A_5 \). [4]

Appropriate books
M.A. Armstrong Groups and Symmetry. Springer–Verlag 1988
R.P. Burn Groups, a Path to Geometry. Cambridge University Press 1987
W. Lederman Introduction to Group Theory. Longman 1976
Nathan Carter Visual Group Theory. Mathematical Association of America Textbooks

VECTORS AND MATRICES

24 lectures, Michaelmas Term

Complex numbers
Review of complex numbers, including complex conjugate, inverse, modulus, argument and Argand diagram. Informal treatment of complex logarithm, \( n \)-th roots and complex powers. de Moivre’s theorem. [2]

Vectors
Review of elementary algebra of vectors in \( \mathbb{R}^3 \), including scalar product. Brief discussion of vectors in \( \mathbb{R}^n \) and \( \mathbb{C}^n \); scalar product and the Cauchy–Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension. Suffix notation: including summation convention, \( \delta_{ij} \) and \( \epsilon_{ijk} \). Vector product and triple product: definition and geometrical interpretation. Solution of linear vector equations. Applications of vectors to geometry, including equations of lines, planes and spheres. [5]

Matrices
Elementary algebra of \( 3 \times 3 \) matrices, including determinants. Extension to \( n \times n \) complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears; kernel and image, rank–nullity theorem (statement only). [4]

Simultaneous linear equations: matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination. [3]

Symmetric, anti-symmetric, orthogonal, hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts. [1]

Eigenvalues and Eigenvectors
Eigenvalues and eigenvectors; geometric significance. [2]

Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general change of basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for \( 2 \times 2 \) matrices. [5]

Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms. [1]

Rotation matrices and Lorentz transformations as transformation groups. [1]

Appropriate books
Alan F Beardon Algebra and Geometry. CUP 2005
NUMBERS AND SETS 24 lectures, Michaelmas Term

Introduction to number systems and logic
Overview of the natural numbers, integers, real numbers, rational and irrational numbers, algebraic and transcendental numbers. Brief discussion of complex numbers; statement of the Fundamental Theorem of Algebra.

Idea of axiomatic systems and proof within mathematics; the need for proof; the role of counterexamples in mathematics. Elementary logic; implication and negation; examples of negation of compound statements. Proof by contradiction. [2]

Sets, relations and functions
Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities. Functions; injections, surjections and bijections. Relations, and equivalence relations. Counting the combinations or permutations of a set. The Inclusion-Exclusion Principle. [4]

The integers
The natural numbers: mathematical induction and the well-ordering principle. Examples, including the Binomial Theorem. [2]

Elementary number theory
Prime numbers: existence and uniqueness of prime factorisation into primes; highest common factors and least common multiples. Euclid’s proof of the infinity of primes. Euclid’s algorithm. Solution in integers of $ax + by = c$.

Modular arithmetic (congruences). Units modulo n. Chinese Remainder Theorem. Wilson’s Theorem; in the form $p! + 1 = a^n$. [8]

The real numbers

Countability and uncountability
Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of $\mathbb{R}$. Non-existence of a bijection from a set to its power set. Indirect proof of existence of transcendental numbers. [4]

DIFFERENTIAL EQUATIONS 24 lectures, Michaelmas Term

Basic calculus
Informal treatment of differentiation as a limit, the chain rule, Leibnitz’s rule, Taylor series, informal treatment of $O$ and $o$ notation and l’Hôpital’s rule; integration as an area, fundamental theorem of calculus, integration by substitution and parts.

Informal treatment of partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule, implicit differentiation. Informal treatment of differentials, including exact differentials. Differentiation of an integral with respect to a parameter. [4]

First-order linear differential equations
Equations with constant coefficients: exponential growth, comparison with discrete equations, series solution; modelling examples including radioactive decay.

Equations with non-constant coefficients: solution by integrating factor. [2]

Nonlinear first-order equations

Higher-order linear differential equations
Complementary function and particular integral, linear independence, Wronskian (for second-order equations), Abel’s theorem. Equations with constant coefficients and examples including radioactive sequences, comparison in simple cases with difference equations, reduction of order, resonance, transients, damping. Homogeneous equations. Response to step and impulse function inputs; introduction to the notions of the Heaviside step-function and the Dirac delta-function. Series solutions including statement only of the need for the logarithmic solution. [8]

Multivariate functions: applications
Directional derivatives and the gradient vector. Statement of Taylor series for functions on $\mathbb{R}^n$. Local extrema of real functions, classification using the Hessian matrix. Coupled first order systems: equivalence to single higher order equations; solution by matrix methods. Non-degenerate phase portraits local to equilibrium points; stability.

Simple examples of first- and second-order partial differential equations, solution of the wave equation in the form $f(x + ct) + g(x - ct)$. [5]

Appropriate books
R.P. Burn Numbers and Functions: steps into analysis. Cambridge University Press 2000
H. Davenport The Higher Arithmetic. Cambridge University Press 1999

Appropriate books
J. Robinson An introduction to Differential Equations. Cambridge University Press, 2004
PART IA

ANALYSIS I  24 lectures, Lent Term

Limits and convergence

Continuity
Continuity of real- and complex-valued functions defined on subsets of $\mathbb{R}$ and $\mathbb{C}$. The intermediate value theorem. A continuous function on a closed bounded interval is bounded and attains its bounds. [3]

Differentiability
Differentiability of functions from $\mathbb{R}$ to $\mathbb{R}$. Derivative of sums and products. The chain rule. Derivative of the inverse function. Rolle’s theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor’s theorem from $\mathbb{R}$ to $\mathbb{R}$; Lagrange’s form of the remainder. Complex differentiation. [5]

Power series
Complex power series and radius of convergence. Exponential, trigonometric and hyperbolic functions, and relations between them. *Direct proof of the differentiability of a power series within its circle of convergence*. [4]

Integration

Appropriate books
† J.C. Burkill A First Course in Mathematical Analysis. Cambridge University Press 1978
D.J.H. Garling A Course in Mathematical Analysis (Vol 1). Cambridge University Press 2013
J.B. Reade Introduction to Mathematical Analysis. Oxford University Press
M. Spivak Calculus. Addison-Wesley/Benjamin-Cummings 2006
David M. Bressoud A Radical Approach to Real Analysis. Mathematical Association of America Textbooks

PROBABILITY  24 lectures, Lent Term

Basic concepts
Classical probability, equally likely outcomes. Combinatorial analysis, permutations and combinations. Stirling’s formula (asymptotics for log $n$! proved). [3]

Axiomatic approach

Discrete random variables

Continuous random variables

Inequalities and limits

Appropriate books
† S. Ross A First Course in Probability. Prentice Hall 2009
VECTOR CALCULUS 24 lectures, Lent Term

Curves in \( \mathbb{R}^3 \)
Parameterised curves and arc length, tangents and normals to curves in \( \mathbb{R}^3 \); curvature and torsion.

Integration in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \)
Line integrals. Surface and volume integrals: definitions, examples using Cartesian, cylindrical and spherical coordinates; change of variables.

Vector operators
Directional derivatives. The gradient of a real-valued function: definition; interpretation as normal to level surfaces; examples including the use of cylindrical, spherical and general orthogonal curvilinear coordinates.

Divergence, curl and \( \nabla \times \nabla \) in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical and general orthogonal curvilinear coordinates. Solenoidal fields, irrotational fields and conservative fields; scalar potentials. Vector derivative identities.

Integration theorems
Divergence theorem, Green’s theorem, Stokes’s theorem, Green’s second theorem: statements; informal proofs; examples; application to fluid dynamics, and to electromagnetism including statement of Maxwell’s equations.

Laplace’s equation
Laplace’s equation in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \): uniqueness theorem and maximum principle. Solution of Poisson’s equation by Gauss’s method (for spherical and cylindrical symmetry) and as an integral.

Cartesian tensors in \( \mathbb{R}^3 \)
Tensor transformation laws, addition, multiplication, contraction, with emphasis on tensors of second rank. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Quotient theorem. Examples including inertia and conductivity.

Appropriate books

DYNAMICS AND RELATIVITY 24 lectures, Lent Term

[Note that this course is omitted from Option (b) of Part IA.]

Basic concepts
Space and time, frames of reference, Galilean transformations. Newton’s laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz.

Newtonian dynamics of a single particle
Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping.

Angular velocity, angular momentum, torque.

Orbits: the \( u(\theta) \) equation; escape velocity; Kepler’s laws; stability of orbits; motion in a repulsive potential (Rutherford scattering).

Rotating frames: centrifugal and Coriolis forces. *Brief discussion of Fourcalt pendulum.*

Newtonian dynamics of systems of particles
Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation.

Rigid bodies
Moments of inertia, angular momentum and energy of a rigid body. Parallel axis theorem. Simple examples of motion involving both rotation and translation (e.g. rolling).

Special relativity
The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in \( (1 + 1) \)-dimensional spacetime. Time dilation and length contraction. The Minkowski metric for \( (1 + 1) \)-dimensional spacetime.

Lorentz transformations in \( (3 + 1) \) dimensions. 4-vectors and Lorentz invariants. Proper time. 4-velocity and 4-momentum. Conservation of 4-momentum in particle decay. Collisions. The Newtonian limit.

Appropriate books
**COMPUTATIONAL PROJECTS**

The Computational Projects course is examined in Part IB. However, introductory practical sessions are offered at the end of Lent Full Term and the beginning of Easter Full Term of the Part IA year (students are advised by email how to register for a session), and lectures are given in the Easter Full Term of the Part IA year. The lectures cover an introduction to algorithms and aspects of the MATLAB programming language. The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July at the end of the Part IA year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at http://www.maths.cam.ac.uk/undergrad/catam/. Full credit may be obtained from the submission of the two core projects and a further two additional projects. Once the manual is available, these projects may be undertaken at any time up to the submission deadlines, which are near the start of the Full Lent Term in the IB year for the two core projects, and near the start of the Full Easter Term in the IB year for the two additional projects.

A list of suitable books can be found in the manual.

**MECHANICS (non-examinable)**

This course is intended for students who have taken only a limited amount of Mechanics at A-level (or the equivalent). The material is prerequisite for Dynamics and Relativity in the Lent Term.

**Lecture 1**

Brief introduction

**Lecture 2: Kinematics of a single particle**


**Lecture 3: Equilibrium of a single particle**

The vector nature of forces, addition of forces, examples including gravity, tension in a string, normal reaction (Newton’s third law), friction. Conditions for equilibrium.

**Lecture 4: Equilibrium of a rigid body**

Resultant of several forces, couple, moment of a force. Conditions for equilibrium.

**Lecture 5: Dynamics of particles**

Newton’s second law. Examples of pulleys, motion on an inclined plane.

**Lecture 6: Dynamics of particles**

Further examples, including motion of a projectile with air-resistance.

**Lecture 7: Energy**

Definition of energy and work. Kinetic energy, potential energy of a particle in a uniform gravitational field. Conservation of energy.

**Lecture 8: Momentum**

Definition of momentum (as a vector), conservation of momentum, collisions, coefficient of restitution, impulse.

**Lecture 9: Springs, strings and SHM**

Force exerted by elastic springs and strings (Hooke’s law). Oscillations of a particle attached to a spring, and of a particle hanging on a string. Simple harmonic motion of a particle for small displacement from equilibrium.

**Lecture 10: Motion in a circle**

Derivation of the central acceleration of a particle constrained to move on a circle. Simple pendulum; motion of a particle sliding on a cylinder.

**Appropriate books**

J. Hebborn and J. Littlewood *Mechanics 1, Mechanics 2 and Mechanics 3 (Edexcel)*, Heinemann, 2000


Anything similar to the above, for the other A-level examination boards.
Part IB

GENERAL ARRANGEMENTS

Structure of Part IB

Sixteen courses, including Computational Projects, are examined in Part IB. The schedules for Complex Analysis and Complex Methods cover much of the same material, but from different points of view: students may attend either (or both) sets of lectures. One course, Optimisation, can be taken in the Easter term of either the first year or the second year. Another course, Variational Principles, can also be taken in either Easter term, but is normally taken in the first year as the material forms a good background for a number of courses in Part IB.

Students are not expected to take all the courses in Part IB, and the structure of the Part IB examination papers allows for considerable flexibility. The Faculty Board guidance regarding choice of courses in Part IB is as follows:

Part IB of the Mathematical Tripos provides a wide range of courses from which students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred workload, bearing in mind that it is better to do a smaller number of courses thoroughly than to do many courses scrappily.

Students might sensibly assess their workload by comparing the numbers of example sheets and lectures they are taking on per term with their previous experience in Part IA (see page 4 of this booklet). The table of dependencies on the next page may also help them with their choice.

Computational Projects

The lectures for Computational Projects will normally be attended in the Easter term of the first year, the Computational Projects themselves being done in the Michaelmas and Lent terms of the second year (or in the summer, Christmas and Easter vacations).

No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of reports. The maximum credit obtainable is 160 marks and there are no alpha or beta quality marks. Credit obtained is added directly to the credit gained on the written papers. The maximum contribution to the final merit mark is thus 160, which is roughly the same (averaging over the alpha weightings) as for a 16-lecture course. The Computational Projects are considered to be a single piece of work within the Mathematical Tripos.

Examinations

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

Each of the four papers is divided into two sections. Candidates may obtain credit for attempts on at most four questions from Section I and at most six questions from Section II.

The number of questions set on each course varies according to the number of lectures given, as shown:

<table>
<thead>
<tr>
<th>Number of lectures</th>
<th>Section I</th>
<th>Section II</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Important note: The number of Section I questions on a 24-lecture course was changed after 2018/19, and the distribution of questions among the papers has also changed.

Examination Papers

Questions on the different courses are distributed among the papers as specified in the following table. The letters S and L appearing in the table denote a question in Section I and a question in Section II, respectively.

<table>
<thead>
<tr>
<th>Course</th>
<th>Paper 1</th>
<th>Paper 2</th>
<th>Paper 3</th>
<th>Paper 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear Algebra</td>
<td>L+S</td>
<td>L</td>
<td>L+L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Groups, Rings and Modules</td>
<td>L</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Analysis and Topology</td>
<td>L</td>
<td>L+S</td>
<td>L</td>
<td>L+S</td>
</tr>
<tr>
<td>Geometry</td>
<td>L+S</td>
<td>L</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Complex Analysis</td>
<td>L+S*</td>
<td>L*</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Complex Methods</td>
<td>L+S*</td>
<td>L*</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Variational Principles</td>
<td>S</td>
<td>S</td>
<td>S</td>
<td>S</td>
</tr>
<tr>
<td>Methods</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Quantum Mechanics</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Electromagnetism</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Fluid Dynamics</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Numerical Analysis</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Statistics</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Markov Chains</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
<tr>
<td>Optimisation</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
<td>L+S</td>
</tr>
</tbody>
</table>

*On Paper 1 and Paper 2, Complex Analysis and Complex Methods are examined by means of common questions (each of which may contain two sub-questions, one on each course, of which candidates may attempt only one (‘either/or’)).
Approximate class boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines in recent years.

For convenience, we define $M_1$ and $M_2$ by

$$M_1 = 30\alpha + 5\beta + m - 120,$$
$$M_2 = 15\alpha + 5\beta + m.$$

$M_1$ is related to the primary classification criterion for the first class and $M_2$ is related to the primary classification criterion for the upper and lower second and third classes.

The second column of each table shows a sufficient criterion for each class (in terms of $M_1$ for the first class and $M_2$ for the other classes). The third and fourth columns show $M_1$ (for the first class) or $M_2$ (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline.

The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

### Part IB 2018

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 721$</td>
<td>722/452,11,12</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 505$</td>
<td>506/386,5,9</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 470$</td>
<td>474/329,7,6</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 200$</td>
<td>201/166,1,4</td>
</tr>
</tbody>
</table>

### Part IB 2019

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 753$</td>
<td>754/449,13,7</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 492$</td>
<td>493/358,6,9</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 330$</td>
<td>336/286,0,10</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 200$</td>
<td>219/189,0,6</td>
</tr>
</tbody>
</table>

Part II dependencies

The relationships between Part IB courses and Part II courses are shown in the following tables. A blank in the table means that the material in the Part IB course is not directly relevant to the Part II course.

The terminology is as follows:

**Essential:** (E) a good understanding of the methods and results of the Part IB course is essential;

**Desirable:** (D) knowledge of some of the results of the Part IB course is required;

**Background:** (B) some knowledge of the Part IB course would provide a useful background.

### Linear Algebra

- Number Theory
- Topics in Analysis
- Coding and Cryptography
- Automata and Form. Lang.
- Statistical Modelling
- Mathematical Biology
- Further Complex Methods
- Classical Dynamics
- Cosmology
- Quantum Inf. and Comp.
- Logic and Set Theory
- Graph Theory
- Galois Theory
- Representation Theory
- Number Fields
- Algebraic Topology
- Linear Analysis
- Analysis of Functions
- Riemann Surfaces
- Algebraic Geometry
- Differential Geometry
- Prob. and Measure
- Applied Prob.
- Princ. of Stats
- Stochastic FM's
- Maths. Machine Learning
- Asymptotic Methods
- Dynamical Systems
- Integrable Systems
- Principles of QM
- Applications of QM
- Statistical Physics
- Electrodynamics
- General Relativity
- Fluid Dynamics II
- Waves
- Numerical Analysis
LINEAR ALGEBRA 24 lectures, Michaelmas Term

Definition of a vector space (over $\mathbb{R}$ or $\mathbb{C}$), subspaces, the space spanned by a subset. Linear independence, bases, dimension. Direct sums and complementary subspaces. Quotient spaces. Linear maps, isomorphisms. Relation between rank and nullity. The space of linear maps from $U$ to $V$, representation by matrices. Change of basis. Row rank and column rank.


Dual of a finite-dimensional vector space, dual bases and maps. Matrix representation, rank and determinant of dual map.


Appropriate books

P.R. Halmos Finite-dimensional vector spaces. Springer 1974

GROUPS, RINGS AND MODULES 24 lectures, Lent Term

Groups


Rings


Modules

Definitions, examples of vector spaces, abelian groups and vector spaces with an endomorphism. Submodules, homomorphisms, quotient modules and direct sums. Equivalence of matrices, canonical form. Structure of finitely generated modules over Euclidean domains, applications to abelian groups and Jordan normal form.

Appropriate books

P.M. Cohn Classic Algebra. Wiley, 2000
P.J. Cameron Introduction to Algebra. OUP
I. Herstein Topics in Algebra. John Wiley and Sons, 1975
P.M. Neumann, G.A. Stoy and E.C. Thomson Groups and Geometry. OUP 1994
Uniform convergence and uniform continuity
The general principle of uniform convergence. A uniform limit of continuous functions is continuous. Uniform convergence and term-wise integration and differentiation of series of real-valued functions. Local uniform convergence of power series. Uniform continuity. Continuous functions on closed bounded intervals are uniformly continuous; Riemann integrability of continuous functions from Analysis I revisited. [3]

Metric spaces

Topological spaces

Connectedness
Definition using open sets and integer-valued functions. Examples. Components. The continuous image of a connected space is connected. Path-connectedness and its relation to connectedness. Connected open sets in Euclidean space are path-connected. Connectedness of products. [3]

Compactness
Definition using open covers. Examples including [0, 1]. Closed subsets of compact spaces are compact. Compact subsets of Hausdorff spaces are closed. The compact subsets of the real line. Continuous images of compact sets are compact. Continuous real-valued functions on a compact space are bounded and attain their bounds. Compactness of products. Compactness of quotient spaces. Sequential compactness. [3]

Differentiation from \( \mathbb{R}^m \) to \( \mathbb{R}^n \)

Appropriate books
1. J.C. Burkill and H. Burkill *A Second Course in Mathematical Analysis*. Cambridge University Press 2002
4. B. Mendelson *Introduction to Topology*. Dover, 1990
COMPLEX ANALYSIS  
16 lectures, Lent Term

Analytic functions
Complex differentiation and the Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of log $z$ and $z^n$. [3]

Contour integration and Cauchy’s theorem

Expansions and singularities

The residue theorem

Appropriate books
A.F. Beardon *Complex Analysis*. Wiley
H.A. Priestley *Introduction to Complex Analysis*. Oxford University Press 2003
I. Stewart and D. Tall *Complex Analysis*. Cambridge University Press 1983

COMPLEX METHODS  
16 lectures, Lent Term

Analytic functions

Contour integration and Cauchy’s Theorem
[Proofs of theorems in this section will not be examined in this course.]

Residue calculus

Fourier and Laplace transforms
Laplace transform: definition and basic properties; inversion theorem (proof not required); convolution theorem. Examples of inversion of Fourier and Laplace transforms by contour integration. Applications to differential equations. [3]

Appropriate books
Stationary points for functions on $\mathbb{R}^n$. Necessary and sufficient conditions for minima and maxima. Importance of convexity. Variational problems with constraints; method of Lagrange multipliers. The Legendre Transform; need for convexity to ensure invertibility; illustrations from thermodynamics. [4]

The idea of a functional and a functional derivative. First variation for functionals, Euler-Lagrange equations, for both ordinary and partial differential equations. Use of Lagrange multipliers and multiplier functions. [3]

Fermat’s principle; geodesics; least action principles, Lagrange’s and Hamilton’s equations for particles and fields. Noether theorems and first integrals, including two forms of Noether’s theorem for ordinary differential equations (energy and momentum, for example). Interpretation in terms of conservation laws. [3]

Second variation for functionals; associated eigenvalue problem. [2]

**VARIATIONAL PRINCIPLES** 12 lectures, Easter Term

**METHODS** 24 lectures, Michaelmas Term

**Self-adjoint ODEs**

Periodic functions, Fourier series: definition and simple properties; Parseval’s theorem. Equations of second order. Self-adjoint differential operators. The Sturm–Liouville equation; eigenfunctions and eigenvalues; reality of eigenvalues and orthogonality of eigenfunctions; eigenfunction expansions (Fourier series as prototype), approximation in mean square, statement of completeness. [5]

**PDEs on bounded domains: separation of variables**

Physical basis of Laplace’s equation, the wave equation and the diffusion equation. General method of separation of variables in Cartesian, cylindrical and spherical coordinates. Legendre’s equation: derivation, solutions including explicit forms of $P_0$, $P_1$ and $P_2$, orthogonality. Bessel’s equation of integer order as an example of a self-adjoint eigenvalue problem with non-trivial weight. Examples including potentials on rectangular and circular domains and on a spherical domain (axisymmetric case only), waves on a finite string and heat flow down a semi-infinite rod. [6]

**Inhomogeneous ODEs: Green’s functions**

Properties of the Dirac delta function. Initial value problems and forced problems with two fixed end points; solution using Green’s functions. Eigenfunction expansions of the delta function and Green’s functions. [3]

**Fourier transforms**

Fourier transforms: definition and simple properties; inversion and convolution theorems. The discrete Fourier transform. Examples of application to linear systems. Relationship of transfer function to Green’s function for initial value problems. [4]

**PDEs on unbounded domains**

Classification of PDEs in two independent variables. Well posedness. Solution by the method of characteristics. Green’s functions for PDEs in 1, 2 and 3 independent variables; fundamental solutions of the wave equation, Laplace’s equation and the diffusion equation. The method of images. Application to the forced wave equation, Poisson’s equation and forced diffusion equation. Transient solutions of diffusion problems: the error function. [6]

**Appropriate books**

D.S. Lemons *Perfect Form*. Princeton University Press 1997

C. Lanczos *The Variational Principles of Mechanics*. Dover 1986

R. Weinstock *Calculus of Variations with applications to physics and engineering*. Dover 1974

I.M. Gel’fand and S.V. Fomin *Calculus of Variations*. Dover 2000

W. Yourgrau and S. Mandelstam *Variational Principles in Dynamics and Quantum Theory*. Dover 2007

S. Hildebrandt and A. Tromba *Mathematics and Optimal Form*. Scientific American Library 1985


Erwin Kreyszig *Advanced Engineering Mathematics*. Wiley
QUANTUM MECHANICS 16 lectures, Michaelmas Term

Physical background
Photoelectric effect. Electrons in atoms and line spectra. Particle diffraction. [1]

Schrödinger equation and solutions

Observables and expectation values
Position and momentum operators and expectation values. Canonical commutation relations. Uncertainty principle. [2]
Observables and Hermitian operators. Eigenvalues and eigenfunctions. Formula for expectation value. [2]

Hydrogen atom

Appropriate books
Feynman, Leighton and Sands vol. 3 Ch 1-3 of the Feynman lectures on Physics. Addison-Wesley 1979
S. Gasiorowicz Quantum Physics. Wiley 2003
A.M. Rae Quantum Mechanics. Institute of Physics Publishing 2002

ELECTROMAGNETISM 16 lectures, Lent Term

Electrostatics

Magnetostatics

Electrodynamics

Electromagnetism and relativity
Review of special relativity; tensors and index notation. Charge conservation. 4-vector potential, gauge transformations. Electromagnetic tensor. Lorentz transformations of electric and magnetic fields. Maxwell’s equations in relativistic form. Lorentz force law. [5]

Appropriate books
D.J. Griffiths Introduction to Electrodynamics. Pearson 2013
E.M. Purcell and D.J. Morin Electricity and Magnetism. Cambridge University Press 2013
A. Zangwill Modern Electromagnetism. Cambridge University Press 2012
J.D. Jackson Classical Electrodynamics. Wiley 1975
P. Lorrain and D. Corson Electromagnetism, Principles and Applications. Freeman 1990
FLUID DYNAMICS 16 lectures, Lent Term

Parallel viscous flow
Plane Couette flow, dynamic viscosity. Momentum equation and boundary conditions. Steady flows including Poiseuille flow in a channel. Unsteady flows, kinematic viscosity, brief description of viscous boundary layers (skin depth).

Kinematics
Material time derivative. Conservation of mass and the kinematic boundary condition. Incompressibility; streamfunction for two-dimensional flow. Streamlines and path lines.

Dynamics
Statement of Navier-Stokes momentum equation. Reynolds number. Stagnation-point flow; discussion of viscous boundary layer and pressure field. Conservation of momentum; Euler momentum equation. Bernoulli’s equation. Vorticity, vorticity equation, vortex line stretching, irrotational flow remains irrotational.

Potential flows
Velocity potential; Laplace’s equation, examples of solutions in spherical and cylindrical geometry by separation of variables. Translating sphere. Lift on a cylinder with circulation. Expression for pressure in time-dependent potential flows with potential forces. Oscillations in a manometer and of a bubble.

Geophysical flows
Linear water waves: dispersion relation, deep and shallow water, standing waves in a container, Rayleigh-Taylor instability, Euler equations in a rotating frame. Steady geostrophic flow, pressure as streamfunction. Motion in a shallow layer, hydrostatic assumption, modified continuity equation. Conservation of potential vorticity, Rossby radius of deformation.

Appropriate books
† D.J. Acheson Elementary Fluid Dynamics. Oxford University Press 1990
G.K. Batchelor An Introduction to Fluid Dynamics. Cambridge University Press 2000
M. van Dyke An Album of Fluid Motion. Parabolic Press

NUMERICAL ANALYSIS 16 lectures, Lent Term

Polynomial approximation

Computation of ordinary differential equations
Euler’s method and proof of convergence. Multistep methods, including order, the root condition and the concept of convergence. Runge-Kutta schemes. Stiff equations and A-stability.

Systems of equations and least squares calculations

Appropriate books
A. Isriles A first course in the Numerical Analysis of Differential Equations. CUP 2009
E. Suli and D.F. Meyers An introduction to numerical analysis. CUP 2003
A. Ralston and P. Rabinowitz A first course in numerical analysis. Dover 2001
M.J.D. Powell Approximation Theory and Methods. CUP 1981
P.J. Davis Interpolation and Approximation. Dover 1975
STATISTICS 16 lectures, Lent Term

Estimation

Hypothesis testing
Simple examples of hypothesis testing, null and alternative hypothesis, critical region, size, power, type I and type II errors, Neyman–Pearson lemma. Significance level of outcome. Uniformly most powerful tests. Likelihood ratio, and use of generalised likelihood ratio to construct test statistics for composite hypotheses. Examples, including t-tests and F-tests. Relationship with confidence intervals. Goodness-of-fit tests and contingency tables. [4]

Linear models

MARKOV CHAINS 12 lectures, Michaelmas Term

Discrete-time chains
Definition and basic properties, the transition matrix. Calculation of n-step transition probabilities. Communicating classes, closed classes, absorption, irreducibility. Calculation of hitting probabilities and mean hitting times; survival probability for birth and death chains. Stopping times and statement of the strong Markov property. Recurrence and transience; equivalence of transience and summability of n-step transition probabilities; equivalence of recurrence and certainty of return. Recurrence as a class property, relation with closed classes. Simple random walks in dimensions one, two and three. Invariant distributions, statement of existence and uniqueness. Mean return time, positive recurrence; equivalence of positive recurrence and the existence of an invariant distribution. Convergence to equilibrium for irreducible, positive recurrent, aperiodic chains and proof by coupling. Long-run proportion of time spent in given state. Time reversal, detailed balance, reversibility; random walk on a graph. [1]

Appropriate books
J.R. Norris Markov Chains. CUP 1997

Appropriate books
G. Casella and J.O. Berger Statistical Inference. Duxbury 2001
OPTIMISATION
12 lectures, Easter Term

Elements of convex optimisation
Convex sets and functions in \( \mathbb{R}^n \), global and constrained optimality. Algorithms for unconstrained convex optimisation: gradient descent, Newton’s algorithm. Introduction to convex optimisation on a convex set, the barrier method. Examples.

Lagrangian methods & duality
General formulation of constrained problems; the Lagrangian sufficiency theorem. Interpretation of Lagrange multipliers as shadow prices. The dual linear problem, duality theorem in a standardized case, complementary slackness, dual variables and their interpretation as shadow prices. Relationship of the primal simplex algorithm to dual problem. Examples.

Linear programming in the nondegenerate case
Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm and the tableau. Examples.

Applications of linear programming
Two person zero-sum games. Network flows; the max-flow min-cut theorem; the Ford-Fulkerson algorithm, the rational case. Network flows with costs, the transportation algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks. The formulation of simple practical and combinatorial problems as linear programming or network problems.

Appropriate books
2. Luenberger Linear and Nonlinear Programming. Addison-Wesley 1984

COMPUTATIONAL PROJECTS
8 lectures, Easter Term of Part IA

Practical sessions are offered and lectures given in the Part IA year. The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July preceding the Part IB year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at http://www.maths.cam.ac.uk/undergrad/catam/. Full credit may be obtained from the submission of the two core projects and a further two additional projects. Once the manual is available, these projects may be undertaken at any time up to the submission deadlines, which are near the start of the Lent Full Term in the IB year for the two core projects, and near the start of the Easter Full Term in the IB year for the two additional projects. A list of suitable books can be found in the CATAM manual.
GENERAL ARRANGEMENTS

Structure of Part II

There are two types of lecture courses in Part II: C courses and D courses. C courses are intended to be straightforward and accessible, and of general interest, whereas D courses are intended to be more demanding. The Faculty Board recommend that students who have not obtained at least a good second class in Part IB should include a significant number of C courses amongst those they choose.

There are 10 C courses and 27 D courses. All C courses are 24 lectures; of the D courses, 20 are 24 lectures and 7 are 16 lectures. The complete list of courses is as follows (an asterisk denotes a 16-lecture course):

<table>
<thead>
<tr>
<th>C courses</th>
<th>D courses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Theory</td>
<td>Logic and Set Theory</td>
</tr>
<tr>
<td>Topics in Analysis</td>
<td>Stochastic Financial Models</td>
</tr>
<tr>
<td>Coding and Cryptography</td>
<td>Mathematics of Machine Learning</td>
</tr>
<tr>
<td>Automata and Formal Lang.</td>
<td>*Asymptotic Methods</td>
</tr>
<tr>
<td>Statistical Modelling</td>
<td>*Integrable Systems</td>
</tr>
<tr>
<td>Mathematical Biology</td>
<td>Algebraic Topology</td>
</tr>
<tr>
<td>Further Complex Methods</td>
<td>Principles of Quantum Mechanics</td>
</tr>
<tr>
<td>Classical Dynamics</td>
<td>*Analysis of Functions</td>
</tr>
<tr>
<td>Cosmology</td>
<td>Statistical Physics</td>
</tr>
<tr>
<td>Quantum Inf. and Comp.</td>
<td>*Riemann Surfaces</td>
</tr>
<tr>
<td></td>
<td>*Electrodynamics</td>
</tr>
<tr>
<td></td>
<td>Probability and Measure</td>
</tr>
<tr>
<td></td>
<td>Fluid Dynamics</td>
</tr>
<tr>
<td></td>
<td>Applied Probability</td>
</tr>
<tr>
<td></td>
<td>Numerical Analysis</td>
</tr>
<tr>
<td></td>
<td>Principles of Statistics</td>
</tr>
</tbody>
</table>

As in Part IB, students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred workload, bearing in mind that it is better to do a smaller number of courses thoroughly than to do many courses scrappily. Students might sensibly assess their proposed workload by comparing the numbers of example sheets and lectures they are taking on per term with their previous experiences in Parts IA and IB.

Computational Projects

In addition to the lectured courses, there is a Computational Projects course.

No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of reports. The maximum credit obtainable is 150 marks and there are no alpha or beta quality marks. Credit obtained is added directly to the credit gained on the written papers. The maximum contribution to the final merit mark is thus 150, which is the same as the maximum for a 16-lecture course. The Computational Projects are considered to be a single piece of work within the Mathematical Tripos.

Examinations

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

There are no restrictions on the number or type of courses that may be presented for examination, but examiners may consider if most of the marks are obtained on only one or two courses. The Faculty Board has recommended to the examiners that no distinction be made, for classification purposes, between quality marks obtained on the C-course questions in Section II and quality marks obtained on D course questions.

On each of the four papers, candidates may obtain credit for attempts on at most six questions in Section I; there is no restriction on the number of questions in Section II that may be attempted for credit.

The number of questions set on each course is determined by the type and length of the course, as shown in the following table:

<table>
<thead>
<tr>
<th>Section I</th>
<th>Section II</th>
</tr>
</thead>
<tbody>
<tr>
<td>C course, 24 lectures</td>
<td>4</td>
</tr>
<tr>
<td>D course, 24 lectures</td>
<td>–</td>
</tr>
<tr>
<td>D course, 16 lectures</td>
<td>–</td>
</tr>
</tbody>
</table>

In Section I of each paper, there are 10 questions, one on each C course.

In Section II of each paper, there are 5 questions on C courses, one question on each of the 20 24-lecture D courses and either one question or no questions on each of the 7 16-lecture D courses, giving a total of 30 or 31 questions on each paper.

The distribution in Section II of the C course questions and the 16-lecture D course questions is shown in the following table.

<table>
<thead>
<tr>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>16-lecture D courses</th>
</tr>
</thead>
<tbody>
<tr>
<td>C courses</td>
<td>16-lecture D courses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number Theory</td>
<td>*</td>
<td>*</td>
<td>Number Fields</td>
<td>*</td>
</tr>
<tr>
<td>Topics in Analysis</td>
<td>*</td>
<td>*</td>
<td>Analysis of Functions</td>
<td>*</td>
</tr>
<tr>
<td>Coding and Cryptography</td>
<td>*</td>
<td>*</td>
<td>Riemann Surfaces</td>
<td>*</td>
</tr>
<tr>
<td>Automata and Form. Lang.</td>
<td>*</td>
<td>*</td>
<td>Maths. Machine Learning</td>
<td>*</td>
</tr>
<tr>
<td>Statistical Modelling</td>
<td>*</td>
<td>*</td>
<td>Asymptotic Methods</td>
<td>*</td>
</tr>
<tr>
<td>Mathematical Biology</td>
<td>*</td>
<td>*</td>
<td>Integrable Systems</td>
<td>*</td>
</tr>
<tr>
<td>Further Complex Methods</td>
<td>*</td>
<td>*</td>
<td>Electrodynamics</td>
<td>*</td>
</tr>
<tr>
<td>Classical Dynamics</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cosmology</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantum Inf. and Comp.</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Approximate class boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines in recent years.

For convenience, we define $M_1$ and $M_2$ by

$$M_1 = 30\alpha + 5\beta + m - 120, \quad M_2 = 15\alpha + 5\beta + m.$$  

$M_1$ is related to the primary classification criterion for the first class and $M_2$ is related to the primary classification criterion for the upper and lower second and third classes.

The second column of each table shows a sufficient criterion for each class (in terms of $M_1$ for the first class and $M_2$ for the other classes). The third and fourth columns show $M_1$ (for the first class) or $M_2$ (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline.

The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

### Part II 2018

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 677$</td>
<td>678/443,10,11 679/449,10,10</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 470$</td>
<td>471/341, 5,11 472/337, 6, 9</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 520$</td>
<td>521/254, 3, 6 536/266, 1,11</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 173$</td>
<td>174/129, 1, 6 186/141, 1, 6</td>
</tr>
</tbody>
</table>

### Part II 2019

<table>
<thead>
<tr>
<th>Class</th>
<th>Sufficient condition</th>
<th>Borderline candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M_1 &gt; 699$</td>
<td>700/460,10,12 701/436,11,11</td>
</tr>
<tr>
<td>2.1</td>
<td>$M_2 &gt; 478$</td>
<td>479/339, 8, 4 480/365, 3,14</td>
</tr>
<tr>
<td>2.2</td>
<td>$M_2 &gt; 530$</td>
<td>531/256, 3, 6 534/264, 1,11</td>
</tr>
<tr>
<td>3</td>
<td>$M_2 &gt; 165$</td>
<td>166/128, 1, 5 182/162, 0, 4</td>
</tr>
</tbody>
</table>
NUMBER THEORY (C)  24 lectures, Michaelmas Term

Review from Part IA Numbers and Sets: Euclid’s Algorithm, prime numbers, fundamental theorem of arithmetic. Congruences. The theorems of Fermat and Euler. [2]
Chinese remainder theorem. Lagrange’s theorem. Primitive roots to an odd prime power modulus. [3]
The mod-$p$ field, quadratic residues and non-residues, Legendre’s symbol. Euler’s criterion. Gauss’ lemma, quadratic reciprocity. [2]
Proof of the law of quadratic reciprocity. The Jacobi symbol. [1]
Distribution of the primes. Divergence of $\sum p^{-1}$. The Riemann zeta-function and Dirichlet series. [5]
Statement of the prime number theorem and of Dirichlet’s theorem on primes in an arithmetic progression. Legendre’s formula. Bertrand’s postulate. [4]
Continued fractions. Pell’s equation. [3]
Primality testing. Fermat, Euler and strong pseudo-primes. [2]
Factorization. Fermat factorization, factor bases, the continued-fraction method. Pollard’s method. [2]

Appropriate books
G.H. Hardy and E.M. Wright  *An Introduction to the Theory of Numbers*. Oxford University Press
T. Nagell  *Introduction to Number Theory*. AMS 1964
H. Davenport  *The Higher Arithmetic*. Cambridge University Press
A. Granville  *Number Theory Revealed: an Introduction*. AMS 2019

TOPICS IN ANALYSIS (C)  24 lectures, Lent Term

Analysis courses from IB will be helpful, but it is intended to introduce and develop concepts of analysis as required.

Discussion of metric spaces; compactness and completeness. Brouwer’s fixed point theorem. Proof(s) in two dimensions. Equivalent formulations, and applications. The degree of a map. The fundamental theorem of algebra, the Argument Principle for continuous functions, and a topological version of Rouche’s Theorem. [6]
Liouville’s proof of the existence of transcendental numbers. The irrationality of $e$ and $\pi$. The continued fraction expansion of real numbers; the continued fraction expansion of $e$. [4]

Appropriate books
E.W. Cheney  *Introduction to Approximation Theory*. AMS, 1999
CODING AND CRYPTOGRAPHY (C) 24 lectures, Lent Term

Part IB Linear Algebra is useful and Part IB Groups, Rings and Modules is very useful.

Introduction to communication channels, coding and channel capacity. [1]
Applications to gambling and the stock market. [1]
Information rate of a Bernoulli source. Capacity of a memoryless binary symmetric channel; Shannon’s noisy coding theorem for such channels. [3]

Appropriate books

2 D. Welch Codes and Cryptography. OUP 1988

AUTOMATA AND FORMAL LANGUAGES (C) 24 lectures, Michaelmas Term

Part IA Numbers and Sets is essential.

Register machines

Regular languages and finite-state automata
Deterministic finite-state automata (DFA) and regular languages. Nondeterminism. Regular expressions. Limitations of finite-state automata: closure properties; the pumping lemma; examples of non-regular languages. Minimization and the use of equivalence relations and quotients to construct minimal DFAs. [9]

Pushdown automata and context-free languages
Context-free grammars and context-free languages: generating languages by replacement rules. Normal forms for context-free languages, the Chomsky normal form; regular languages are context-free. Limitations of context-free grammars: the pumping lemma for context-free languages; examples of non-context-free languages. Pushdown automata. [5]

Appropriate books

J.E. Hopcroft, R. Motwani and J.D. Ullman Introduction to automata theory, languages and computation. 3rd edn. Pearson 2006
P.T. Johnstone Notes on logic and set theory (Chapter 4). CUP 1987
D.C. Kozen Automata and computability. Springer 1997
R.I. Soare Recursively enumerable sets and degrees: a study of computable functions and computably generated sets. Springer (Perspectives in mathematical logic) 1987
PART II

STATISTICAL MODELLING (C)  24 lectures, Michaelmas Term

Part IB Statistics is essential. About two thirds of this course will be lectures, with the remaining hours as practical classes, using R in the CATAM system. R may be downloaded at no cost via http://cran.r-project.org

Introduction to the statistical programming language R
Graphical summaries of data, e.g. histograms. Matrix computations. Writing simple functions. Simulation. [2]

Linear models

Overview of basic inferential techniques

Exponential dispersion families and generalised linear models (glm)

Examples in R
Linear and generalised linear models. Interpretation of models, inference and model selection. [6]

Mathematical Biology (C)  24 lectures, Lent Term

Part II Dynamical Systems is useful.

Introduction to the role of mathematics in biology

Systems without spatial structure: deterministic systems

Stochastic systems
Discrete stochastic models of birth and death processes. Master equations and Fokker-Planck equations. The continuum limit and the importance of fluctuations. Comparison of deterministic and stochastic models, including implications for extinction/invasion. Simple random walk and derivation of the diffusion equation. [6]

Systems with spatial structure: diffusion and reaction-diffusion systems

Appropriate books
L. Edelstein-Keshet Mathematical Models in Biology. SIAM classics in applied mathematics reprint, 2005

Appropriate books
J. Faraway Practical Regression and Anova in R. http://cran.r-project.org/doc/contrib/Faraway-PRA.pdf
A.C. Davison Statistical Models. CUP 2008
J. Albert and M. Rizzo R by Example. Springer 2012
S. Chaterjee and J.S. Simonoff Handbook of Regression Analysis. Wiley 2013
A. Agresti Foundations of Linear and Generalized Linear Models. Wiley 2015
FURTHER COMPLEX METHODS (C) 24 lectures, Lent Term

Complex variable
Revision of complex variable. Analyticity of a function defined by an integral (statement and discussion only). Analytic and meromorphic continuation.


Multivalued functions: definitions, branch points and cuts, integration; examples, including inverse trigonometric functions as integrals and elliptic integrals.

Special functions
Gamma function: Euler integral definition; brief discussion of product formulae; Hankel representation; reflection formula; discussion of uniqueness (e.g. Wielandt’s theorem). Beta function: Euler integral definition; relation to the gamma function. Riemann zeta function: definition as a sum; integral representations; functional equation; *discussion of zeros and relation to π(x) and the distribution of prime numbers*.

Differential equations by transform methods
Solution of differential equations by integral representation; Airy equation as an example. Solution of partial differential equations by transforms; the wave equation as an example. Causality. Nyquist stability criterion.

Second order ordinary differential equations in the complex plane
Classification of singularities, exponents at a regular singular point. Nature of the solution near an isolated singularity by analytic continuation. Fuchsian differential equations. The Riemann P-function, hypergeometric functions and the hypergeometric equation, including brief discussion of monodromy.

Appropriate books
4. B. Spain and M.G. Smith *Functions of Mathematical Physics*. Van Nostrand 1970

CLASSICAL DYNAMICS (C) 24 lectures, Michaelmas Term

Review of Newtonian mechanics

Lagrange’s equations

Motion of a rigid body
Kinematics of a rigid body. Angular momentum, kinetic energy, diagonalization of inertia tensor. Euler top, conserved angular momentum, Euler equations and their solution in terms of elliptic integrals. Lagrange top, steady precession, nutation.

Hamilton’s equations

Hamiltonian systems in nonlinear phase spaces, e.g. classical spin in a magnetic field. 2-D motion of ideal point vortices. *Connections between Lagrangian/Hamiltonian dynamics and quantum mechanics.*

Appropriate books
5. F. Scheck *Mechanics: from Newton’s laws to deterministic chaos*. Springer 2010
COSMOLOGY (C) 24 lectures, Michaelmas Term

The expanding universe

The hot universe

Structure formation

QUANTUM INFORMATION AND COMPUTATION (C) 24 lectures, Lent Term

Introduction
Why quantum information and computation. The basic idea of polynomial vs exponential computational complexity.

Quantum mechanics and quantum information
Basic principles of quantum mechanics and Dirac notation in a finite-dimensional setting. Composite systems and tensor products, projective measurements. Two-dimensional systems: qubits and Pauli operations. Definition of an entangled state.

Quantum states as information carriers
The no-cloning theorem. Optimal discrimination of non-orthogonal pure states; the Helstrom bound. Local quantum operations. The no-signalling theorem.

Quantum teleportation and dense coding
Bell states and basic properties; quantum dense coding. Exposition of quantum teleportation. Consistency of quantum teleportation with the no-signalling and no-cloning theorems.

Quantum cryptography
Cryptographic key distribution and the one-time pad. Quantum key distribution: the BB84 protocol. Sketch of the security of the BB84 protocol against individual attacks. Brief discussion of implementations of quantum key.

Basic principles of quantum computing

Basic quantum algorithms

Grover’s quantum searching algorithm
Introduction to search problems and the complexity class NP. Exposition of Grover’s quantum searching algorithm.

Shor’s quantum factoring algorithm
Exposition of Shor’s quantum factoring algorithm (proofs of classical number-theory ingredients not examinable).

Appropriate books
B. Ryden Introduction to cosmology. Addison-Wesley 2003
A. Liddle An introduction to modern cosmology. Wiley 2003
E.R. Harrison Cosmology: The science of the universe. CUP 2000
S. Weinberg Cosmology. OUP 2008 (A more advanced text)

M. Nielsen and I. Chuang Quantum Computation and Quantum Information. CUP 2000
B. Schumacher and M. Westmoreland Quantum Processes, Systems, and Information. CUP 2010
LOGIC AND SET THEORY (D) 24 lectures, Lent Term
No specific prerequisites.

Ordinals and cardinals

Posets and Zorn’s lemma
Partially ordered sets; Hasse diagrams, chains, maximal elements. Lattices and Boolean algebras. Complete and chain-complete posets; fixed-point theorems. The axiom of choice and Zorn’s lemma. Applications of Zorn’s lemma in mathematics. The well-ordering principle.

Propositional logic

Predicate logic
The predicate calculus with equality. Examples of first-order languages and theories. Statement of the completeness theorem; *sketch of proof*. The compactness theorem and the Löwenheim-Skolem theorems. Limitations of first-order logic. Model theory.

Set theory
Set theory as a first-order theory; the axioms of ZF set theory. Transitive closures, epsilon-induction and epsilon-recursion. Well-founded relations. Mostowski’s collapsing theorem. The rank function and the von Neumann hierarchy.

Consistency
«Problems of consistency and independence».

Appropriate books
A. Hajnal and P. Hamburger Set Theory. LMS Student Texts number 48, CUP 1999
P.T. Johnstone Notes on Logic and Set Theory. Cambridge University Press 1987
D. van Dalen Logic and Structure. Springer-Verlag 1994

GRAPH THEORY (D) 24 lectures, Lent Term
No specific prerequisites.

Introduction

Connectivity and matchings
Matchings in bipartite graphs; Hall’s theorem and its variants. Connectivity and Menger’s theorem.

Extremal graph theory
Long paths, long cycles and Hamilton cycles. Complete subgraphs and Turán’s theorem. Bipartite subgraphs and the problem of Zarankiewicz. The Erdős-Stone theorem; "sketch of proof".

Eigenvalue methods
The adjacency matrix and the Laplacian. Strongly regular graphs.

Graph colouring
Vertex and edge colourings; simple bounds. The chromatic polynomial. The theorems of Brooks and Vizing. Equivalent forms of the four colour theorem; the five colour theorem. Heawood’s theorem for surfaces; the torus and the Klein bottle.

Ramsey theory
Ramsey’s theorem (finite and infinite forms). Upper bounds for Ramsey numbers.

Probabilistic methods
Basic notions; lower bounds for Ramsey numbers. The model $G(n, p)$; graphs of large girth and large chromatic number. The clique number.

Appropriate books
B.Bollobás Modern Graph Theory. Springer 1998
R.Diestel Graph Theory. Springer 2000
D.West Introduction to Graph Theory. Prentice Hall 1999
GALOIS THEORY (D)  24 lectures, Michaelmas Term

Groups, Rings and Modules is essential.

Field extensions, tower law, algebraic extensions; irreducible polynomials and relation with simple algebraic extensions. Finite multiplicative subgroups of a field are cyclic. Existence and uniqueness of splitting fields. [6]

Existence and uniqueness of algebraic closure. [1]

Separability. Theorem of primitive element. Trace and norm. [3]

Normal and Galois extensions, automorphic groups. Fundamental theorem of Galois theory. [3]


Solvability by radicals. Insolubility of general quintic equations and other classical problems. [3]

Artin’s theorem on the subfield fixed by a finite group of automorphisms. Polynomial invariants of a finite group; examples. [2]

Appropriate books

E. Artin Galois Theory. Dover Publications
B. L. van der Waerden Modern Algebra. Ungar Pub 1949
S. Lang Algebra (Graduate Texts in Mathematics). Springer-Verlag New York Inc
I. Kaplansky Fields and Rings. The University of Chicago Press

REPRESENTATION THEORY (D)  24 lectures, Michaelmas Term

Linear Algebra, and Groups, Rings and Modules are essential.

Representations of finite groups

Character theory

Arithmetic properties of characters
Divisibility of the order of the group by the degrees of its irreducible characters. Burnside’s $p^aq^b$ theorem. [2]

Tensor products
Tensor products of representations and products of characters. The character ring. Tensor, symmetric and exterior algebras. [3]

Representations of $S^1$ and $SU_2$
The groups $S^1$, $SU_2$ and $SO(3)$, their irreducible representations, complete reducibility. The Clebsch-Gordan formula. Compact groups. [4]

Further worked examples
The characters of one of $GL_2(F_q)$, $S_n$ or the Heisenberg group. [3]

Appropriate books

J.L. Alperin and R.B. Bell Groups and representations. Springer 1995
J-P. Serre Linear representations of finite groups. Springer-Verlag 1977
NUMBER FIELDS (D) 16 lectures, Lent Term

Part IB Groups, Rings and Modules is essential and Part II Galois Theory is desirable.

Definition of algebraic number fields, their integers and units. Norms, bases and discriminants. [3]
Ideals, principal and prime ideals, unique factorisation. Norms of ideals. [3]
Dedekind’s theorem on the factorisation of primes. Application to quadratic fields. [2]
Minkowski’s theorem on convex bodies. Statement of Dirichlet’s unit theorem. Determination of units in quadratic fields. [2]
Ideal classes, finiteness of the class group. Calculation of class numbers using statement of the Minkowski bound. [2]
Discussion of the cyclotomic field and the Fermat equation or some other topic chosen by the lecturer. [3]

Appropriate books
J. Esmonde and M.R. Murty Problems in Algebraic Number Theory. Springer 1999
D.A. Marcus Number Fields. Springer 1977

ALGEBRAIC TOPOLOGY (D) 24 lectures, Michaelmas Term

Part IB Analysis and Topology is essential.

The fundamental group
Homotopy of continuous functions and homotopy equivalence between topological spaces. The fundamental group of a space, homomorphisms induced by maps of spaces, change of base point, invariance under homotopy equivalence. [3]

Covering spaces
Covering spaces and covering maps. Path-lifting and homotopy-lifting properties, and their application to the calculation of fundamental groups. The fundamental group of the circle; topological proof of the fundamental theorem of algebra. *Construction of the universal covering of a path-connected, locally simply connected space*. The correspondence between connected coverings of $X$ and conjugacy classes of subgroups of the fundamental group of $X$. [5]

The Seifert–Van Kampen theorem
Free groups, generators and relations for groups, free products with amalgamation. Statement *and proof* of the Seifert–Van Kampen theorem. Applications to the calculation of fundamental groups. [4]

Simplicial complexes
Finite simplicial complexes and subdivisions; the simplicial approximation theorem. [3]

Homology
Simplicial homology, the homology groups of a simplex and its boundary. Functorial properties for simplicial maps. *Proof of functoriality for continuous maps, and of homotopy invariance*. [4]

Homology calculations
The homology groups of $S^n$, applications including Brouwer’s fixed-point theorem. The Mayer–Vietoris theorem. *Sketch of the classification of closed combinatorial surfaces*; determination of their homology groups. Rational homology groups; the Euler–Poincaré characteristic and the Lefschetz fixed-point theorem. [5]

Appropriate books
M. A. Armstrong Basic Topology. Springer 1983
W. Massey A basic course in algebraic topology. Springer 1991
LINEAR ANALYSIS (D) 24 lectures, Michaelmas Term

Part II Linear Algebra, and Analysis and Topology are essential.

Normed and Banach spaces. Linear mappings, continuity, boundedness, and norms. Finite-dimensional normed spaces.

The Baire category theorem. The principle of uniform boundedness, the closed graph theorem and the inversion theorem; other applications.


Inner product spaces and Hilbert spaces; examples and elementary properties. Orthonormal systems, and the orthogonalization process. Bessel’s inequality, the Parseval equation, and the Riesz–Fischer theorem. Duality; the self duality of Hilbert space.

Bounded linear operations, invariant subspaces, eigenvectors; the spectrum and resolvent set. Compact operators on Hilbert space; discreteness of spectrum. Spectral theorem for compact Hermitian operators.

Appropriate books

ANALYSIS OF FUNCTIONS (D) 16 lectures, Lent Term

Part II Linear Analysis and Part II Probability and Measure are essential.

Lebesgue integration theory

Review of integration: simple functions, monotone and dominated convergence; existence of Lebesgue measure; definition of $L^p$ spaces and their completeness. The Lebesgue differentiation theorem. Mollification by convolution, continuity of translation and separability of $L^p$ when $p \neq \infty$.

Banach and Hilbert space analysis


Fourier analysis

Definition of Fourier transform in $L^1$; the Riemann–Lebesgue lemma. Fourier inversion theorem. Extension to $L^2$ by density and Plancherel’s isometry. Duality between regularity in real variable and decay in Fourier variable.

Generalized derivatives and function spaces


Appropriate books
RIEMANN SURFACES (D) 16 lectures, Michaelmas Term

Part IB Complex Analysis is essential, and Analysis and Topology is desirable.

The complex logarithm. Analytic continuation in the plane; natural boundaries of power series. Informal examples of Riemann surfaces of simple functions (via analytic continuation). Examples of Riemann surfaces, including the Riemann sphere, and the torus as a quotient surface. [4]

Analytic, meromorphic and harmonic functions on a Riemann surface; analytic maps between two Riemann surfaces. The open mapping theorem, the local representation of an analytic function as $z \mapsto z^k$. Complex-valued analytic and harmonic functions on a compact surface are constant. [2]

Germ of an analytic map between two Riemann surfaces; the space of germs as a covering surface (in the sense of complex analysis). The monodromy theorem (statement only). The analytic continuation of a germ over a simply connected domain is single-valued. [3]

The degree of a map between compact Riemann surfaces; Branched covering maps and the Riemann-Hurwitz relation (assuming the existence of a triangulation). The fundamental theorem of algebra. Rational functions as meromorphic functions from the sphere to the sphere. [3]

Meromorphic periodic functions; elliptic functions as functions from a torus to the sphere. The Weierstrass $P$-function. [3]

Statement of the Uniformization Theorem; applications to conformal structure on the sphere, to tori, and the hyperbolic geometry of Riemann surfaces. [1]

Appropriate books

A.F.Beardon A Primer on Riemann Surfaces. Cambridge University Press, 2004
G.A.Jones and D.Singerman Complex functions: an algebraic and geometric viewpoint. Cambridge University Press, 1987

ALGEBRAIC GEOMETRY (D) 24 lectures, Lent Term

Groups Rings and Modules is essential.

Affine varieties and coordinate rings. Projective space, projective varieties and homogenous coordinates. Rational and regular maps. [4]

Discussion of basic commutative algebra. Dimension, singularities and smoothness. [4]

Conics and plane cubics. Quadric surfaces and their lines. Segre and Veronese embeddings. [4]

Curves, differentials, genus. Divisors, linear systems and maps to projective space. The canonical class. [8]

Statement of the Riemann-Roch theorem, with applications. [4]

Appropriate books

F. Kirwan Complex Algebraic Curves. Cambridge University Press, 1992
M. Reid Undergraduate Algebraic Geometry. Cambridge University Press 1989
B. Hassett Introduction to Algebraic Geometry. Cambridge University Press, 2007
R. Hartshorne Algebraic Geometry, chapters I and 4. Springer 1997
DIFFERENTIAL GEOMETRY (D) 24 lectures, Lent Term

Smooth manifolds in \( \mathbb{R}^n \), tangent spaces, smooth maps and the inverse function theorem. Examples, regular values, Sard’s theorem (statement only). Transverse intersection of submanifolds. [4]

Manifolds with boundary, degree mod 2 of smooth maps, applications. [3]

Curves in 2-space and 3-space, arc-length, curvature, torsion. The isoperimetric inequality. [2]

Smooth surfaces in 3-space, first fundamental form, area. [1]

The Gauss map, second fundamental form, principal curvatures and Gaussian curvature. Theorema Egregium. [3]


Parallel transport and geodesics for surfaces in 3-space. Geodesic curvature. [2]


Global theorems on curves: Fenchel’s theorem (the total curvature of a simple closed curve is greater than or equal to \( 2\pi \)); the Fary-Milnor theorem (the total curvature of a simple knotted closed curve is greater than \( 4\pi \)). [3]

PROBABILITY AND MEASURE (D) 24 lectures, Michaelmas Term

Measure spaces, \( \sigma \)-algebras, \( \pi \)-systems and uniqueness of extension, statement and proofs of Carathéodory’s extension theorem. Construction of Lebesgue measure on \( \mathbb{R} \). The Borel \( \sigma \)-algebra of \( \mathbb{R} \). Existence of non-measurable subsets of \( \mathbb{R} \). Lebesgue-Stieltjes measures and probability distributions. Independence of events, independence of \( \sigma \)-algebras. The Borel-Cantelli lemmas. Kolmogorov’s zero-one law. [6]


\( L^2 \) as a Hilbert space. Orthogonal projection, relation with elementary conditional probability. Variance and covariance. Gaussian random variables, the multivariate normal distribution. [2]

The strong law of large numbers, proof for independent random variables with bounded fourth moments. Measure preserving transformations, Bernoulli shifts. Statements and proofs of maximal ergodic theorem and Birkhoff’s almost everywhere ergodic theorem, proof of the strong law. [4]

The Fourier transform of a finite measure, characteristic functions, uniqueness and inversion. Weak convergence, statement of Lévy’s convergence theorem for characteristic functions. The central limit theorem. [2]

Appropriate books

P.M.H. Wilson Curved Spaces. CUP, January 2008
B. O’Neill Elementary Differential Geometry,. Hacour 2nd ed 1997

Appropriate books

P. Billingsley Probability and Measure. Wiley 1995
D. Williams Probability with Martingales. Cambridge University Press
PART II

APPLIED PROBABILITY (D)  
24 lectures, Lent Term

Markov Chains is essential

Finite-state continuous-time Markov chains: basic properties. Q-matrix (or generator), backward and forward equations. The homogeneous Poisson process and its properties (thinning, superposition). Birth and death processes.


Applications: the $M/M/1$ and $M/M/\infty$ queues. Burke’s theorem. Jackson’s theorem for queuing networks. The $M/G/1$ queue.

Renewal theory: renewal theorems, equilibrium theory (proof of convergence only in discrete time). Renewal-reward processes. Little’s formula.

Spatial Poisson processes in $d$ dimensions. The superposition, mapping, and colouring theorems. Rényi’s theorem. Applications including Olbers’ paradox.

Appropriate books

J.R. Norris Markov Chains. CUP 1997
J.F.C. Kingman Poisson processes. OUP 1992

PRINCIPLES OF STATISTICS (D)  
24 lectures, Michaelmas Term

Part IB Statistics is essential

The Likelihood Principle

Bayesian Inference
Prior and posterior distributions. Conjugate families, improper priors, predictive distributions. Asymptotic theory for posterior distributions. Point estimation, credible regions, hypothesis testing and Bayes factors.

Decision Theory
Basic elements of a decision problem, including loss and risk functions. Decision rules, admissibility, minimax and Bayes rules. Finite decision problems. Stein estimator.

Multivariate Analysis

Nonparametric Inference and Monte Carlo Techniques

Appropriate books

STOCHASTIC FINANCIAL MODELS (D) 24 lectures, Michaelmas Term
Methods, Statistics, Probability and Measure, and Markov Chains are desirable.

Utility and mean-variance analysis
Utility functions; risk aversion and risk neutrality. Portfolio selection with the mean-variance criterion; the efficient frontier when all assets are risky and when there is one riskless asset. The capital-asset pricing model. Reservation bid and ask prices, marginal utility pricing. Simplest ideas of equilibrium and market clearing. State-price density. [5]

Martingales
Conditional expectation, definition and basic properties. Stopping times. Martingales, supermartingales, submartingales. Use of the optional sampling theorem. [3]

Dynamic models
Introduction to dynamic programming; optimal stopping and exercising American puts; optimal portfolio selection. [3]

Pricing contingent claims
Lack of arbitrage in one-period models; hedging portfolios; martingale probabilities and pricing claims in the binomial model. Extension to the multi-period binomial model. Axiomatic derivation. [4]

Brownian motion
Introduction to Brownian motion; Brownian motion as a limit of random walks. Hitting-time distributions; changes of probability. [3]

Black–Scholes model
The Black–Scholes formula for the price of a European call; sensitivity of price with respect to the parameters; implied volatility; pricing other claims. Binomial approximation to Black–Scholes. Use of finite-difference schemes to compute prices. [6]

Appropriate books
J. Hull Options, Futures and Other Derivative Securities. Prentice-Hall 2003

MATHEMATICS OF MACHINE LEARNING (D) 16 lectures, Lent Term
Part IA Probability is essential. Part IB Statistics and IB Optimisation are useful.

Introduction to statistical learning

Statistical learning theory

Computation for empirical risk minimisation

Popular machine learning methods

Appropriate books
C. Giraud Introduction to High-Dimensional Statistics. CRC press, 2015
T. Hastie, R. Tibshirani and J. Friedman The Elements of Statistical Learning, 2nd ed.. Springer, 2001
M. Wainwright High-Dimensional Statistics: A Non-Asymptotic Viewpoint.. CUP, 2019
S. Shalev-Shwartz and S. Ben-David Understanding Machine Learning: From Theory to Algorithms. CUP, 2014
ASYMPTOTIC METHODS (D) 16 lectures, Lent Term

Either Complex Methods or Complex Analysis is essential, Part II Further Complex Methods is useful.

Asymptotic expansions
Definition (Poincare) of $\phi(z) \sim \sum a_n z^{-n}$; examples; elementary properties; uniqueness; Stokes' phenomenon. [4]

Asymptotics behaviour of functions defined by integrals

Asymptotic behaviour of solutions of differential equations
Asymptotic solution of second-order linear differential equations, including Liouville–Green functions (proof that they are asymptotic not required) and WKBJ with the quantum harmonic oscillator as an example. [4]

Recent developments
Further discussion of Stokes' phenomenon. *Asymptotics 'beyond all orders'*. [1]

Appropriate books
A. Erdelyi Asymptotic Expansions. Dover 1956
E.J. Hinch Perturbation Methods. CUP 1991
J.D. Murray Asymptotic Analysis. Springer 1984

DYNAMICAL SYSTEMS (D) 24 lectures, Michaelmas Term

General introduction
The notion of a dynamical system and examples of simple phase portraits. Relationship between continuous and discrete systems. Reduction to autonomous systems. Initial value problems, uniqueness, finite-time blowup, examples. Flows, orbits, invariant sets, limit sets and topological equivalence. [3]

Fixed points of flows
Linearization. Classification of fixed points in $\mathbb{R}^2$, Hamiltonian case. Effects of nonlinearity; hyperbolic and non-hyperbolic cases; Stable-manifold theorem (statement only), stable and unstable manifolds in $\mathbb{R}^2$. Phase-plane sketching. [3]

Stability
Lyapunov, quasi-asymptotic and asymptotic stability of invariant sets. Lyapunov and bounding functions. Lyapunov’s 1st theorem; La Salle’s invariance principle. Local and global stability. [2]

Periodic orbits in $\mathbb{R}^2$
The Poincaré index; Dulac’s criterion; the Poincaré–Bendixson theorem (*and proof*). Nearly Hamiltonian flows.
Stability of periodic orbits; Floquet multipliers. Examples; van der Pol oscillator. [5]

Bifurcations in flows and maps

Chaos
Sensitive dependence on initial conditions, topological transitivity. Maps of the interval, the sawtooth map, horseshoes, symbolic dynamics. Period three implies chaos, the occurrence of $N$-cycles, Sharkovsky’s theorem (statement only). The tent map. Unimodal maps and Feigenbaum’s constant. [6]

Appropriate books
D.K. Arrowsmith and C.M. Place Introduction to Dynamical Systems. CUP 1990
P.G. Drazin Nonlinear Systems. CUP1992
A. Glendinning Stability, Instability and Chaos. CUP1994
D.W. Jordan and P. Smith Nonlinear Ordinary Differential Equations. OUP 1999
J. Guckenheimer and P. Holmes Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer, second edition 1986
INTEGRABLE SYSTEMS (D)  
16 lectures, Michaelmas Term

Part III Methods, and Complex Methods or Complex Analysis are essential; Part II Classical Dynamics is desirable.

Hamiltonian formulation of soliton equations. [2]
Painlevé equations and Lie symmetries: Symmetries of differential equations, the ODE reductions of certain integrable nonlinear PDEs, Painlevé equations. [3]

Appropriate books

7. MJ Ablowitz and P Clarkson Solitons, Nonlinear Evolution Equations and Inverse Scattering. CUP 1991

PRINCIPLES OF QUANTUM MECHANICS (D)  
24 lectures, Michaelmas Term

Part III Quantum Mechanics is essential.

Dirac formalism
Bra and ket notation, operators and observables, probability amplitudes, expectation values, complete commuting sets of operators, unitary operators. Schrödinger equation, wave functions in position and momentum space. [3]
Time evolution operator, Schrödinger and Heisenberg pictures, Heisenberg equations of motion. [2]

Harmonic oscillator
Analysis using annihilation, creation and number operators. Significance for normal modes in physical examples. [2]

Multiparticle systems
Composite systems and tensor products, wave functions for multiparticle systems. Symmetry or anti-symmetry of states for identical particles, Bose and Fermi statistics, Pauli exclusion principle. [5]

Perturbation theory
Time-independent theory; second order without degeneracy, first order with degeneracy. [2]
Angular momentum

Translations and rotations
Unitary operators corresponding to spatial translations, momenta as generators, conservation of momentum and translational invariance. Corresponding discussion for rotations. Reflections, parity, intrinsic parity. [4]

Time-dependent perturbation theory
Interaction picture. First-order transition probability, the golden rule for transition rates. Application to atomic transitions, selection rules based on angular momentum and parity, "absorption, stimulated and spontaneous emission of photons". [3]

Quantum basics
Quantum data, qubits, no cloning theorem. Entanglement, pure and mixed states, density matrix. Classical determinism versus quantum probability, Bell inequality for singlet two-electron state, GHZ state. [2]

Appropriate books

5. S. Weinberg Lectures on Quantum Mechanics. CUP, 2nd ed., 2015
APPLICATIONS OF QUANTUM MECHANICS (D) 24 lectures, Lent Term

Principles of Quantum Mechanics is essential.

Variational Principle
Variational principle, examples.

Bound states and scattering states in one dimension
Bound states, reflection and transmission amplitudes. Examples. Relation between bound states and transmission amplitude by analytic continuation.

Scattering theory in three dimensions

Electrons in a magnetic field
Vector potential and Hamiltonian. Quantum Hamiltonian, inclusion of electron spin, gauge invariance, Zeeman splitting. Landau levels, effect of spin, degeneracy and filling effects, use of complex variable for lowest Landau level. Aharonov-Bohm effect.

Particle in a one-dimensional periodic potential
Discrete translation group, lattice and reciprocal lattice, periodic functions. Bloch's theorem, Brillouin zone, energy bands and gaps. Floquet matrix, eigenvalues. Band gap in nearly-free electron model, tight-binding approximation.

Crystalline solids

Appropriate books
D.J. Griffiths Introduction to Quantum Mechanics. 2nd edition, Pearson Education 2005

STATISTICAL PHYSICS (D) 24 lectures, Lent Term

Part IB Quantum Mechanics and “Multiparticle Systems” from Part II Principles of Quantum Mechanics are essential.

Fundamentals of statistical mechanics

Classical gases

Quantum gases

Thermodynamics

Phase transitions

Appropriate books
F. Mandl Statistical Physics. Wiley 1988
F. Reif Fundamentals of Thermal and Statistical Physics. McGraw–Hill 1965
A.B. Pippard Elements of Classical Thermodynamics. Cambridge University Press, 1957
K. Huang Introduction to Statistical Physics. Taylor and Francis 2001
PART II

ELECTRODYNAMICS (D) 16 lectures, Michaelmas Term

IB Electromagnetism and IA Dynamics and Relativity are essential. IB Methods is desirable.

Classical Field Theory

Electromagnetic Radiation

Electromagnetism in Media

Appropriate books
A. Zangwill Modern Electrodynamics. CUP 2012
J.D. Jackson Electrodynamics. Wiley 1999

GENERAL RELATIVITY (D) 24 lectures, Lent Term

Part IB Methods and Variational Principles are very useful.

Brief review of Special Relativity
Notion of proper time. Equation of motion for free point particle derivable from a variational principle. Noether’s theorem. [1]

Introduction and motivation for General Relativity

Tensor calculus

Vacuum field equations
Spherically symmetric spacetimes, the Schwarzschild solution. Birkhoff’s theorem *with proof*. Rays and orbits, gravitational red-shift, light deflection, perihelion advance. Shapiro time delay. [4]

Einstein Equations coupled to matter

Linearized theory

Gravitational collapse and black holes
Non-singular nature of the surface \( r = 2M \) in the Schwarzschild solution using Finkelstein and Kruskal coordinates. The idea of an event horizon and the one-way passage of timelike geodesics through it. Qualitative account of idealized spherically symmetric collapse. The final state: statement of Israel’s Theorem. *Qualitative description of Hawking radiation.* [2]

Appropriate books
S.M. Carroll Spacetime and Geometry. Addison-Wesley 2004
FLUID DYNAMICS II (D) 24 lectures, Michaelmas Term

Methods and Fluid Dynamics are essential. It is recommended that students attend the associated Laboratory Demonstrations in Fluid Dynamics, which take place in the Michaelmas term.

Governing equations for an incompressible Newtonian fluid
Stress and rate-of-strain tensors and hypothesis of linear relation between them for an isotropic fluid; equation of motion; conditions at a material boundary; dissipation; flux of mass, momentum and energy; the Navier-Stokes equations. Dynamical similarity; steady and unsteady Reynolds numbers. [4]

Unidirectional flows
Couette and Poiseuille flows; the Stokes layer; the Rayleigh problem. [2]

Stokes flows
Flow at low Reynolds number; linearity and reversibility; uniqueness and minimum dissipation theorems. Flow in a corner; force and torque relations for a rigid particle in arbitrary motion; case of a rigid sphere and a spherical bubble. [4]

Flow in a thin layer
Lubrication theory; simple examples; the Hele-Shaw cell; gravitational spreading on a horizontal surface. [3]

Generation and confinement of vorticity
Vorticity equation; vortex stretching; flow along a plane wall with suction; flow toward a stagnation point on a wall; flow in a stretched line vortex. [3]

Boundary layers at high Reynolds number
The Euler limit and the Prandtl limit; the boundary layer equation for two-dimensional flow. Similarity solutions for flow past a flat plate and a wedge. «Discussion of the effect of acceleration of the external stream, separation.» Boundary layer at a free surface; rise velocity of a spherical bubble. [6]

Stability of unidirectional inviscid flow
Instability of a vortex sheet and of simple jets (e.g. vortex sheet jets). [2]

Appropriate books
G.K. Batchelor An Introduction to Fluid Dynamics. Cambridge University Press 2000

WAVES (D) 24 lectures, Lent Term

Part IB Methods is essential and Part IB Fluid Dynamics is very helpful.

Sound waves
Equations of motion of an inviscid compressible fluid (without discussion of thermodynamics). Mach number. Linear acoustic waves; wave equation; wave-energy equation; plane waves; spherically symmetric waves. [4]

Elastic waves
Momentum balance; stress and infinitesimal strain tensors and hypothesis of a linear relation between them for an isotropic solid. Wave equations for dilatation and rotation. Compressional and shear plane waves; simple problems of reflection and transmission; Rayleigh waves. [5]

Dispersive waves
Rectangular acoustic wave guide; Love waves; cut-off frequency. Representation of a localised initial disturbance by a Fourier integral (one-dimensional case only); modulated wave trains; stationary phase. Group velocity as energy propagation velocity; dispersing wave trains. Water waves; internal gravity waves. [6]

Ray theory
Group velocity from wave-crest kinematics; ray tracing equations. Doppler effect; ship wave pattern. Cases where Fermat’s principle and Snell’s law apply. [4]

Non-linear waves

Appropriate books
J. Billingham and A.C. King Wave Motion: Theory and application. Cambridge University Press 2000
M.J. Lighthill Waves in Fluids. Cambridge University Press 1978
G.B. Whitham Linear and Nonlinear Waves. Wiley 1999
NUMERICAL ANALYSIS (D) 24 lectures, Michaelmas Term

Part IB Numerical Analysis is essential and Analysis and Topology, Linear Algebra and Complex Methods or Complex Analysis are all desirable.

Finite difference methods for the Poisson’s equation
Approximation of $\nabla^2$ by finite differences. The accuracy of the five-point method in a square. Higher order methods. Solution of the difference equations by iterative methods, including multigrid. Fast Fourier transform (FFT) techniques.[5]

Finite difference methods for initial value partial differential equations
Difference schemes for the diffusion equation and the advection equation. Proof of convergence in simple cases. The concepts of well posedness and stability. Stability analysis by eigenvalue and Fourier techniques. Splitting methods.[6]

Spectral methods

Iterative methods for linear algebraic systems

Computation of eigenvalues and eigenvectors
The power method and inverse iteration. Transformations to tridiagonal and upper Hessenberg forms. The QR algorithm for symmetric and general matrices, including shifts.[3]

Appropriate books

COMPUTATIONAL PROJECTS

The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July preceding the Part II year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at http://www.maths.cam.ac.uk/undergrad/catam/. Each project is allocated a number of units of credit. Full credit may obtained from the submission of projects with credit totalling 30 units. Credit for submissions totalling less than 30 units is awarded proportionately. There is no restriction on the choice of projects. Once the manual is available, the projects may be done at any time up to the submission deadline, which is near the beginning of the Easter Full Term.

A list of suitable books can be found in the manual.