



UNIVERSITY OF
CAMBRIDGE

Faculty of Mathematics

MATHEMATICAL TRIPOS

2019/2020

GUIDE TO COURSES

IN PART IA

This booklet provides an introduction for new students, giving an outline of the first year with informal and non-technical descriptions of the courses.

It is intended to supplement the more formal descriptions contained in the booklet *Schedules of Lecture Courses and Form of Examinations*.

These and other Faculty documents for students taking the Mathematical Tripos are available from <http://www.maths.cam.ac.uk/undergrad/>

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1 Introduction

The *Mathematical Tripos* consists of Parts IA, IB and II, taken in successive years, with an optional fourth year, Part III, taken by students who do sufficiently well. Those who successfully complete three years are eligible to graduate with a BA honours degree, while those who go on to complete the additional fourth year graduate with both BA honours and MMath degrees.

Unlike many degree courses, the Mathematical Tripos is not modular (at least in its first three years). It is tightly structured, with no choice in the first year, some choice in the second year, and a very wide choice in the third year. The examinations (especially in the second and third years) are cross-sectional, meaning that instead of each lecture course having a dedicated examination paper, each examination paper has questions on many lecture courses. The flexibility that this allows students is considered by the Faculty to be one of the great strengths of the Tripos.

This booklet provides an introduction for new students, with an outline of Part IA and informal descriptions of the courses. The descriptions are intended to be comprehensible without much prior knowledge and to convey something of the flavour of each course (corresponding booklets are also available for Parts IB and II).

You will also find it helpful to consult the booklet *Schedules of Lecture Courses and Form of Examinations*, <http://www.maths.cam.ac.uk/undergrad/course/schedules.pdf>, usually known as *the Schedules*. It describes the structure of the Tripos formally, in more technical detail, and it is the definitive reference for matters of course content and assessment, for students, lecturers and examiners. The introductory sections, in particular, are worth a careful read alongside this booklet.

Your college Director of Studies will be able to provide further advice and guidance on all aspects of the Mathematical Tripos.

2 The first year in outline

Students are admitted to study one of two options in Part IA: (a) Pure and Applied Mathematics; or (b) Mathematics with Physics. (You will have chosen one of these options when you applied.) There is no choice within each option: you are expected to follow all the courses.

- For option (a) there are eight 24-lecture courses (four in Michaelmas Term and four in Lent Term) and four 3-hour examination papers, with two courses examined on each paper.
- For option (b), the lecture courses Numbers & Sets and Dynamics & Relativity are replaced by the complete Physics course from Part IA of the Natural Sciences Tripos, which has lectures in Michaelmas, Lent and Easter Terms and assessed practical work throughout the year. Paper 4 of the Mathematics examination is replaced by the Physics paper from Natural Sciences.
- Students taking options (a) and (b) are classed together, as a single group, following the examinations at the end of the year.
- Options (a) and (b) do not extend into the second year. Those taking option (a) will (usually) continue to Part IB Mathematics; those taking option (b) may do the same, or they may change to Part IB Natural Sciences.

Option (b) is designed for students who have a strong interest in mathematics but who may wish to change to Part IB of the Natural Sciences Tripos (and take the Physics A / Physics B / Mathematics options) after the first year. It provides an excellent mathematical background for students who plan to study theoretical or experimental physics: their greater mathematical knowledge, compared with students who come to Physics through Part IA of the Natural Sciences Tripos, can be a significant benefit.

Changing from option (b) to option (a) is usually feasible if it is done early enough, but the courses cover ground rapidly; if this is something you are considering then you should discuss it with your Director of Studies as soon as possible. Changing from option (a) to option (b) is likely to be more complicated, because of the additional assessed practical work.

Michaelmas Term: non-examinable mechanics course

In Michaelmas Term there is a short non-examinable course, Introduction to Mechanics. This is intended to provide catch-up material for those students who have taken only a limited amount of Mechanics at A-level (or the equivalent). You should discuss with your Director of Studies whether it might be sensible for you to attend these lectures.

Easter Term courses

A number of lecture courses are given in Easter Term, none of them examinable in Part IA.

- Lectures on Computational Projects (CATAM). It is essential to attend these in your first year, to prepare for the project work that will be submitted and assessed during your second year.
- Two courses examinable in Part IB can be attended in either your first or second year (or both): Variational Principles, and Optimisation.
- There is a non-examinable course of 8 lectures on Concepts in Theoretical Physics that should be of interest to all students. The intention is to give an introductory flavour of some of the most important areas of modern theoretical physics.

One advantage of attending IB Easter Term courses in your first year is that material prepared well in advance can sink in much better than material prepared just before the exams. An additional benefit is that the Easter Term courses can provide helpful background for other courses in Part IB, e.g. the content of Variational Principles will connect well with topics covered in many of the other applied courses. It is common for Directors of Studies to advise their students to attend lectures in the Easter Term of their first year, work on examples sheets over the summer, and then have supervisions at the very start of their second year. Priority, of course, is given to preparing for the Part IA examination.

For the Computational Projects course, the CATAM manual should be available some time in July. You should bear in mind that the computational work can be time-consuming and so an early start is strongly recommended. The two core projects require no mathematical knowledge from Part IB courses, so you can make substantial progress over the summer before the start of your second year. The faculty CATAM laboratory is usually open over the vacations, but check, just in case. Many colleges have suitable computing facilities which you can use at any time.

Changes since last year

There are no changes to the content of Part IA for 2019/20 compared to 2018/19.

Additional activities and lectures

Most students will find that there is enough mathematics in Part IA to keep them busy (or very busy!), and the Faculty places no expectations on students beyond keeping up with the first-year lectures, examples sheets and supervisions. There are many other educational and recreational opportunities to enjoy at university, though mathematics itself can hopefully be recreational.

For those who do want something extra or something a bit different from the mathematics in Part IA, the first choice should be the many excellent lectures provided by the student mathematics societies. Some students reach the end of their time at Cambridge and realise too late that they have missed the opportunity of hearing leading experts talking in an accessible way about some of the most important new ideas in mathematics and theoretical physics.

There is also an excellent and entertaining set of (non-examinable) lectures on the history of mathematics, given conveniently at 4.00 pm in the Michaelmas and Lent Terms.

You are allowed to attend any lecture given in the University (provided it does not require limited laboratory or other facilities). You may well find something interesting and accessible in some other Tripos. Unfortunately, there is no complete list of all the lectures given in the University; you have to look on the faculty web site of the subject you are interested in to see what is on offer.

You could also attend lectures in Part IB or even Part II of the Mathematical Tripos in your first year. However, it is unlikely that you would have time to do supervision work on such courses, and it is generally undesirable to be supervised before taking the courses properly later on. The Part IB course Groups, Rings and Modules is timetabled in Lent Term so that it can be attended by first year students, and if you felt very comfortable with the workload in the Michaelmas Term, you might consider attending this course. However, your Director of Studies will probably advise you to concentrate on learning the Part IA courses thoroughly; that way, you would need less revision time in the Easter Term and this would allow you to take the courses provided then (Variational Principles, Optimisation, or the non-examinable Concepts in Theoretical Physics).

3 Informal descriptions of courses in Part IA

Each lecture course has an official syllabus or *schedule* that sets out formally, and in technical terms, the material to be covered. See <http://www.maths.cam.ac.uk/undergrad/course/schedules.pdf>

This section, by contrast, provides an *informal* description of each lecture course given by the Faculty of Mathematics that is examinable in Part IA. Students taking option (b) Mathematics with Physics should refer to <http://www.phy.cam.ac.uk/teaching> for a description of the examinable material that replaces the courses Numbers & Sets and Dynamics & Relativity from option (a).

Each description below ends with a summary of the learning outcomes for the course. The full learning outcome for Part IA is that you should understand the material described in the formal syllabuses given in the Schedules booklet and be able to apply it to the sorts of problems that can be found on Tripos papers from earlier years.

Vectors and Matrices

24 lectures, Michaelmas Term

The course starts with revision of complex numbers. It then introduces some more advanced ideas, including de Moivre's theorem, which may be new to you. It moves on to generalise to higher (possibly complex) dimensions the familiar idea of a vector. A very important tool, suffix notation, is introduced and used for vector algebra. This is followed by the application of vector methods to geometry.

The remainder of the course is taken up with matrices: algebraic manipulation; applications to solution of simultaneous equations; geometrical applications; and eigenvectors and eigenvalues.

The material in this course is absolutely fundamental to nearly all areas of mathematics.

Learning outcomes

By the end of this course, you should:

- be able to manipulate complex numbers and be able to solve geometrical problems using complex numbers;
- be able to manipulate vectors in \mathbb{R}^3 (using suffix notation and summation convention where appropriate), and to solve geometrical problems using vectors;
- be able to manipulate matrices and determinants, and understand their relation to linear maps and systems of linear equations;
- be able to calculate eigenvectors and eigenvalues and understand their relation with diagonalisation of matrices and canonical forms.

Analysis I

24 lectures, Lent Term

Analysis involves the rigorous investigation of limits and calculus. You need to study analysis to have a firm foundation for techniques you already know, such as basic differentiation and integration. This not only allows you to understand exactly when these techniques can be used, but also allows you to generalise them to more complicated situations.

The sorts of questions that you will be asking in this course are: 'what does it mean to say that a sequence or a function tends to a limit?'; 'what is the exact definition of a derivative or an integral?'; 'which functions can be differentiated and which can be integrated?'; 'what conditions are needed for a Taylor series to be valid?'

In later courses on analysis, differentiation and integration of functions of more than one variable are investigated.

In Analysis I, you will encounter the ' ϵ - δ ' method of characterising the properties of functions. This is the basis of rigorous thought in this area of mathematics, and will repay you handsomely for all the work you put into understanding it.

Learning outcomes

By the end of this course, you should:

- be able to apply the basic techniques of rigorous analysis and be familiar with examples of ‘good behaviour’ and ‘bad behaviour’ in basic analysis;
- know the definition of a limit and be able to establish the convergence or divergence of simple real and complex sequences and series;
- understand the completeness of the real line and be able to derive the basic properties of continuous real-valued functions;
- be able to establish the rules for differentiation, and to prove and apply the mean value theorem;
- be acquainted with complex power series and be able to determine the radius of convergence in simple cases;
- know the definition of the Riemann integral, be able to test simple functions for integrability, and establish the rules for integration.

Differential Equations

24 lectures, Michaelmas Term

The main aim is to develop the skill of representing real (physical or biological) situations by means of differential (or difference) equations. The course follows smoothly from the A-level syllabus, starting with revision of differentiation and integration.

A particularly important sort of differential equation is one which is linear and has constant coefficients. These equations are unusual in that they can be solved exactly (the solutions are exponential or trigonometric functions). Many of the equations of physics are of this sort: the equations governing radioactive decay, Maxwell’s equations for electromagnetism and the Schrödinger equation in quantum mechanics, for example.

In other cases, it is useful to try to represent solutions which cannot be obtained explicitly by means of phase-plane diagrams. Sometimes a particular solution describing some important situation is known although the general solution is not. In this case, it is often important to determine whether this solution is typical, or whether a small change in the conditions will lead to a very different solution. In the latter case, the solution is said to be unstable. This property is determined by linearising the original equation to obtain an equation with constant coefficients of the sort discussed above. Sometimes, the solutions are so unstable that they are called *chaotic*.

The very important idea of partial differentiation is also introduced in the course. This is the analogue of familiar differentiation to functions which depend on more than one variable. The approach is mainly geometrical and one of the applications is determining the stationary points of, for example, a function that gives height above sea-level and classifying them into maxima (mountain peaks), minima (valley bottoms) and saddle points (cols or passes).

Learning outcomes

By the end of this course, you should:

- understand the theory of, and be able to solve (in simple cases), linear differential or difference equations, and standard types of non-linear equations;
- calculate partial derivatives and use the chain rule;
- find and classify stationary points of functions of more than one variable;
- be able to investigate the stability of solutions of differential or difference equations.

Probability

24 lectures, Lent Term

From its origin in games of chance and the analysis of experimental data, probability theory has developed into an area of mathematics with many varied applications in physics, biology and business.

The course introduces the basic ideas of probability and should be accessible to students who have no previous experience of probability or statistics. While developing the underlying theory, the course should strengthen students' general mathematical background and manipulative skills by its use of the axiomatic approach. There are links with other courses, in particular Vectors and Matrices, the elementary combinatorics of Numbers and Sets, the difference equations of Differential Equations and calculus of Vector Calculus and Analysis. Students should be left with a sense of the power of mathematics in relation to a variety of application areas.

After a discussion of basic concepts (including conditional probability, Bayes' formula, the binomial and Poisson distributions, and expectation), the course studies random walks, branching processes, geometric probability, simulation, sampling and the central limit theorem. Random walks can be used, for example, to represent the movement of a molecule of gas or the fluctuations of a share price; branching processes have applications in the modelling of chain reactions and epidemics. Through its treatment of discrete and continuous random variables, the course lays the foundation for the later study of statistical inference.

Learning outcomes

By the end of this course, you should:

- understand the basic concepts of probability theory, including independence, conditional probability, Bayes' formula, expectation, variance and generating functions;
- be familiar with the properties of commonly-used distribution functions for discrete and continuous random variables;
- understand and be able to apply the central limit theorem.
- be able to apply the above theory to 'real world' problems, including random walks and branching processes.

Groups

24 lectures, Michaelmas Term

In university mathematics, *algebra* is the study of abstract systems of objects whose behaviour is governed by fixed rules or *axioms*. An example is the set of real numbers, governed by the rules of addition and multiplication. One of the simplest forms of abstract algebraic systems is a group, which is roughly a set of objects and a rule for multiplying them together. Groups arise all over mathematics, particularly where there is symmetry.

The course introduces groups and their properties. The emphasis is on both the general theory and a range of examples, such as groups of symmetries and groups of linear transformations.

Learning outcomes

By the end of this course, you should:

- be familiar with elementary properties of abstract groups, including the theory of mappings between groups;
- understand the group-theoretic perspective on symmetries in geometry.

Vector Calculus

24 lectures, Lent Term

This course is about functions of more than one variable. It is an ‘applied’ course, meaning that you are expected to be able to apply techniques, but not necessarily to prove rigorously that they work – that will come in future analysis courses.

In the first part of the course, the idea of integration is extended from \mathbb{R} to \mathbb{R}^2 and \mathbb{R}^3 (with an obvious extension to higher dimensions): integrals along the x -axis are replaced by integrals over curves, surfaces and volumes.

Then the idea of differentiation is extended to vectors (div, grad and curl), which is a basic tool in many areas of theoretical physics (such as electromagnetism and fluid dynamics).

Two important theorems are introduced, namely the divergence theorem and Stokes’s theorem; in both cases, an integral over a region (in \mathbb{R}^3 and in \mathbb{R}^2 , respectively) is converted to an integral over the boundary of the region.

All the previous ideas are then applied to Laplace’s equation $\nabla^2\phi = 0$ and the related Poisson’s equation, which are amongst the most important equations in all of mathematics and physics.

Finally, the notion of a vector is generalised to that of a *tensor*. A vector can be thought of as a 3×1 matrix that carries physical information: namely, magnitude and direction. This information is preserved when the axes are rotated only if the components change according to a certain rule. Very often, it is necessary to describe physical quantities using a 3×3 matrix (or even a $3 \times 3 \times 3 \dots$ ‘matrix’). Such a quantity is called a tensor if its components transform according to a certain rule when the axes are rotated. This rule means that the physical information embodied in the tensor is preserved.

Learning outcomes

By the end of this course, you should:

- be able to manipulate, and solve problems using, vector operators;
- be able to calculate line, surface and volume integrals in \mathbb{R}^3 , using Stokes theorem and the divergence theorem where appropriate;
- be able to solve Laplace’s equation in simple cases, and be able to prove standard uniqueness theorems for Laplace’s and related equations.
- understand the notion of a tensor and the general properties of tensors in simple cases.

Numbers and Sets

24 lectures, Michaelmas Term

This course is concerned not so much with teaching you new parts of mathematics as with explaining how the language of mathematical arguments is used. We will use simple mathematics to develop an understanding of how results are established.

Because you will be exploring a broader and more intricate range of mathematical ideas at university, you will need to develop greater skills in understanding arguments and in formulating your own. These arguments are usually constructed in a careful, logical way as proofs of propositions. We begin with clearly stated and plausible assumptions or *axioms* and then develop a more and more complex theory from them. The course, and the lecturer, will have succeeded if you finish the course able to construct valid arguments of your own and to examine critically those that are presented to you. Example sheets and supervisions will play a key role in achieving this. These skills will form the basis for the later courses, particularly those devoted to Pure Mathematics.

In order to give examples of arguments, we will take two topics: sets and numbers. Set theory provides a basic vocabulary for much of mathematics. We can use it to express in a convenient and precise shorthand the relationships between different objects. Numbers have always been a fascinating and fundamental part of Mathematics. We will use them to provide examples of proofs, algorithms and counter-examples.

Initially we will study the natural numbers $1, 2, 3, \dots$ and especially *mathematical induction*. Then we expand to consider integers and arithmetic leading to codes like the RSA code used on the internet. Finally we move to rational, real and complex numbers where we lay the logical foundations for analysis. (Analysis is the name given to the study of, for example, the precise meaning of differentiation and integration and the sorts of functions to which these processes can be applied.)

Learning outcomes

By the end of this course, you should:

- understand the need for rigorous proof in mathematics, and be able to apply various different methods, including proof by induction and contradiction, to propositions in set theory and the theory of numbers;
- know the basic properties of the natural numbers, rational numbers and real numbers;
- understand elementary counting arguments and the properties of the binomial coefficients;
- be familiar with elementary number theory and be able to apply your knowledge to the solution of simple problems in modular arithmetic;
- understand the concept of countability and be able to identify typical countable and uncountable sets.

Dynamics and Relativity

24 lectures, Lent Term

This course assumes knowledge from A-level mechanics (or the equivalent). If you are unsure whether you have the necessary background, then you should attend at least the first lecture of the non-examinable Introduction to Mechanics course in the Michaelmas Term.

This course is the first look at theoretical physics. The course is important not just for the material it contains; it is also important because it serves as a model for the mathematical treatment of all later courses in theoretical physics.

The first 17 lectures are on classical dynamics. The basis of the treatment is the set of laws due to Newton that govern the motion of a particle under the action of forces, and which can be extended to solid bodies. The approach relies heavily on vector methods.

One of the major topics is motion in a gravitational field. This is not only an important application of techniques from this course and the Differential Equations course, it is also of historical interest: it was in order to understand the motions of the planets that Isaac Newton developed calculus.

With the advent of Maxwell's equations in the late nineteenth century came a comfortable feeling that all was well in the world of theoretical physics. This complacency was rudely shaken by Michelson's attempt to measure the velocity of the Earth through the surrounding aether by comparing the speed of light measured in perpendicular directions. The surprising result was that it makes no difference whether one is travelling towards or away from the light source; the velocity of light is always the same. Various physicists suggested a rule of thumb (time dilation and length contraction) which would account for this phenomenon, but it was Einstein who deduced the underlying theory, special relativity, from his considerations of the Maxwell equations.

In this short introduction, the last 7 lectures of this course, there is time only to develop the framework in which the theory can be discussed (the amalgamation of space and time into Minkowski space-time) and tackle simple problems involving the kinematics and dynamics of particles.

Learning outcomes

By the end of this course, you should:

- appreciate the axiomatic nature of, and understand the basic concepts of, Newtonian mechanics;
- be able to apply the theory of Newtonian mechanics to simple problems including the motion of particles, systems of particles and rigid bodies, collisions of particles and rotating frames;
- be able to calculate orbits under a central force and investigate their stability;
- be able to tackle problems in rotating frames;
- be able to solve relativistic problems involving space-time kinematics and simple dynamics.

4 Descriptions of non-examinable courses

Introduction to Mechanics

10 lectures, Michaelmas Term

This course covers the background material in mechanics required for the Dynamics and Relativity course in the Lent Term (and for later courses in applied mathematics and theoretical physics). It is intended for students who have taken only a limited amount of mechanics at A-level (or the equivalent), e.g. students who have not taken mechanics as part of Further Maths A-level. You should attend at least the first lecture if you are unsure whether you have covered the right material. Each of the lectures will discuss an important topic, such as conservation of momentum or conservation of energy, including worked examples, and each topic will be announced beforehand, so you can decide whether you should attend. The course should not require a significant investment of time.

Concepts in Theoretical Physics

8 lectures, Easter Term

This course is intended to provide a mathematical, but not highly detailed, account of eight topics in theoretical physics. Each lecture will be self-contained, although there may be a few cross-references between them where appropriate. The style will be informal: there is no prescribed syllabus and there will be no work to take away. It should be of interest to all students, especially to provide some relief from revision.

Some of the lectures will be wide-ranging, looking at general features of the mathematical equations used in physics and the surprising power of *action principles*. Other lectures will focus upon a particular subject. The aim throughout is to survey some essential features of each topic, using mathematics that you know, from foundations to unsolved problems. Overall, the course will explore the mathematical physics of the very small (quantum mechanics and elementary particles), the very large (general relativity and cosmology), and the very complex (chaos, organised complexity and statistical physics).

5 Computational Projects (CATAM)

The lectures for this course should be attended in the Easter Term of the first year.

The Computational Projects course (CATAM) consists mainly of practical projects, with an emphasis on understanding the physical and mathematical problems being modelled rather than on the details of computer programming. Projects must be written up and submitted during the second year (with deadlines just after the start of Lent and Easter Terms) and marks contribute to the total result for the Part IB examination. Lectures are given in the Easter Term of the first year to introduce some of the mathematical and practical aspects of the various projects. This allows an early start to be made on CATAM over the summer, which is strongly recommended. More details are available in the Part IB Computational Projects Manual, which is available on-line <http://www.maths.cam.ac.uk/undergrad/catam/IB>. The main programming language is Matlab.

Learning outcomes

By the end of this course, you should:

- be able to programme using a traditional programming language;
- understand the limitations of computers in relation to solving mathematical problems;
- be able to use a computer to solve problems in both pure and applied mathematics involving, for example, solution of ordinary differential equations and manipulation of matrices.

6 Informal descriptions of Part IB Easter Term courses

The following courses are lectured in the Easter Term and examined in Part IB.

Optimisation

12 lectures, Easter Term

This course may be taken in the Easter Term of either the first or the second year.

A typical problem in optimisation is to find the cheapest way of supplying a set of supermarkets from a set of warehouses: in more general terms, the problem is to find the minimum (or maximum) value of a quantity when the variables are subject to certain constraints. Many real-world problems are of this type and the theory discussed in the course are practically extremely important as well as being interesting applications of ideas introduced earlier in Numbers and Sets and Vectors and Matrices.

The theory of Lagrange multipliers, linear programming and network analysis is developed. Topics covered include the simplex algorithm, the theory of two-person games and some algorithms particularly well suited to solving the problem of minimising the cost of flow through a network.

Learning outcomes

By the end of this course, you should:

- understand the nature and importance of convex optimisation;
- be able to apply Lagrangian methods to solve problems involving constraints;
- be able to solve problems in linear programming by methods including the simplex algorithm and duality;
- be able to solve network problems by methods using, for example, the Ford–Fulkerson algorithm and min-cut max-flow theorems.

Variational Principles

12 lectures, Easter Term

This course may be taken in the Easter Term of either the first year or the second year; however it contains helpful background material for many of the other applied courses in Part IB.

The techniques developed in this course are of fundamental importance throughout physics and applied mathematics, as well as in many areas of pure and applicable mathematics.

The first part of the course considers stationary points of functions on \mathbb{R}^n and extends the treatment in IA Differential Equations to deal with *constraints* using the method of *Lagrange multipliers*. This allows one to determine e.g. the stationary points of a function on a surface in \mathbb{R}^3 .

The second part of the course deals with *functionals* (and functional derivatives) and enables one to find the path that minimises the distance between two points on a given surface (a *geodesic*), the path of a light ray that gives the shortest travel time (satisfying *Fermat's Principle*), or the minimum energy shape of a soap film.

Many fundamental laws of physics (in Newtonian mechanics, relativity, electromagnetism or quantum mechanics) can be expressed as variational principles in a profoundly elegant and useful way that brings underlying symmetries to the fore.

Learning outcomes

By the end of this course, you should:

- understand the concepts of a functional, and of a functional derivative;
- be able to apply constraints to variational problems;
- appreciate the relationship between variational statements, conservation laws and symmetries in physics.