

### 3 Fluid and Solid Mechanics

#### 3.6 Particle Drift in a Periodic Flow Field (4 units)

*This project builds on material in the Part IB Fluid Dynamics course.*

A one-dimensional periodic flow in a fluid has velocity  $u$  in the  $x$ -direction only, given by

$$u = \alpha \cos k(x - ct). \quad (1)$$

A material fluid element subject to this motion will have trajectory  $X(t)$  satisfying

$$\frac{dX}{dt} = \alpha \cos k(X(t) - ct). \quad (2)$$

**Question 1** Explain why, without loss of generality, distance and time units may be chosen so that  $k = 2\pi$  and  $c = 1$ , giving

$$\frac{dX}{dt} = a \cos 2\pi(X(t) - t). \quad (3)$$

How is  $a$  related to  $\alpha$ ?

**Question 2** Solve (3) numerically for a representative set of values of  $a$ , taking  $X(0) = 0$ . Describe your results qualitatively, and plot the solutions against time. You can use your own ODE integrator, or alternatively one such as the MATLAB function `ode45`. In either case you should justify the accuracy of your results (for example, by considering results produced with different step-sizes or tolerances). What if  $X(0) \neq 0$ ?

**Question 3** Verify from your numerical results that for  $|a|$  sufficiently small, there is a time-averaged mean ‘drift’ velocity of  $\frac{1}{2}a^2$ . Include details of your method.

**Question 4** Give a *physical* interpretation of the interaction between the flow and the material element. Do not confine your answer only to small  $|a|$ .

*Hint:* You may wish to consider a graph of  $\frac{dX}{dt}$  against  $X$ .

**Question 5** Analyse mathematically the above system, using any approach you see fit, e.g. in the case of question 3 you might seek an approximate solution for small  $|a|$ .