## 3 Fluid and Solid Mechanics

### 3.6 Particle Drift in a Periodic Flow Field

This project builds on material in the Part IB Fluid Dynamics course.
A one-dimensional periodic flow in a fluid has velocity $u$ in the $x$-direction only, given by

$$
\begin{equation*}
u=\alpha \cos k(x-c t) . \tag{1}
\end{equation*}
$$

A material fluid element subject to this motion will have trajectory $X(t)$ satisfying

$$
\begin{equation*}
\frac{d X}{d t}=\alpha \cos k(X(t)-c t) . \tag{2}
\end{equation*}
$$

Question 1 Explain why, without loss of generality, distance and time units may be chosen so that $k=2 \pi$ and $c=1$, giving

$$
\begin{equation*}
\frac{d X}{d t}=a \cos 2 \pi(X(t)-t) . \tag{3}
\end{equation*}
$$

How is $a$ related to $\alpha$ ?

Question 2 Solve (3) numerically for a representative set of values of $a$, taking $X(0)=0$. Describe your results qualitatively, and plot the solutions against time. You can use your own ODE integrator, or alternatively one such as the Matlab function ode45. In either case you should justify the accuracy of your results (for example, by considering results produced with different step-sizes or tolerances). What if $X(0) \neq 0$ ?

Question 3 Verify from your numerical results that for $|a|$ sufficiently small, there is a time-averaged mean 'drift' velocity of $\frac{1}{2} a^{2}$. Include details of your method.

Question 4 Give a physical interpretation of the interaction between the flow and the material element. Do not confine your answer only to small $|a|$.
Hint: You may wish to consider a graph of $\frac{d X}{d t}$ against $X$.
Question 5 Analyse mathematically the above system, using any approach you see fit, e.g. in the case of question 3 you might seek an approximate solution for small $|a|$.

