UNIVERSITY OF CAMBRIDGE

A brief introduction to the Cambridge Mathematics Course

First Year (Part IA)

About the course

The Cambridge mathematics course is called the Mathematical Tripos.

In the first year, there are 8 lecture courses, which is two lectures a day for two terms, covering a wide range of mathematics. Students take all 8 courses, which serve as a platform for the later years.

There are courses in:

- **abstract algebra**, which is the study of mathematical structures, such as sets, vector spaces and groups;
- **analysis**, which is the study of the foundations of calculus;
- **number theory,** in which equations involving integers are investigated;
- **differential equations**, in which equations involving rates of change are investigated;
- **mathematical methods,** which provide the basis for mathematical applications; for example, to theoretical physics;
- **Newtonian dynamics and special relativity**, in which the laws of Newton and Einstein are formulated mathematically;
- **probability,** which is (probably) what you think it is.

At the end of the year, there are four three-hour examinations.

Equations

- Here is a definition from the Analysis course. It says that you can draw a *continuous* function f without taking the pencil off the paper.

$$\text{Given } \epsilon > 0,\ \exists\delta \text{ such that } |f(x) - f(a)| < \epsilon\ \forall\ x \in (a-\delta, a+\delta).$$

- Here is an equation from the Vector Calculus course. It says that the amount that stuff expands in a fixed volume is equal to the amount of stuff crossing the boundary of the volume.

$$\int_V \nabla \cdot \boldsymbol{F} dV = \int_{\partial V} \boldsymbol{F} \cdot d\boldsymbol{S}$$

- This result from Probability says that random things tend to be Normally distributed if there are enough of them.

$$\lim_{n\to\infty} P(\sqrt{n}(S_n - \mu)/\sigma) \to \Phi(z)$$

- Here is an equation from Group Theory. It says, for example, that if you shuffle a pack of cards (same shuffle) 80,658,175,170,943,878,571,660, 636,856,403,766,975,289,505,440,883,277,824,000,000,000,000 times, the pack returns to its original state (try it!).

$$g^{|G|} = e$$

- The relativistic rocket equation, from Dynamics and Special Relativity

$$V = c \tanh\left(\frac{v_e}{c} \ln \frac{m_0}{m_1}\right)$$

tells us how fast a rocket goes if it expels a mass $m_0 - m_1$ of fuel at speed v_e.

Second Year (Part IB)

About the course

In the second year, there are 17 lecture courses, and a computational project.

Students decide how many courses to take: unusually (maybe uniquely) there is no fixed number for examination purposes. Students generally study more courses than they prepare for examination, to give themselves a good grounding for the third year.

There are more courses in:

- **abstract algebra;**
- **analysis;**
- **mathematical methods.**

There are new courses, including:

- **geometry of curved spaces;**
- **quantum mechanics;**
- **electromagnetism;**
- **statistics.**

At the end of the year, there are four three-hour examinations, and reports are submitted for the computational projects course.

Equations

- The Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\phi + V\phi = i\hbar\frac{\partial\phi}{\partial t}$$

expresses the conservation of energy in quantum mechanical systems.

- Maxwell's equations

$$F^{ab}{}_{,b} = \mu_0 J^a \qquad F_{[ab,c]} = 0$$

are the fundamental equations of electromagnetism; solutions tell us, for example, how light propagates.

- The basic equation of complex analysis, due to Cauchy (as are most other equations in the subject), is

$$\oint f(z)dz = 0$$

which is an integral round a closed path in the complex plane.

- The Cayley-Hamilton theorem for a matrix A

$$P(\lambda) \equiv \det(\lambda I - A) \Longrightarrow P(A) = 0$$

asserts that any matrix satisfies its own characteristic equation.

- In statistics, the Rao-Blackwell theorem is a statement about expected loss:

$$E(L(\delta_1(X))) \leq E(L(\delta(X)))$$

Third Year (Part II)

In the third year, there are 35 lecture courses, and a computational project. As in the second year, students decide how many courses to take: usually three or four a term. Again, there is no fixed number for examination purposes.

The courses include some whose content may be guessed at from the titles, such as:

- **Number Theory,**
- **Coding and Cryptography,**
- **Mathematical Biology,**
- **Cosmology,**
- **Logic and Set Theory,**
- **Principles of Statistics,**
- **Waves**

and some whose content remains obscure unless you know about these things:

- **Galois Theory** (advanced group theory in which it is proved that you can't in general solve a quintic equation);
- **Algebraic Topology** (in which properties of similar shapes- such as doughnuts and teacups- are classified);
- **Asymptotic Methods** (how functions behave at large values of their arguments);
- **General Relativity** (a theory of gravity);
- **Stochastic Financial Models** (how to predict unpredictable markets).

At the end of the year, there are four three-hour examinations, and reports are submitted for the computational projects course.

Equations

- $\theta = 2\arcsin\frac{1}{3}$ is the angle of the wake made by a ship or a duck, which is derived in the Waves course.
- The Einstein equations

  $$R_{ab} - \frac{1}{2}Rg_{ab} = \frac{8\pi G}{c^4}T_{ab}$$

  are solved in General Relativity.

- the Prime Number theorem, discussed in the Number Theory course

  $$\pi(x) \sim \frac{x}{\log x}$$

  approximates the number of prime numbers less than a given number x.

- In Coding and Cryptography, the RSA, which is one of the first public-key cryptosystems, is derived:

  $$c \equiv m^e \mod n \qquad m \equiv c^d \mod n$$

- The Riemann hypothesis

  $$\zeta(z) = 0 \Longrightarrow \Re z = \tfrac{1}{2} \quad (\text{or } z = -2m)$$

  gets a mention, but not a proof, in Further Complex Methods.

- Black and Scholes received Nobel prizes for their celebrated equation

  $$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

  which is derived by our third-year students in Stochastic Financial Models.

Fourth Year (Part III) - optional, leading to MMath

Part III is the jewel in the crown of our course. It goes back to 1769, when it was known as 'The Smith's Prize examination',and is recognised as a world-leading taught Master's course in mathematics and one of the best ways of preparing for graduate work in mathematics or theoretical physics.

The course Is exciting and varied as no other mathematics course.

In recent times Part III has developed into a course offering around 80 different courses (you would normally choose between six and eight) more than 100 possible topics for the optional dissertation in which students have to review recent research in an area of their choice. Courses on offer span the whole range of Mathematics and its applications, Theoretical Physics and Probability and Statistics, and aim to introduce students to the latest developments In the field, in preparation for research. Part III provides an essential link in maintaining a buzz of mathematical excitement all the way up from first year undergraduates to research students and academic staff.

Cambridge mathematics undergraduates who wish stay on to do Part III. currently around 90, are joined by other Cambridge students who have done physics, and by around 170 students from other universities in the UK and the rest of the world. With students from many different backgrounds, you will have the opportunity to experience high level mathematics within a truly rich environment.

You will have the opportunity to be taught by some of the world's best mathematicians.

Topics are at the cutting edge of mathematical research, and often taught by the people who introduced them or who have made the greatest strides in recent research in the field.

Random examples include: '**String Theory',** by the theoretical physicist whose work foreshadowed M-Theory; **'The Unified Method for Boundary Value Problems'**, by the mathematician who first introduced it; '**Geometric Group Theory', 'Advanced Financial Models'. 'Algebraic Number Theory·, 'Quantum Information Theory', 'Origin and Evolution of Galaxies'. 'Analysis of Operators', 'Black Holes', 'Computability and Logic', 'Fluid Dynamics of Climate', 'Algebraic Topology'** and **'Biostatistics'.**

You will have excellent career prospects.

Most students taking Part III go on to further research, either in Cambridge or in leading universities elsewhere in the UK and overseas. Those who do not are snapped up by employers in a wide variety of fields. from the financial sector to Industry, GCHQ, other parts of the public sector and many more. The rigour and breadth of the course are widely known and much appreciated. So enjoy the Maths in the knowledge that it will also be great for your career!

Equations

There are many truly marvellous equations in Part III of the Mathematical Tripos, but the margin of this leaflet is too narrow to contain them.