UNIVERSITY OF CAMBRIDGE
Faculty of Mathematics

MATHEMATICAL READING LIST

This list of interesting mathematics books and internet sites is mainly intended for sixth-formers planning to take a degree in mathematics. However, everyone who likes mathematics should take a look: some of the items are very suitable for less experienced readers and even the most hardened mathematician will probably find something new here.

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INTRODUCTION

The range of mathematics books now available is enormous. This list just contains a few suggestions which you should find helpful if you wish to choose some mathematical reading. Do not feel you should read all or most of these books: there is far too much here for even the most dedicated reader! Just pick what appeals to you. Any reading you do will certainly prove useful, and it is also perfectly alright not to have read any of the books in this list!

The books here are divided into three groups: historical and general (which aim to give a broad idea of the scope and development of the subject); recreational, from problem books (which aim to keep your brain working) to technical books (which give you insight into a specific area of mathematics and include mathematical discussion); and textbooks (which cover a topic in advanced mathematics of the kind that you will encounter in your first year at university). Do not feel that you should only read the difficult ones: medicine is only good for you if it is hard to take, but this is not true for mathematics books. t

All the books on the list should be obtainable from your local library, though you may have to order them. Most are available (relatively) cheaply in paperback or as kindle editions and so would make good additions to your Christmas list. Some may be out of print, but still obtainable from libraries.

You might also like to look on the web for mathematics sites. Good starting points are:

NRICH (http://nrich.maths.org.uk)

which is a web-based interactive mathematics club; in addition there is:

Plus (http://plus.maths.org.uk),

which is a web-based mathematics journal. Both these sites are based in Cambridge.

1 DOING MATHEMATICS

How to Solve It George Polya (Penguin, 1990)

An old gem. First published in 1945, this book is an invaluable and timeless guide to mathematical problem-solving. The Fields medallist Terry Tao describes it as the book from which he himeslf learnt. This is the paperback edition, with an interesting and entertaining foreword by Ian Stewart. The Kindle edition (2014) has a foreword by John H. Conway. Highly recommended.

How to study for a maths degree Lara Alcock (OUP, 2013)

This sounds like the sort of book that could be terrible, but it turns out to be rather good. What is written on the cover tells you accurately what is inside, so there is no need to say any more. Definitely worth a look.

How to Think like a Mathematician Kevin Houston (CUP, 2009)

There is lots of good mathematics in this book (including many interesting exercises) as well as lots of good advice. How can you resist a book the first words of which (relating to the need for accurate expression) are:

Question: How many months have 28 days?

Mathematician's answer: All of them.

2 HISTORICAL AND GENERAL

One of the most frequent complaints of mathematics undergraduates is that they did not realise until too late what was behind all the material they wrote down in lectures: Why was it important? What were the problems which demanded this new approach? Who did it? There is much to be learnt from a historical approach, even if it is fairly non-mathematical.

Mathematics: Queen and Servant of Science E.T. Bell (Spectrum, 1996)

Another old gem. An absorbing account of pure and applied mathematics from the geometry of Euclid to that of Riemann, and its application in Einstein's theory of relativity. The twenty chapters cover such topics as: algebraroups, ge, number theory, logic, probability, infinite sets and the foundations of mathematics, rings, matrices, transformations, gometry, and topology. As Martin Gardner says in the foreword: "This continues to be one of the finest of all introductions to the rich diversity of those fantastic structures that mathematicians invent, explore, and apply with such mysterious success to the huge unfathomable world outside the little organic computers at the top of their heads".

Mathematicians: An Outer View of the Inner World Mariana Cook (Princeton University Press, 2009)
Another, more modern, biographical approach. This book gives a compelling and immediate introduction to some of the most amazing mathematicians of our time, not just through a glimpse of their brilliant mathematical work, but also of their experience as fathers, daughters, husbands, wives... Each portrait is personal and in the voice of the mathematicians themselves. You will find out what inspired them to pursue maths, and no doubt be inspired yourself to participate in the joy of mathematical discovery.

A Russian Childhood S. Kovalevskaya (trans. B. Stillman) (Springer, 1978)
Sonya Kovalevskaya was the first woman in modern times to hold a lectureship at a European university: in 1889 she was appointed a professor at the University of Stockholm, in spite of the fact that she was a woman (with an unconventional private life), a foreigner, a socialist and a practitioner of the new Weierstrassian theory of analysis. Her memories of childhood are non-mathematical but fascinating. She discovered in her nursery the theory of infinitesimals: times being hard, the walls had been papered with pages of mathematical notes.

Alan Turing, the Enigma A. Hodges (Vintage, 2014)
A great biography of Alan Turing, a pioneer of modern computing. The title has a double meaning: the man was an enigma himself, and the German code that he was instrumental in cracking was generated by the Enigma machine. The book is largely non-mathematical, but there are no holds barred when it comes to describing his major achievement, now called a Turing machine, with which he demonstrated that a famous conjecture by Hilbert is false.

In Code: A Mathematical Journey Sarah Flannery and David Flannery (Workman Publishing, 2001)
This is the story of Sarah Flannery, who at age 16 won the titles of 1999 Irish Young Scientist of the Year and European Young Scientist of the Year for her innovative work on cryptography. An inspiring read just for letting onself be carried along by the enthusiasm of mathematical discovery and exploration, narrated by an adolescent with freshness and honesty, without hiding setbacks or frustration on the way. Plenty of actual maths too: enjoy the accessible explanation of RSA Public Key cryptography, and the many mathematical puzzles that intrigued Sarah during her mathematical journey (do have a crack at them before looking at the solutions!).

The Man Who Knew Infinity R. Kanigel (Abacus, 1992)
The life of Ramanujan, the self-taught mathematical prodigy from a village near Madras. He sent Hardy samples of his work from India, which included rediscoveries of theorems already well known in the West and other results which completely baffled Hardy. Some of his estimates for the number of ways a large integer can be expressed as the sum of integers are extraordinarily accurate, but seem to have been plucked out of thin air.

A Mathematician's Apology G.H. Hardy (CUP, 1992)
Hardy was one of the best mathematicians of the first part of this century. Always an achiever (his New Year resolutions one year included proving the Riemann hypothesis, making 211 not out in the fourth test at the Oval, finding an argument for the non-existence of God which would convince the general public, and murdering Mussolini), he led the renaissance in mathematical analysis in England. Graham Greene knew of no writing (except perhaps Henry James's Introductory Essays) which conveys so clearly and with such an absence of fuss the excitement of the creative artist. There is an introduction by C.P. Snow.

The man who loved only numbers Paul Hoffman (Fourth Estate, 1999)
An excellent biography of Paul Erdös, one of the most prolific mathematicians of all time. Erdös wrote over 1500 papers (about 10 times the normal number for a mathematician) and collaborated with 485 other mathematicians. He had no home; he just descended on colleagues with whom he wanted to work, bringing with him all his belongings in a suitcase. Apart from details of Erdös's life, there is plenty of discussion of the kind of problems (mainly number theory) that he worked on.

Surely You're Joking, Mr Feynman R.P. Feynman (Arrow Books, 1992)
Autobiographical anecdotes from one of the greatest theoretical physicists of the last century, which became an immediate best-seller. You learn about physics, about life and (most puzzling of all) about Feynman. Very amusing and entertaining.

Fermat's Last Theorem Simon Singh (Fourth Estate, 2002)
This story of Andrew Wiles's proof of Fermat's Last Theorem is a firm favourite. Simon Singh does a great job of explaining some of the very technical mathematics at the heart of Andrew Wile's proof, including along the way all sorts of mathematical ideas and anecdotes. Of course most mathematical research is not as obsessive and shrouded in secrecy as this particular search, but the book reads almost as an exciting thriller (with entertaining digressions) and conveys true passion for the beauty of mathematics. You can watch the author talk about the mathematics involved in the book on *Numberphile*, at
https://youtu.be/qiNcEguuFSA

The Simpsons and Their Mathematical Secrets Simon Singh (Bloomsbury, 2013)
"There is tons of maths hidden in the Simpsons", as Simon Singh says. The sheer love of mathematics by the producers of this popular series (mostly mathematicians) shines through. Great fun, if you're a fan of the Simpsons and Futurama. For this book too there is a companion video, at
https://youtu.be/bk_Kjpl2AaA

The Music of the Primes Marcus du Sautoy (Harper-Collins, 2003)
This is a wide-ranging historical survey of a large chunk of mathematics with the Riemann Hypothesis acting as a thread tying everything together. The Riemann Hypothesis is one of the big unsolved problems in mathematics – in fact, it is one of the Clay Institute million dollar problems – though unlike Fermat's last theorem it is unlikely ever to be the subject of pub conversation.
Du Sautoy's book is insightful and attractively written. Some of the maths is tough but the history and storytelling paint a convincing (and appealing) picture of the world of professional mathematics.

Finding Moonshine: a mathematician's journey through symmetry Marcus Du Sautoy (Fourth Estate, 2008)
This book has had exceptionally good reviews (even better than Du Sautoy's Music of the Primes listed above). The title is self-explanatory. The book starts with a romp through the history and winds up with some very modern ideas. You even have the opportunity to discover a group for yourself and have it named after you.

Symmetry and the Beautiful Universe Leon M. Lederman and Christopher T. Hill (Prometheus, 2004)
The notion of symmetry is central to understanding the laws of physics governing the universe. This book succeeds in making some of the most subtle and profound concepts of modern physics accessible to a general audience, with minimal use of mathematical equations. It will take you, through the unifying idea of symmetry, from classical mechanics and discussions of inertia in the solar system and Newton's laws, to Noether's theorem and the connection between symmetry and conservation laws, to Einstein's relativity, the standard model and Higgs boson. The journey is peppered with original and entertaining stories and analogies. Hugely recommended, though beware of the Kindle edition, which is apparently full of misprints!

Closing the Gap: The Quest to Understand Prime Numbers Vicky Neale (OUP, 2017)
Some famous mathematical problems have the knack of being very easy to state, but very difficult to solve. Fermat's last theorem is an example. The twin prime conjecture, on which this book focuses, is another. This conjecture states that there are infinitely many prime numbers whose difference is 2. As you can guess by its name, a proof still hasn't been found. Neale does a great job of taking the reader through attempts at getting closer to proving the conjecture, explaining on the way the building blocks of number theory and cutting edge mathematical ideas in a way that is both approachable and rigorous. She conveys the excitement of mathematical research and collaboration, including the online 'massive collaborative' Polymath project initiated just a few years ago by Tim Gowers, which led to new proofs of related mathematical problems. The book also has plenty of problems to make you think, which is always fun, and very good for you!

Euler's Pioneering Equation Robin Wilson (OUP, 2018)
Euler's equation,$e^{i\pi} + 1 = 0$ connects five of the most important numbers in mathematics, and has been voted the most beautiful mathematical theorem on several occasions. MRI scans have shown that this elegant and deeply meaningful equation affect the brains of mathematicians in the same way as a work of art! Robin Wilson explores each of the numbers in turns, through their historical mathematical development, then devotes the last chapter to the equation itself. You will learn many ways of approximating π, anecdotes and 'near misses' by great past mathematicians, applications and proofs. A book for the curious, and an enjoyable introduction to some fundamental mathematical ideas.

3 MATHEMATICAL ADVENTURES AND RECREATIONAL

You can find any number of puzzle books in the shops and some which are both instructive and entertaining are listed here. Other books in this section do not attempt to set the reader problems, but to give an appetising introduction to important areas of, or recent advances in, mathematics.

http://www.cut-the-knot.org Alexander Bogomolny (since 1996)
This web site is absolutely brilliant. If you haven't seen it before, you should take a look immediately. It is like a mathematical labyrinth: you can wander through it for hours (years, probably), following different links. It covers a huge range of mathematics, much of which is elementary (which is not the same as saying it is easy) and all of which is interesting and beautifully presented — see, for example, the 103 essentially different proofs of Pythagoras's theorem.

http://www.dimensions-math.org Etienne Ghys (since 2006)
A mathematical film: a fascinating and very accessible journey though geometry and topology (where a teacup and a doughnut have the same shape), touching on maps, tilings, transformations in space and fractals, that takes you gradually to the fourth dimension. On the way you'll pass through Flatland, encounter famous mathematicians and artists, and see abstract mathematical concepts in beautiful graphics by mathematical artist and engineer Jos Leys. And, if you're up to some more difficult mathematics, check out the connections between knots and the Lorenz attractor, and even the Riemann hypothesis, at `www.josleys.com/articles/ams_article/Lorenz3.htm`

The Colossal Book of Mathematics M. Gardner (Norton 2004)
Over 700 pages of Gardner for under 20 pounds is an astonishing bargain. You will be hooked by the very first topic in the book if you haven't seen it before (and probably even if you have): a diophantine problem involving a monkey and some coconuts — can't say more without writing a spoiler. At the beginning, about 60 other books by Martin Gardner are listed, none of which will disappoint.

Problem Solving Through Recreational Mathematics Bonnie Averbach & Orin Chein (Dover 2000)
One can never have enough maths puzzles! This is another great collection, from easier ones to some that will leave you stumped, through quite enough variety to please all tastes, and to give an introduction to all the main areas of mathematics while you have fun making your way through it. Lots of practice problems, and hints and solutions to most puzzles.

Game, Set and Math. I. Stewart (Penguin, 1997)
Stewart is one of the best current writers of mathematics (recreational or otherwise). This collection (which includes a calculation which shows why you need only be marginally the better player to win a tennis match — whence the title) was originally written in French: some of the puns seem to have suffered in translation, but the *joie de vivre* shines through. You might also like Stewart's book on Chaos, *Does God Play Dice?* (Penguin, 1990). Excellent writing again but, unlike the chaos books mentioned below, no colour pictures. The title is a quotation from Einstein, who believed (probably incorrectly) that the answer was no; he thought that theories of physics should be deterministic, unlike quantum mechanics which is probabilistic.

To Infinity and Beyond Eli Maor (Princeton, 1991)
Not much hard mathematics here, but lots of interesting mathematical ideas (prime numbers, irrationals, the continuum hypothesis, Olber's paradox (why is the sky dark at night?) and the expanding universe to name but a few), fascinating history and lavish illustrations. The same author has also written a whole book about one number (*e The Story of a Number*), also published by Princeton (1994).

Cakes, Custard and Category Theory: Easy recipes for understanding complex maths Eugenia Cheng (Profile Books Ltd, 2015)
Entertaining, innovative, and packed with infectious enthusiasm and unexpectedly mathematical recipes. The baking metaphors may seem a little forced at times, but on the whole work well.

A Mathematical Mosaic Ravi Vakil (Brendan Kelly, 2008)
This is a bit unusual. It's is a brilliant collection of some of the most beautiful and intriguing mathematical ideas and problems, from Combinatorics to Game Theory, Geometry, Galois Theory and much more, weaved into a mosaic that shows their connection. Ravi Vakil won several mathematical competitions and olympiads, and his perspective comes from his personal problem solving adventure. The style is friendly and fun. The maths wonderful. The author starts by stating "Mathematics is an intensely social discipline" and proceeds to thank all those who have shared their love of mathematics with him, especially "the young men and women who agreed to share their unique perspective on the joys of mathematics for the seven profiles included in this book".
Don't be discouraged by the profiles of exceptional young mathematicians – they *are* exceptional!

4 READABLE MATHEMATICS

The MαTH βOOK Clifford A Pickover (Sterling, 2009)
The subtitle is 'From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics'. Each left hand page has a largely non-mathematical description of one of the great results in mathematics and each right hand page has a relevant illustration. There is just enough mathematical detail to allow you to understand the result and pursue it (if you fancy it), via google. The book is beautifully produced. The illustration for the page on Russell and Whitehead's Principia Mathematica, said here to be the 23rd most important non-fiction book of the 20th century, is the proposition occurring several hundred pages into the book, that $1 + 1 = 2$.

The Mathematical Experience P.J. Davis & R. Hersh (Penguin, 1990)
This gives a tremendous foretaste of the excitement of discovering mathematics. A classic.

Mathematics: a very short introduction Timothy Gowers (OUP, 2002)
Gowers is a Fields Medallist (the Fields medal is the mathematical equivalent of the Nobel prize), so it is not at all surprising that what he writes is worth reading. What is surprising is the ease and charm of his writing. He touches lightly many areas of mathematics, some that will be familiar (Pythagoras) and some that may not be (manifolds) and has something illuminating to say about all of them. The book is small and thin: it will fit in your pocket. You should get it.

Solving Mathematical Problems Terence Tao (OUP, 2006)
Tao is another Fields Medallist. He subtitles this little book 'a personal perspective' and there is probably no one better qualified to give a personal perspective on problem solving: at 13, he was the youngest ever (by some margin) gold medal winner in International Mathematical Olympiad. There are easy problems (as well as hard problems) and good insights throughout. The problems are mainly geometric and algebraic, including number theory (no calculus).

Archimedes' Revenge P. Hoffman (Penguin, 1991)
This is not a difficult read, but it covers some very interesting topics: for example, why democracy is mathematically unsound, Turing machines and travelling salesmen. Remarkably, there is no chapter on chaos.

The Pleasures of Counting T.W. Körner (CUP, 1996)
A brilliant book. There is something here for anyone interested in mathematics and even the most erudite professional mathematicians will learn something new. Some of the chapters involve very little technical mathematics (the discussion of cholera outbreaks which begins the book, for example) while others require the techniques of a first or second year undergraduate course. However, you can skip through the technical bits and still have an idea what is going on. You will enjoy the account of Braess's paradox (a mathematical demonstration of the result, which we all know to be correct, that building more roads can increase journey times), the explanation of why we should all be called Smith, and the account of the Enigma code–breaking. These are just a few of the topics Körner explains with enviable clarity and humour.

Calculus for the Ambitious T.W. Körner (CUP, 2014)
You can and should supplement your sixth-form calculus with Körner's latest offering. You will find here some familiar ideas seen from unfamiliar angles and almost certainly much that is unfamiliar; multivariable calculus for example (when functions depend on more than one variable).
This excerpt from introduction gives you a flavour of the style: *When leaving a party, Brahms is reported to have said 'If there is anyone here whom I have not offended tonight, I beg their pardon.' If any logician, historian of mathematics, numerical analyst, physicist, teacher of pedagogy or any other sort of expert picks up this book to see how I have treated their subject, I can only repeat Brahms' apology.*

Logical Labyrinths Raymond S. Smullyan (CRC Press, 2008)
This book is a fun and engaging collection of logical puzzles, combined with a rigouous mathematical introduction to logic. The carefully graded and entertaining progression leads you to the more formal logical reasoning through a journey that is always challenging enough but manageable and rewarding. Enjoy!

Luck, Logic, and White Lies: The Mathematics of Games Jörg Bewersdorff (A K Peters Ltd, 2004)
Learn about (some of) the maths behind risk, uncertainty and gambling. Debunk some popular myths, and improve your chances of winning at games! Very readable, clear and practical. A good introduction.

Insights into Game Theory: An Alternative Mathematical Experience Ein-Ya Gura & Michael M. Maschler (CUP, 2008)
This book arose from Ein-Ya Gura's PhD dissertation. It provides an introduction to the field of Game Theory - the mathematical analysis of competitive strategies - for an audience without a background in higher mathematics. Although the book avoids formal mathematical notation, rigorous proofs are given of some of the major results of the field. And you can also use the many exercises provided to help consolidate the material.

What is Mathematics? R. Courant & H. Robbins (OUP, 1996)
A new edition of a classic, revised by Ian Stewart. It has chapters on numbers (including ∞), logic, duality, soap-films, etc. The subtitle (*An elementary approach to ideas and methods*) is rather optimistic: challenging would be a more appropriate adjective, though interesting or instructive would do equally well. Stewart has resisted the temptation to tamper: he has simply updated where appropriate — for example, he discusses the solution to the four–colour problem and the proof of Fermat's Last Theorem.

Beyond Numeracy J. A. Paulos (Penguin, 1991)
Bite-sized essays on fractals, game-theory, countability, convergence and much more. It is a sequel to his equally entertaining, but less technical, *Numeracy.*

Neural Networks and Deep Learning Michael Nielsen (Determination Press, 2015, available in web-only format at http://neuralnetworksanddeeplearning.com.)
This a free online interactive book, that will teach you the core concepts behind what enables a computer to learn from observational data. It uses a hands-on approach through many examples of code, whilst retaining a focus on the mathematical aspect of the core principles. The style is accessible and compelling, and the interactive elements throughout help develop intuition as well deepen understanding. This book is a great starting point for anybody who is interested in this area of AI with applications in image recognition, speech recognition, natural language processing, and many areas of everyday life.

From Here to Infinity Ian Stewart (OUP, 1996)
This is a revised version of *Problems in Mathematics (1987)*; revised of necessity, as the author says, because some of the problems now have solutions — an indication of the speed at which the frontiers of mathematics are receding. Topics discussed include solving the quintic, colouring, knots, infinitesimals, computability and chaos. In the preface, it is guaranteed that the very least you will get from the book is the understanding that mathematical research is not just a matter of inventing new numbers; what you will in fact get is an idea of what real mathematics is.

What's Happening in the Mathematical Sciences D. MacKenzie and B. Cipra (AMS, biennial publication, since 1993)
This is a really excellent series, It contains low(ish)-level discussions, with lots of pictures and photographs, of some of the most important recent discoveries in mathematics. Volumes 1 and 2 cover advances in map–colouring, computer proofs, knot theory, travelling salesmen, and much more. Volume 3 (1995–96) has, among other things, articles on Wiles' proof of Fermat's Last Theorem, the investigation of twin primes which led to the discovery that the Pentium chip was flawed, codes depending on large prime numbers and the Enormous Theorem in group theory (the theorem is small but the proof, in condensed form, runs to 5000 pages). Volume 9 (2012–13) includes an article on the CERN experiment that confirmed the Higgs Boson. The series is now up to Volume 12, which contains a three-part series on the maths behind the battle to beat COVID-19. MAA members get a discount. Exciting stuff.

The Penguin Dictionary of Curious and Interesting Numbers D. Wells (Penguin, 1997)
A brilliant idea. The numbers are listed in order of magnitude with historical and mathematical information. Look up 1729 to see why it is 'among the most famous of all numbers'. Look up 0.7404 ($= \pi/\sqrt{18}$) to discover that this is the density of closely-packed identical spheres in what is believed by many mathematicians (though it was at that time an unproven hypothesis) and is known by all physicists and greengrocers to be the optimal packing. Look up Graham's number (the last one in the book), which is inconceivably big: even written as a tower of powers $(9 \uparrow (9 \uparrow (9 \cdots)))$ it would take up far more ink than could be made from all the atoms in the universe. It is an upper bound for a quantity in Ramsey theory whose actual value is believed to be about 6. A book for the bathroom to be dipped into at leisure. You might also like Wells's *The Penguin Dictionary of Curious and Interesting Geometry* (Penguin, 1991) which is another book for the bathroom. It is not just obscure theorems about triangles and circles (though there are plenty of them); far-reaching results such as the hairy ball theorem (you can't brush the hair flat everywhere) and fixed point theorems are also discussed.

Curves for the Mathematically Curious: An Anthology of the Unpredictable, Historical, Beautiful, and Romantic Julian Havil (Princeton University Press, 2019)
This book opens with a quote by Felix Klen (1958): "*What is a curve?* Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions.", so prepare to be challenged! Meet some of the most famous and beautiful mathematical curves, and through them explore many different fields of mathematics, from statistics to cryptography, to analysis, to geometry, and much more. Lots of historical nuggets and fun facts too, besides the maths.

50 Visions of Mathematics Sam Parc (ed.) (OUP, 2014)
Written to celebrate the 50^{th} anniversary of the Institute of Mathematics and its Applications (IMA), this is a collection of 50 brief articles by different authors, introducing some of the many applications of mathematics in a variety of areas, brief biographies of some of the look at maths in the same way again. major mathematicians of the twentieth century, some fun maths, and some mathematical philosophy. All clear, engaging, sometimes humorous, with equations kept to a minimum (but further reading suggested to follow up where you interest has been piqued), and brilliant pictures. You'll find references to everyday life (how GPS satellites work, medical imaging, and many more) as well as references to fictional characters (for example, in 'The Simpson's rule'). There is an engaging and highly original article about Pythagora's theorem, cutting edge modern mathematics introduced in an accessible way, the mathematical explanation of the annoying traffic jams that occur for no apparent reason, even 'proof by pizza'. After reading this book, you may revise your idea of what applied mathematics is.

Reaching for Infinity S. Gibilisco (Tab/McGraw-Hill, 1990)
A short and comfortable, though mathematical, read about different sorts of infinity. It has theorems, too, which are good for you. An example: $\aleph_0 + \aleph_1 = \aleph_1$. This probably needs a bit of explanation. Loosely speaking: $\aleph_0$ (pronounced 'aleph' zero) is the number of integers (which is the same as the number of rational numbers) and $\aleph_1$ is the next biggest infinity. There is another infinity, $c = 2^{\aleph_0}$, which is the number of real numbers. The continuum hypothesis says that $c = \aleph_1$, but it was not realised until 1963 that this cannot be proved or disproved.

The Computational Beauty of Nature: Computer Explorations of Fractals, Chaos, Complex Systems, and Adaptation Gary W Flake (MIT Press, 2000)
This book covers a wide breadth of topics, including Gödel's Incompleteness theorem, Cellular Automata, Genetic Algorithms, and Neural Networks, as well as – as expected – Chaos and Fractals. It doesn't require much mathematics at all to start with, but introduces advanced mathematical concepts from scratch with clear explanations and helpful example, and conveys the fascinating links between these and the natural world. You can have fun using the numerous examples of pseudo-code to write some code and play around with each topic. A great introduction to many fascinating topics, and, especially, to using computer experiments as a tool to explore mathematical ideas.

Chaos J. Gleick (Minerva/Random House, 1997)
Sometimes, at interview, candidates are asked whether they have read any good mathematics books recently. There was a time when nine out of ten candidates who expressed a view named this one. Surely they couldn't all have been wrong? Still a very enjoyable introduction, with a focus on the history of chaos theory.

Chaos and Fractals: An Elementary Introduction David P. Feldman (OUP, 2012)
An enjoyable and very clear introduction to chaos and fractals that requires at the start no mathematics beyond basics algebra. This approach enables the author to take you gently from basics concepts to advanced mathematical ideas such as statistical stability, fractal dimension, Lyapunov exponent and chaotic flows in phase space. The many exercises help clarify and understand the wealth of mathematical topics tackled. Even though this is essentially a textbook, it is not dry at all: the writing is like a conversation, which peppers rigorous maths with illuminating opinions about the development of theories, and nuggets of quirky information about the mathematicians behind them.

Algorithms Unlocked Thomas H. Cormen (MIT, 2013)
"What are algorithms and why should you care?" asks this book. Mathematicians (aspiring and confirmed) should particularly care, given the usefulness of computers in many areas of mathematics, and also the overlap between what could be described as 'algorithmic thinking' and 'mathematical thinking'. Donal Knuth analysed exactly this overlap in an article in *The American Mathematical Monthly*. He concluded that the overlap is pretty much complete, aside from two main concepts: "complexity", i.e. economy of operation (algorithmic only), and "dealing with infinity" (mathematical only). If you're interested in problem solving with computers, but also the basic principles behind cryptography, or the fundamental ideas of data compression, this book is an engaging and readable introduction. It answers many interesting questions, including: "What does it mean to produce a correct solution to a problem?"

5 READABLE THEORETICAL PHYSICS

A Short History of Nearly Everything Bill Bryson (Black Swan 2004)
This one is for those of you who are more polymath than pure math. The book describes the Universe and everything in it.

The Strangest Man: The Hidden Life of Paul Dirac Graham Farmelo (Faber and Faber 2009)
Paul Dirac was one of the great scientists of the 20th century. His work unifying quantum mechanics and special relativity gave rise to the Dirac equation, often argued to be the most beautiful equation in physics. The man was almost as strange as his theories. You can read about both in this engaging biography. Special relativity, quantum mechanics, and the Dirac equation are covered in many courses throughout the Cambridge undergraduate degree.

Black Hole Blues Janna Levin (Vintage 2016)
A long time ago, in a galaxy far far away, two black holes collided. The collision was so violent it caused a rupture in the spacetime continuum, emitting ripples that we call gravitational waves. These waves travelled unimpeded through the universe for 1.4 billion years until, on September 14th 2015, we felt them. We will teach you about Einstein's theory of general relativity and gravitational waves in the third year. In the meantime, this book is great place to start.

The Particle at the End of the Universe Sean Carroll (Oneworld Publications 2013)
Not a great deal has changed on the elementary particle scene since this absorbing survey was written: it was just in time to report first sightings of the Z and W particles. It even reports, with (as it turned out) well-founded scepticism on claims to have seen the top quark. The final chapter makes the all-important link between particle physics (physics on the smallest scale) and cosmology (physics on the largest scale). The energies required to study the latest batch of elementary particles are so great that the Big Bang is the only feasible 'laboratory'.

The New Quantum Universe T. Hey & P. Walters (CUP, 2003)
The discovery of the Higgs boson in 2013 closed one chapter of particle physics and opened the next. This book recounts the story of quantum particles, from atoms to electrons and nuclei, to the wonders of quarks and gluons and W-bosons and, ultimately, the Higgs. The underlying mathematics will be covered in the fourth year of the maths course at Cambridge.

A Brief History of Time Stephen Hawking (Bantam Publishing 1988)
An oldie but a goodie. We couldn't finish this list without including the classic book from Cambridge's most famous physicist of recent times. Want to know why black holes aren't black? Or what imaginary time means? Or what happened at the big bang? This book won't really tell you. But it may pique your interest enough to stay on to the fourth year of the maths course where we will teach all of these things.

6 READABLE TEXTBOOKS

There is not much point in trying to cover a lot of material from the first year undergraduate mathematics course you are just about to start, but there is a great deal of point in trying to familiarise yourself with the sort of topics you are going to encounter. It is also a good plan to get used to working on your own; reading mathematics text books is an art not much practised in schools. Many of the following have exercises and answers and some have solutions.

Advanced Problems in Mathematics: Preparing for University S.T.C. Siklos (OpenBook Publishers, 2016)
This is a combined and much improved version by Stephen Siklos of his two previous booklets on STEP problems: *Advanced Problems in Core Mathematics* (2003) and *Advanced Problems in Mathematics* (1996). It is recommended as preparation for any undergraduate mathematics course, even for students who do not plan to take the Sixth Term Examination Paper (STEP- the examination normally used as a basis for conditional offers to Cambridge). It contains a selection of STEP–like problems complete

with hints, discussion and full solutions. There is a lovely introduction with two fully worked examples. The problems are different from most A-level questions, being much longer ('multi-step' is the current terminology) and sometimes covering material from apparently unconnected areas of mathematics. They are more like the sort of problems that you encounter in a university mathematics course, although they are based on the syllabuses of school mathematics. Working through one or both of these booklets would be an excellent way of getting your mathematics up to speed again after the summer break. The book is ***free to download*** from

http://www.openbookpublishers.com

or can be bought as a paperback.

Towards Higher Mathematics: A Companion Richard Earl (Cambridge University Press, 2017)

This is a nice book, with some very useful introductory advice, and a wealth of exercises for practice. Using the language of sixth form mathematics to introduce substantial extension material beyond the school syllabus, it enables students to bridge the gap between school-level and university-level mathematics. The wide choice of topics covered is ideal preparation for any undergraduate mathematics course. The more than 1500 carefully graded exercises are accompanied by hints included in the text, and solutions available online.

The Art of Statistics: Learning from Data David Spiegelhalter (Pelican Books, 2019)

This is a great introduction to Statistics, aimed at the general reader, but also with plenty of material going deeper into important topics for the more mathematically-inclined. It covers all the topics you would expect from an introductory course in Statistics, and takes you in graded steps from fairly basic to really challenging concepts. There are no mathematical equations and formulae, aside from the very useful and accessible technical glossary at the end. Important statistical concepts are explained using real life scenarios and case studies, building on intuition with cogent logical reasoning and helpful visualisations. You'll learn to distinguish 'good data' from 'bad data', and spot and call out deliberately abused data. On the way, you'll be intrigued by seeing how statistics can help identify the luckiest survivor from the Titanic, whether the mass-murderer Harold Shipman could have been caught earlier, or whether the body dug out in Leicester's car park really was Richard III. Expect to be taken through interesting stories, analysing the evidence in the way Sherlock Holmes would.

Mathematical Methods for Science Students G. Stephenson (Longman, 1973)

This starts with material you already know and advances cautiously in traditional directions. You may not be bowled over with excitement but you will appreciate the careful explanations, the many examples and exercises and the generally sympathetic approach. You may prefer an entirely problems-based approach, in which case *Worked examples in Mathematics for Scientists and Engineers* (Longman, 1985) by the same author is for you.

Mathematical Methods for Physics and Engineering K F Riley, M P Hobson & S J Bence (Cambridge University Press 1998)

Most of A-level pure mathematics consists of what could be called 'mathematical methods' — i.e. techniques you can use in other areas (such as mechanics and statistics). The continuation of this material forms a basic part of every university course (and would count as applied mathematics!). This book is a strong recommendation for any such course.

A Concise Introduction to Pure Mathematics Martin Liebeck (Chapman& Hall/CRC Mathematic, 2010)

This is really excellent. Liebeck provides a simple, nicely explained, appetizer to a wide variety of topics (such as number systems, complex numbers, prime factorisation, number theory, infinities) that would be found in any first year course. His approach is rigorous but he stops before the reader can get too bogged down in detail. There are worked examples (e.g. 'Between any two real numbers there is an irrational') and exercises, which have the same light touch as the text.

What is Mathematical Analysis? John Baylis (MacMillan, 1991)
This book (now out of print, but available from libraries) is part of a series which is supposed to bridge the gap between school and university. It covers some serious analysis (the intermediate value theorem, limits, differentiation and integration) in a most accessible style: it never gets hard, though you will need to study carefully. The layout could be nicer, but do not be put off: you can discover here a function which is continuous at every irrational number but discontinuous (it jumps) at every rational number. Who would have thought that was possible?

Groups: A Path to Geometry R.P. Burn (CUP, 1987)
Permutations, groups, matrices, complex numbers and, above all (or rather, behind all), geometry.

Yet Another Introduction to Analysis V. Bryant (CUP, 1990)
Yes, another; but a very good one. And it has solutions to the many problems. Analysis is the study of all those things you think you already know how to do (such as differentiation, integration), from first principles. This book goes through functions, continuity, series and calculus at a brisk trot; essential material for any mathematician.

A First Course in Mechanics Mary Lunn (OUP, 1991)
A bridge between the sort of mechanics you meet at A-level and the sort you are going to meet at university; not just a bridge, but also a good bit of road on the far side.

Understanding Probability Henk Tijm (CUP, 2012)
This book will give you a very accessible introduction to probability through a wealth of fun examples and exercises. The explanations are refreshingly clear and though-provoking, adding much even to well-known classic problems. You won't need much mathematics to read the first part, which provides an intutive approach to probability. The second part is more technical. Fully discussed solutions are provided at the end of the book for all odd-numbered problems.

YOUR NOTES