UNIVERSITY OF CAMBRIDGE
Faculty of Mathematics

Mathematical Tripos
Part III Guide to Courses 2015-2016

The Faulkes Institute of Geometry, completed in January 2002

Mathematical Tripos
Part III Lecture Courses in 2015-2016

Department of Pure Mathematics & Mathematical Statistics

Department of Applied Mathematics & Theoretical Physics

Notes and Disclaimers.

- Students may take any combination of lectures that is allowed by the timetable. The examination timetable corresponds to the lecture timetable and it is therefore not possible to take two courses for examination that are lectured in the same timetable slot. There is ***no*** requirement that students study only courses offered by one Department.
- The code in parentheses after each course name indicates the term of the course (M: Michaelmas; L: Lent; E: Easter), and the number of lectures in the course. Unless indicated otherwise, a 16 lecture course is equivalent to 2 credit units, while a 24 lecture course is equivalent to 3 credit units. Please note that certain courses are *non-examinable*, and are indicated as such after the title. Some of these courses may be the basis for Part III essays.
- At the start of some sections there is a paragraph indicating the desirable previous knowledge for courses in that section. On one hand, such paragraphs are not exhaustive, whilst on the other, not all courses require all the pre-requisite material indicated. However you are strongly recommended to read up on the material with which you are unfamiliar if you intend to take a significant number of courses from a particular section.
- The courses described in this document apply only for the academic year 2015-16. Details for subsequent years are often broadly similar, but *not* necessarily identical. The courses evolve from year to year.
- Please note that while an attempt has been made to ensure that the outlines in this booklet are an accurate indication of the content of courses, the outlines do *not* constitute definitive syllabuses. The lectures and associated course materials as offered in this academic year define the syllabus. Each course lecturer has discretion to vary the material covered.
- Some courses have no writeup available at this time, in which case you will see "No description available" in place of a description. Course descriptions will be added to the online version of the Guide to Courses as soon as they are provided by the lecturer. Until then, the descriptions for the previous year (available at http://www.maths.cam.ac.uk/postgrad/mathiii/courseguide.html) may be helpful in giving a rough idea of course content, but beware of the comments in the preceding item on what defines the syllabus.

Table of Contents

Algebra

Commutative Algebra (M24)

C.J.B. Brookes

The aim of the course is to give an introduction to the theory of commutative Noetherian rings and modules, a theory that is an essential ingredient in algebraic geometry, algebraic number theory and representation theory.

Topics I hope to fit in will be the theory of ideals for Noetherian and Artinian rings; localisations and completions; integral closure, valuation rings and Dedekind rings; dimension theory; differential operators

Pre-requisites

It will be assumed that you have attended a first course on ring theory, eg IB Groups, Rings and Modules. Experience of other algebraic courses such as II Representation Theory, Galois Theory or Number Fields will be helpful but not necessary.

Literature

1. M.F. Atiyah and I.G. Macdonald, Introduction to commutative algebra, Addison-Wesley, 1969.
2. N. Bourbaki, Commutative algebra, Elements of Mathematics, Springer, 1989 .
3. I. Kaplanksy, Commutative rings, University of Chicago Press, 1974.
4. H. Matsumura, Commutative ring theory, Cambridge Studies 8, Cambridge University Press, 1989.
5. M.Reid, Undergraduate Commutative Algebra, LMS student texts 29, Cambridge University Press, 1995.
6. R.Y. Sharp, Steps in commutative algebra, LMS Student Texts 19, Cambridge University Press, 1990.

   The basic text is Atiyah and Macdonald but it doesn't go into much detail and many results are left to the exercises. Sharp fills in some of the detail but neither book goes far enough. Both Kaplanksy and Matsumura cover the additional material though Matsumura is a bit tough as an introduction. Reid's book is a companion to one on algebraic geometry and that influences his choice of topics and examples. Bourbaki is encyclopaedic.

Additional support

Four examples sheets will be provided, with supporting examples classes.

Lie Algebras and Their Representations (M24)

David Stewart

Lie algebras were introduced by Sophus Lie as a way to study what we now call Lie Groups. The latter can be thought of as smooth groups. Then Lie algebras arise by looking at infinitesimal transformations, specifically, the tangent space at the identity. We'll go through these concepts in some detail, but actually the definition of a Lie algebra (which will be given in approximately three lines) is simply a vector space with a certain anticommutative multiplication which satisfies some version of associativity. So for the most part, all the geometry of the Lie group can be exorcised and we can get down to the algebraic arguments which will give us a complete picture of the finite-dimensional complex representations of finite-dimensional semisimple Lie algebras. But we'll do more than that, giving a classification of the complex simple Lie algebras by root data, covering all the structure theory necessary to get us there.

Lie theory comes in many flavours and is important in finite group theory (with 26 exceptions all nonabelian finite simple groups come from Lie theoretic objects), number theory (notably the Langlands programme), physics (e.g. quantum), differential equations, integrable systems ... Underpinning all Lie theoretical objects are root systems. In some way this course can be seen as an introduction to those most fundamental of mathematical objects, as motivated by Lie algebras.

Desirable Previous Knowledge

You need to be happy with the notion of a vector space but that's more-or-less it. I'm planning to illustrate many of the theorems by showing how they go wrong over fields of positive characteristic, so a basic familiarity with the existence of such fields would be good. Having taken some course on representation theory in the past would be a plus, only so that terms like 'completely reducible' are familiar.

Reading to complement course material

1. Representation theory, Fulton and Harris. Springer. This is a beautiful book written in a fun, chatty style with plenty of examples, motivation, and pictures. It tells a good story. It is the main source of the lecture notes and would be a great complement to the course. It also has stuff on representations of the symmetric groups. If you are thinking of staying on in algebra, it would be a great purchase.
2. Introduction to Lie algebras. Erdmann and Wilson. Springer. Very accessible. Every 'i' dotted and 't' crossed.
3. Introduction to Lie algebras and representation theory. Humphreys. Springer. A good book, taking a more algebraic approach.

Additional support

Four examples sheets will be provided and four associated examples classes will be given.

Representation Theory (L24)

Stuart Martin

The representation theory of the symmetric group S_n is a classical subject that, from the foundational work of Frobenius, Schur and Young, has developed into a richly diverse area, with important connections across algebra, computer science, statistical mechanics and theoretical physics.

This course is essentially an introduction to the algebraic combinatorics that underpins the representation theory of S_n. I hope to cover a selection of classical topics such as Specht modules, Young symmetrizers, Young tableaux, the branching rule, Schur functions, the Robinson-Schensted-Knuth correspondence, the Jacobi-Trudi identity, the hook-length formula, the Littlewood -Richardson rule, the Murnaghan-Nakayama rule, Schützenberger's involution and jeu de taquin, etc. I also give an account of a new approach to computing the complex finite-dimensional irreducible representations of S_n, developed by Anatoly Vershik and Andrei Okounkov (see the book [1] below). The main tool here is the so-called Gelfand-Tsetlin (GZ) algebra, which is a certain commutative subalgebra of $\mathbf{C}S_n$. If time allows, some of the following more recent topics will be included: Hecke algebras, partition algebras, (Young-)Jucys-Murphy elements (as generators of the GZ-algebra), Schur positivity, Macdonald polynomials, non-commutative symmetric functions.

- Representations of S_n. The Young diagram, Young tableaux and Young poset.
- Review of classical theory: Schur's lemma and its corollaries. Characters. Conjugacy classes. Orthogonality of characters. Wiring diagrams and Coxeter relations.

- The Okounkov-Vershik construction. Branching graph and Gelfand-Tsetlin bases. Centre of the group algebra. Jucys-Murphy elements. JM elements saitisfy generic Hecke algebra relations. Hook-length formula and hook-walks.
- Schur functions via semistandard Young tableaux (SSYT). Cauchy identity. Robinson-Schensted-Knuth (RSK) correspondence. Fomin's growth diagrams. Gelfand-Tsetlin patterns. q-binomials.
- The ring of symmetric functions and its various bases. Lindstrom's lemma and Jacobi-Trudi identity. The classical definition of Schur polynomials. The determinantal formula for the number of SSYTs.
- (If time) q-analogue of the determinantal formula and of the hook-length formula. Reverse plane partitions. Hillman-Grassl correspondence.
- (If time) Plane partitions, non-crossing paths, rhombus tilings, perfect matchings, and pseudoline arrangements. Viennot's shadow construction for RSK.
- Green's theorem. P-equivalence and Knuth's equivalence. Schützenberger's jeu de taquin. The Littlewood-Richardson rule and its variants.
- The Murnaghan-Nakayama rule. The Frobenius characteristic map. The characters of the symmetric group.

Pre-requisites

Prerequisites are minimal. Undergraduate representation theory (semisimplicity of the complex group algebra, completeness of characters over **C**), permutation representations. Group theory (symmetric groups and general linear groups and their conjugacy classes).

Preliminary Reading

1. T. Ceccherini-Silberstein, F. Scarabotti and F. Tolli, Representation theory of the symmetric groups: the Okounkov-Vershik approach, character formulas and partition algebras, CUP 2010.
2. W. Fulton, Young tableaux, Cambridge University Press, 1997.
3. W. Fulton and J. Harris, Representation theory, a first course, GTM 129, Springer, 1991.
4. G.D. James, The Representation Theory of the Symmetric Group, LNM 682, Springer 1978.
5. B.E. Sagan, The Symmetric Group: representations, combinatorial algorithms and symmetric functions (2nd edn), GTM 203, Springer 2001.
6. R.P. Stanley, Enumerative Combinatorics, Volume 2 (Chapter 7), CUP 2001.

Literature

1. A. Garsia, Young's seminormal representation, Murphy elements and content evaluations, 2003. (See Garsia's UCSD webpage)
2. A. Garsia, Alfred Young's construction of the irreducible representations of S_n, 2014. Available online.
3. A. Lascoux. Young's representations of the symmetric group. Available online at http:// phalanstere.univ-mlv.fr/∼ al/ARTICLES/ProcCrac.ps.gz
4. A.M. Vershik and A. Okounkov, A new approach to the representation theory of the symmetric groups, II (in Russian), Selecta Math. (N.S.) **2**, No 4, 581–605 (1996). Revised English version appears at http://arxiv.org/pdf/math/0503040v3.pdf
5. P. Py, On representation theory of the symmetric groups, J. Math. Sci. (N.Y.) 129 (2005), no 2 3806–3813.
6. A Young, The Collected Papers of Alfred Young, Math. Expositions No. 21 Univ. of Toronto. Toronto Press (1873–1940).

Additional support

Two sheets of examples will be provided backed up by two or three classes.

Infinite Groups and Decision Problems (L16)

Jack Button and Maurice Chiodo

Overview

The aim of this course is to investigate the theory of infinite groups. We will discuss ways of defining an infinite group via a finite description, and look at various classical constructions of these. We will then turn our attention to incomputability in group theory, giving several problems which are algorithmically incomputable in these finite descriptions of infinite groups.

Course description

Infinite groups (8 lectures with Jack Button): Review of basic definitions and results. Free groups and normal forms; free products. Nielsen-Schreier theorem and index formula. Group (and semigroup) presentations. Amalgamated free products and HNN extensions, normal and/or reduced forms, Britton's lemma and applications. Subgroups of finite index; Higman's group.
Decision problems (8 lectures with Maurice Chiodo): Turing machines, recursive and recursively enumerable sets, the halting set $\mathbb{K}$. Recursive presentations of groups, the word and isomorphism problems with basic properties and examples. Post's construction of a finitely presented semigroup with unsolvable word problem. Modular machines and their equivalence to Turing machines. A finitely presented group with unsolvable word problem. Higman's embedding theorem. The Adian-Rabin construction, unrecognisability of Markov properties, a universal finitely presented group.

Pre-requisites

It will be assumed that you have attended a first course in group theory. Some experience with algebraic topology (fundamental group, covering spaces), would be useful; the required background can instead be found in the preliminary reading listed below. In addition, Part III Computability Theory (Michaelmas) would be helpful, but is not essential.

Preliminary Reading

Any introductory text in group theory, of which there are plenty. The necessary algebraic topology can certainly be found in either of:

1. J. M. Lee, *Introduction to Topological Manifolds.* (GTM 202), Springer-Verlag, second edition, 2011: Chapters 7, 10, 11, 12.
2. A. Hatcher, *Algebraic topology.* Cambridge University Press, 2001: Chapter 0 and Sections 1.1, 1.2, 1.3, 1.A. Also available at

   `http://www.math.cornell.edu/~hatcher/AT/ATpage.html`

Literature

1. R. Lyndon and P. Schupp,*Combinatorial Group Theory.* Springer-Verlag, 2001: Sections I.1, II.1, II.2, IV.1, IV.2.
2. D. E. Cohen,*Combinatorial Group Theory: A Topological Approach.* London Mathematical Society Student Texts, Cambridge University Press, 1989: Sections 9.1–9.6.

3. C. F. Miller III, *Decision Problems For Groups-Survey and Reflections.* Algorithms and classification in combinatorial group theory (Berkeley, CA, 1989), Math. Sci. Res. Inst. Publ., **23**, Springer, New York, 1–59 (1992): Section 3. Also available at

   `http://www.ms.unimelb.edu.au/~cfm/papers/paperpdfs/msri_survey.all.pdf`

4. J. Rotman, *An Introduction To The Theory Of Groups.* (GTM 148), Springer, fourth edition, 1995: p.404–430.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Analysis

Analysis of Partial Differential Equations (M24)

David Stuart

The purpose of this course is to introduce some techniques and methodologies which are used in the mathematical treatment of Partial Differential Equations (PDE). The theory of PDE is nowadays a huge area of active research, and it goes back to the very birth of mathematical analysis in the 18th and 19th century. It lies at a crossroads with physics and many areas of pure and applied mathematics.

The course begins with an introduction to four prototype linear equations: Laplace's equation, the heat equation, the wave equation and Schrödinger's equation. Emphasis will be given to the modern functional analytic techniques, relying on the notion of Cauchy problem and estimates, rather than explicit solutions, although the interaction with classical methods (such as the fundamental solution and Fourier representation) will be discussed. The following basic unifying concepts will be studied: well-posedness, energy estimates, elliptic regularity, characteristics, propagation of singularities, group velocity, and the maximum principle. The course will end with a discussion of some of the open problems in PDE.

Pre-requisites

There are no specific pre-requisites beyond a standard undergraduate analysis background, in particular a familiarity with measure theory and integration. The course will be mostly self-contained and can be used as a first introductory course in PDE for students wishing to continue with some specialised PDE Part III courses in the lent and easter terms. In particular, having attended the 2014 course "Partial differential equations" in Part II is *not* a pre-requisite.

Preliminary Reading

The following article gives an overview of the field of PDE:

1. Klainerman, S., *Partial Differential Equations*, Princeton Companion to Mathematics (editor T. Gowers), Princeton University Press, 2008.

Literature

1. Some lecture notes are available online at: `http://cmouhot.wordpress.com/teachings/`.

   The following textbooks are excellent references:

2. Evans, L. C., *Partial Differential Equations*, Springer, 2010.
3. Brezis, H., *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2010.
4. John, F., *Partial Differential Equations*, Springer, 1991.

Additional Information

This course is also intended for doctoral students of the Centre for Analysis (CCA), who will also be involved in additional assigments, presentations and group work. Part III students do not do these, and they will be assessed in the usual way by exam at the end of the academic year. Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term. There will be one office hour a week.

Functional Analysis (M24)

András Zsák

This course covers many of the major theorems of abstract Functional Analysis. It is intended to provide a foundation for several areas of pure and applied mathematics. We will cover the following topics:

Hahn–Banach Theorem on the extension of linear functionals. Locally convex spaces.

Duals of the spaces $L_p(\mu)$ and $C(K)$. The Radon–Nikodym Theorem and the Riesz Representation Theorem.

Weak and weak-* topologies. Theorems of Mazur, Goldstine, Banach–Alaoglu. Reflexivity and local reflexivity.

Hahn–Banach Theorem on separation of convex sets. Extreme points and the Krein–Milman theorem. Partial converse and the Banach–Stone Theorem.

Banach algebras, elementary spectral theory. Commutative Banach algebras and the Gelfand representation theorem. Holomorphic functional calculus.

Hilbert space operators, C^*-algebras. The Gelfand–Naimark theorem. Spectral theorem for commutative C^*-algebras. Spectral theorem and Borel functional calculus for normal operators.

Some additional topics time permitting. For example, the Fréchet–Kolmogorov Theorem, weakly compact subsets of $L_1(\mu)$, the Eberlein–Šmulian and the Krein–Šmulian theorems, the Gelfand–Naimark–Segal construction.

Pre-requisites

Thorough grounding in basic topology and analysis. Some knowledge of basic functional analysis and basic measure theory (much of which will be recalled either in lectures or via handouts). In Spectral Theory we will make use of basic complex analysis. For example, Cauchy's Theorem, Cauchy's Integral Formula and the Maximum Modulus Principle.

Literature

1. Allan, Graham R. *Introduction to Banach spaces and algebras (prepared for publication by H. Garth Dales).* Oxford University Press, 2011.
2. Bollobás, Béla *Linear analysis: an introductory course.* Cambridge University Press, 1990.
3. Rudin, Walter *Real & Complex Analysis.* McGraw-Hill, 1987.
4. Rudin, Walter *Functional Analysis.* McGraw-Hill, 1990.
5. Taylor, S. J. *Introduction to measure and integration.* Cambridge University Press 1973.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term. There will be some material as well as examples sheets and announcements available at `www.dpmms.cam.ac.uk/~az10000/`

Elliptic Partial Differential Equations (L24)

Costante Bellettini and Otis Chodosh

This course is intended as an introduction to the theory of elliptic partial differential equations. Elliptic equations play an important role in geometric analysis and a strong background in linear elliptic equations provides a foundation for understanding other topics including minimal submanifolds, harmonic maps, and general relativity. We will discuss both classical and weak solutions to elliptic equations, considering when solutions to the Dirichlet problem exist and are unique and considering the regularity of solutions. This involves establishing maximum principles, Schauder estimates, and other estimates on solutions. As time permits, we will discuss other topics including the De Giorgi-Nash theory, which can be used to prove the Harnack inequality and establish Hölder continuity for weak solutions, and quasilinear elliptic equations.

Pre-requisites

Lebesgue integration, Lebesgue spaces, Sobolev spaces, and basic functional analysis.

Literature

1. David Gilbarg and Neil S. Trudinger, Elliptic Partial Differential Equations of Second Order. Springer-Verlag (1983).
2. Lawrence Evans, Partial Differential Equations. AMS (1998)
3. Qing Han and Fanghua Lin, Elliptic partial differential equations. Courant Lecture Notes, Vol. 1 (2011).

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Topics in Ergodic Theory (L24)

Péter Varjú

Ergodic theory studies dynamical systems that are endowed with an invariant measure. There are many examples of such systems that originate from other branches of mathematics. This led to a fruitful interplay between ergodic theory and other fields.

I will explain some basic elements of ergodic theory, such as recurrence, ergodic theorems and mixing properties. I will also talk about some applications of the theory, such as Furstenberg's proof of Szemerédi's theorem, continued fractions and Weyl's equidistribution theorem for polynomials.

Pre-requisites

Measure theory, point-set topology and basic functional analysis.

Literature

Einsiedler, Ward, *Ergodic Theory with a view towards Number Theory*, Springer, 2011.

Nonlinear Wave Equations (L24)

Non-Examinable (Part III Level)

Jonathan Luk

We will discuss the local and global theories for quasilinear wave equations and their applications to physical theories including fluid mechanics and general relativity. The following topics will be covered:

1. Quantitative behaviour of solutions to the linear wave equation in Minkowski spacetime
2. Energy methods and the local theory for quasilinear wave equations
3. Application in general relativity: local well-posedness of the Einstein equations
4. Examples of subcritical nonlinear wave equations
5. Small data global theory in higher dimensions
6. The null condition and the small-data global theory in three dimensions
7. The weak null condition
8. Further applications

Pre-requisites

Some exposure to partial differential equations, Fourier analysis and differential geometry will be useful but we will develop most of the necessary tools within the course.

Literature

We will not follow any specific texts but students may find the literature listed below useful:

1. H. Ringström, *Non-linear wave equations*, available at

   `http://www.math.kth.se/~hansr/nlw.pdf`

2. C. Sogge, *Lectures in nonlinear wave equations* 2nd edition. International Press, 2011.
3. S. Klainerman, *Lecture notes in analysis*, available at

   `https://web.math.princeton.edu/~seri/homepage/courses/Analysis2011.pdf`

Lectures notes will also be provided online.

Additional support

Examples sheets will be provided and example classes will be given.

Optimal Transportation (L24)

Non-Examinable (Graduate Level)

Dr Garling

The problem of optimal transportation (moving material distributed as a measure μ to a site with distribution ν, at minimal cost) was introduced by Gaspard Monge in 1781. Major advances were made by Leonid Kantorovich in the 1940s, for which he won the Nobel Prize for Economics in 1975. In recent years there have been further major developments and applications, leading to a Fields Medal for Cédric Villani in 2010. This course will develop the fundamental theory, and the analysis needed for it.

Topics include:

Polish spaces, measures on them and the convergence of measures.

Convex sets and functions, Legendre transforms and related ideas.

Extreme points and Choquet theory.

Optimal transportation and Wasserstein metrics.

Pre-requisites

Some linear analysis, including the Hahn-Banach theorem.

Measure theory, to the level of the Part II course.

Preliminary Reading

The introduction to [1], Chapter 3 of [2] and the Preface and Introduction of [3] each give a good idea of what it is all about.

Literature

1. Gangbo, W. and McCann, R.J. *The geometry of optimal transportation*, Acta Math., **177**, 113-161 (1995). Also available at

   //http://people.math.gatech.edu/ gangbo/publications/geo.pdf

2. Villani, C., *Topics in optimal transportation.* AMS, 2003.
3. Villani, C., *Optimal transport old and new.* Springer, 2009.

Function Spaces (L24)

Non-Examinable (Graduate Level)

Sophia Demoulini

Review (as necessary, including Measure Theory and Lebesgue spaces, Riesz representations on spaces of continuous functions, Egorov, Lusin BV spaces in 1 dim)

Hardy-Littlewood Principle Calderon-Zygmund decomposition Weak and Strong (p,q) operators Covering Theorems (Vitali, Besicovich) Absolute Continuity Differentiation Hahn and Lebesgue Decomposition

(Possibly: Introduction to Fractional Integral Operators) Hardy-Littlewood-Sobolev theorem)

Sobolev Spaces Hausdorff Measure - Isodiametric inequality Capacity Area and Coarea formulae Trace Capacity Space of functions of Bounded Variation in n-dimensions Coarea for BV Reduced boundary and Gauss-Green Theorem for BV

Pre-requisites

Measure theory, Lebesgue Integration, as in Probability and Measure, Part II Basic Functional Analysis (Hahn-Banach Theorem, Banach spaces)

Literature

1. Evans and Gariepy *Measure theory and Fine Properties of Functions* CRC Press 1992.
2. *Lecture Notes* by James Kilbane https://www.dpmms.cam.ac.uk/ jk511/ Also available at

   `https://www.dpmms.cam.ac.uk/~jk511/`

Topics in Geometric Analysis (L16)

Non-Examinable (Graduate Level)

Neshan Wickramasekera

The course will aim to cover some basic principles in modern geometric analysis, taking as examples the variational theories of minimal submanifolds and harmonic maps. Topics to be discussed (some lightly): existence results including the classical Plateau problem, monotonicity formulae, ϵ-regularity and compactness theories, tangent cones and the question of their uniqueness, size and stratification of singular sets, and regularity of singular sets.

Pre-requisites

Some familiarity with geometry of submanifolds of Euclidean spaces, rectifiable sets, measure theory including Hausdorff measure, and the theory of second order quasilinear ellipitic PDEs will be very helpful.

Literature

1. D. Gilbarg & N. Trudinger, *Elliptic partial differential equations of second order.*

2. L. Simon,

   Lectures on Geometric Measure Theory.

3. L. Simon,

   Theorems on regularity and singularity of energy minimizing maps.

4. M. Struwe,

   Plateau's problem and the calculus of variations.

5. N. Wickramsekera,

   Regularity of stable minimal hypersurfaces: recent advances in the theory and applications. Surveys in Diff. Geom, Vol 19 (2014), pp. 231–301. available at:

   `http://intlpress.com/site/pub/pages/journals/items/sdg/content/vols/0019/0001/index.html`

Advanced Topics in Many-Particle Systems (E16)

Non-Examinable (Graduate Level)

Clément Mouhot

This non-examinable course will present some mathematical tools and concepts for the rigorous derivation and study of nonlinear partial differential equations arising from many-particle limits: Vlasov transport equations, Boltzmann collision equations, nonlinear diffusion, quantum Hartree equations. . . Depending on time and interest it could include: notions of master equation and empirical measures, stability estimate in optimal transport distance, coupling method, chaos and entropic chaos, hydrodynamic limit of lattice systems.

Pre-requisites

Basics in functional analysis, partial differential equations and probability.

Literature

1. H. Spohn *Large Scale Dynamics of Interacting Particles.* Springer 1991.
2. C. Kipnis & C. Landim *Scaling Limits of Interacting Particle Systems.* Springer 1999.
3. F. Golse *The Mean-Field Limit for the Dynamics of Large Particle Systems*, Journeées Équations aux dérivées partielles Forges-les-Eaux, 2-6 juin 2003.
4. F. Bolley *Optimal coupling for mean field limits*, arXiv:10093855.
5. S. Mischler & C. Mouhot *Kac's program in kinetic theory*, Inventiones mathematicae 2013, vol 193, pp 1-147.
6. P.-E. Jabin *A review of the mean field limits for Vlasov equations*, Kinetic and Related Models 2014, vol 7, pp 661 - 711.

Combinatorics

Combinatorics (M16)

Prof I.B.Leader

The flavour of the course is similar to that of the Part II Graph Theory course, although we shall not rely on many of the results from that course.

We shall study collections of subsets of a finite set, with special emphasis on size, intersection and containment. There are many very natural and fundamental questions to ask about families of subsets; although many of these remain unsolved, several have been answered using a great variety of elegant techniques.

We shall cover a number of 'classical' extremal theorems, such as those of Erdős-Ko-Rado and Kruskal-Katona, together with more recent results concerning isoperimetric inequalities and intersecting families. The aim of the course is to give an introduction to a very active area of mathematics.

We hope to cover the following material.

Set Systems

Definitions. Antichains; Sperner's lemma and related results. Shadows. Compression operators and the Kruskal-Katona theorem. Intersecting families; the Erdős-Ko-Rado theorem.

Isoperimetric Inequalities

Harper's theorem and the edge-isoperimetric inequality in the cube. Inequalities in the grid. The classical isoperimetric inequality on the sphere. The 'concentration of measure' phenomenon. Applications.

Intersecting Families

Katona's t-intersecting theorem. The Ahlswede-Khachatrian theorem. Restricted intersections. The Kahn-Kalai counterexample to Borsuk's conjecture.

Desirable Previous Knowledge

The only prerequisites are the very basic concepts of graph theory.

Introductory Reading

1. Bollobás, B., Combinatorics, C.U.P. 1986.

Extremal Graph Theory (M16)

Andrew Thomason

Extremal graph theory is, broadly speaking, the study of graph properties and their dependence on the values of graph parameters. The simplest example is the well-known theorem of Turán. We develop the basic theory for graphs, and extend to hypergraphs. The recent idea of hypergraph containers shows how hypergraph tools can give new information even for graphs. The following material is expected to be in the course, with further topics being explored if time permits.

The Erdős-Stone theorem and stability. Szemerédi's Regularity Lemma, with applications.

Hypergraphs. Erdős's r-partite theorem. Instability. The theorem of de Caen.

Containers for regular and irregular graphs. Hypergraph containers. The number of H-free and induded H-free graphs. Further applications.

Pre-requisites

A knowledge of the basic concepts, techniques and results of graph theory, such as that afforded by the Part II Graph Theory course.

Literature

No book covers the course but the following can be helpful.

1. B. Bollobás, *Modern graph theory*, Graduate Texts in Mathematics **184**, Springer-Verlag, New York (1998), xiv+394 pp.
2. N. Alon and J. Spencer, *The Probabilistic Method*, Wiley, 3rd ed. (2008)

Additional support

Three examples classes will be offered, based on examples sheets. Moreover there will be a revision class during the Easter Term.

Techniques in Non-Abelian Additive Combinatorics (L16)

W. T. Gowers

This course will be an introduction to some (but by no means all) of the central ideas of additive combinatorics. It will begin by showing how discrete Fourier analysis on the cyclic group $\mathbb{Z}_N$ can be used to prove interesting theorems in combinatorial number theory, but the main emphasis will be on generalizing these techniques to arbitrary finite Abelian and then non-Abelian groups. While the course will contain some interesting theorems, its main aim is to teach the methods of proof, which make it possible to solve problems that would otherwise be out of reach.

I hope to include all of the following, but may have to leave out certain topics if I run out of time. Approximate numbers of lectures are given in brackets (but these are guesses and may turn out to be overoptimistic).

Discrete Fourier analysis in finite Abelian groups. Roth's theorem. Bogolyubov's method. [3]

Matrices, box norms, singular values, quasirandom regular bipartite graphs. [3]

Review of basic results of representation theory. [1]

Discrete Fourier analysis in general finite groups. Quasirandom groups. Bounds for the sizes of product-free sets. [3]

Use of arithmetic geometry. The Schwartz-Zippel lemma. Dvir's theorem. The Lang-Weil theorem (statement only). Interleaved products in $\mathrm{SL}(2,q)$. [3]

Fourier analysis for matrix-valued functions on general finite groups. The stability of near-representations. [3]

Pre-requisites

You should be comfortable with the terminology of graph theory (for example, knowing what a bipartite graph is). You should know enough linear algebra to understand the statement that a Hermitian matrix has an orthonormal basis of eigenvectors with real eigenvalues. A familiarity with the basic ideas of representation theory, such as the definition of an irreducible representation and Schur's theorem, would be helpful, but I shall include a quick review of the concepts I need.

Literature

There is no obvious book for this course. The book Additive Combinatorics, by Terence Tao and Van Vu, is a very comprehensive introduction to additive combinatorics and would complement the course well, but its overlap with the material above is rather slight.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Probabilistic Combinatorics and Its Applications (L16)

Béla Bollobás

For the past fifty years or so, probability theory has been strongly influenced by combinatorics: a great variety of results in probability theory have been inspired by combinatorial problems. In this course we shall present some of these results, together with a number of applications.

The topics to be covered are likely to include the following.

Correlation inequalities, including those of Harris, van den Berg and Kesten, and the Four Functions Inequality.

Isoperimetric inequalities, with emphasis on the (high-dimensional discrete) cube and grid.

Talagrand's inequalities.

Entropy (Shannon and von Neumann); inequalities of Shearer and Balister and Bollobás.

Concentration of probability, including the Azuma–Hoeffding Inequality, the inequalities of Freedman and McDiarmid, and transportation inequalities.

The influence of a random variable: Fourier analytic methods and the KKL (Kahn, Kalai, Linial) Inequality.

As applications, I shall prove some basic results about the chromatic number of random graphs, the travelling salesman problem, percolation theory, and monotone cellular automata, and hope to present some very recent results of Balogh, Morris and others.

Almost all the material in the course will be self-contained, although I may use one or two results from the other combinatorics courses, for example the container theorem of Balogh, Morris and Samotij, and Saxton and Thomason.

Much of the material will be based on research papers, some of which are very recent, but the more standard parts will be supported by printed notes.

The Upper Tail and Concentration in Combinatorics (L16)

Non-Examinable (Graduate Level)

Paul Smith

If X is a sum of n indicator random variables, how close is X typically to its mean? If the indicator random variables are independent, then the answer is that X is typically very close to its mean: with high probability, X is contained in an interval of length $O(\sqrt{n})$. Moreover, the probability that X lies outside the interval is exponentially small. This result, known as Chernoff's inequality, is an example of a phenomenon known as *concentration of measure.*

We are not usually so lucky as to have independent random variables, but the concentration of measure phenomenon often still holds if the random variables are only 'sufficiently close' to being independent. The aim of this course is to present a toolbox of techniques for proving such concentration results, with a focus on *upper tail inequalities*, which are bounds on the probability a random variable is much larger than its mean. The techniques will be illustrated with applications to variety of problems in probabilistic combinatorics, random graphs, and discrete probability.

The topics covered will include some of the following: sums of independent random variables and Chernoff's inequalities; generalization to martingales: Azuma–Hoeffding, Talagrand, Freedman; further upper tail inequalities: Kim–Vu, Lipschitz-type methods, the deletion method and other combinatorial methods; Janson's inequality; Stein–Chen Poisson approximation; examples of non-concentration and scaling limits.

Pre-requisites

You should be familiar with elementary graph theory and probability. Knowledge of discrete time martingales will be useful, but these will be treated informally and a quick recap will be given at the start of the course. The course will complement, but not overlap, the course on Random Graphs and Percolation given by Professor Bollobás.

Literature

There is no single book or paper containing all of what we hope to cover, but the union of the following three comes close. The beautiful book by Alon and Spencer is the classic introduction to the subject.

1. N. Alon and J. Spencer, *The Probabilistic Method*, third ed., Wiley, 2008.
2. B. Bollobás, *Random Graphs*, second ed., Cambridge, 2001.
3. S. Janson and A. Ruciński, *The infamous upper tail*, Random Structures Algorithms **20** (2002), no. 3, 317–342.

Topics in Ramsey Theory (L16)

Non-Examinable (Graduate Level)

B. P. Narayanan

Ramsey theory is concerned with the general question of whether, in a large amount of disorder, one can find regions of order. A typical example is van der Waerden's theorem, which states that whenever we partition the natural numbers into finitely many classes there is a class that contains arbitrarily long arithmetic progressions.

The flavour of the course is combinatorial. Ramsey theory is remarkably attractive: we study questions that are very natural and easy to appreciate, but whose answers rely on a great variety of beautiful methods.

The first half of this course will be an introduction to Ramsey theory and will cover some of the classical results in the area such as Ramsey's theorem, the Canonical Ramsey theorems, van der Waerden's theorem and the Hales–Jewett theorem.

The second half of the course will focus on more recent developments. Some of the topics I hope to cover include recent work in geometric Ramsey theory, the properties of non-Ramsey graphs, and finally, connections to the theory of ultrafilters and ergodic-theory.

Pre-requisites

There are almost no prerequisites; the course will start with a review of Ramsey's theorem, so even prior knowledge of this is not essential. However, students familiar with the material from a first course in graph theory are bound to find the course easier.

Appropriate Books

The first half of the course will cover material available in the following books.

1. *Combinatorics*, B. Bollobás, Cambridge University Press 1986.
2. *Ramsey Theory*, R. Graham, B. Rothschild and J. Spencer, John Wiley 1990.

Geometry and Topology Courses

Algebraic Geometry (M24)

Caucher Birkar

This course is intended to serve as an introduction to modern algebraic geometry. Algebraic geometry is about studying the solutions of systems of polynomial equations. However, much of this study involves geometric intuition and advanced algebraic techniques. The methods of algebraic geometry are so fruitful that they are applied to subjects far beyond algebraic geometry such as number theory, analytic and differential geometry, topology, mathematical physics, mathematical logic, cryptography, etc.

Topics I hope to cover: sheaves, schemes, varieties, morphisms, divisors, differential forms, cohomology, duality, Riemann-Roch theorem, quotient by group actions, algebraic groups, etc.

Pre-requisites

Previous familiarity with algebraic geometry is not necessary but it would be very helpful. If you have not encountered algebraic geometry before, it is recommended that prior to the start of the course you browse through chapter I of [H] or through [S]. On the other hand, commutative algebra is used systematically.

Related courses

The part III *commutative algebra* is strongly recommended.

Literature

[AM] M. Atiyah, I. Macdonald. *Introduction to commutative algebra.* Westview Press, 1994.

[H] R. Hartshorne. *Algebraic geometry.* Springer, 1977. (Much of the course is based on chapters II-III of this book.)

[S] I. Shafarevich. *Basic algebraic geometry I.* Springer, 1994.

Additional support

Four examples sheets will be provided and two associated examples classes will be given.

Algebraic Topology (M24)

Ivan Smith

Algebraic Topology assigns algebraic invariants to topological spaces; it permeates modern pure mathematics. This course will focus on (co)homology, with an emphasis on applications in differential geometry and the topology of manifolds. Some basic homological algebra and, time permitting, some homotopy theory will be included. The course will not assume any knowledge of algebraic topology, but will go quite fast in order to reach more interesting material, so some previous exposure to simplicial homology and / or the fundamental group would certainly be helpful. Topics to be covered include:

- singular homology and cohomology, degrees of maps, cellular (co)homology, cup-product and Künneth theorem;
- vector bundles, the Thom isomorphism theorem, the Euler class;

- topology of manifolds, Poincaré duality, cup-length and critical points, the Lefschetz fixed point theorem.

The course will emphasise examples and computations; it will be accompanied by four question sheets with associated Examples Classes, which will again involve applying the general theory to do explicit calculations and solve geometric problems.

Pre-requisites

Basic topology: topological spaces, compactness and connectedness, at the level of Sutherland's book. Some knowledge of the fundamental group would be helpful though not a requirement. The Part III Differential Geometry course will also contain useful, relevant material.

Hatcher's book is especially recommended for the course, but there are many other suitable texts.

Literature

1. Bott, R. and Tu, L. *Differential forms in algebraic topology.* Springer, 1982.
2. Hatcher, A. *Algebraic Topology.* Cambridge Univ. Press, 2002.
3. May, P. *A concise course in algebraic topology.* Univ. of Chicago Press, 1999.
4. Sutherland, W. *Introduction to metric and topological spaces.* Oxford Univ. Press, 1999.

Differential Geometry (M24)

P.M.H. Wilson

This course is intended as an introduction to modern differential geometry. It can be taken with a view to further studies in Geometry and Topology and should also be suitable as a supplementary course if your main interests are for instance in Analysis or Mathematical Physics. A tentative syllabus is as follows.

- *Local Analysis and Differential Manifolds.* Definition and examples of manifolds, smooth maps. Tangent vectors and vector fields, tangent bundle. Geometric consequences of the implicit function theorem, submanifolds. Lie Groups. Differential 1-forms, cotangent bundle.
- *Vector Bundles.* Structure group. The example of Hopf bundle. Bundle morphisms and automorphisms. Exterior algebra of differential forms. Tensors. Orientability of manifolds. Partitions of unity and integration on manifolds, Stokes Theorem; de Rham cohomology. Lie derivative of tensors. Connections on vector bundles and covariant derivatives: covariant exterior derivative, curvature. Bianchi identity, orthogonal connections.
- *Riemannian Geometry.* Connections on the tangent bundle, torsion. Bianchi's identities for torsion free connections. Riemannian metrics, Levi–Civita connection, Christoffel symbols, geodesics. Riemannian curvature tensor and its symmetries, second Bianchi identity, sectional curvatures. Ricci tensor and Einstein metrics. Ricci and scalar curvatures. Schur's theorem.

The main references for this course are the books listed below and some printed notes on the lecturer's home page.

Pre-requisites

An essential pre-requisite is a working knowledge of linear algebra (including bilinear forms) and multivariate calculus. Exposure to some of the ideas of classical differential geometry might also be found useful.

Literature

1. D. Barden, C. Thomas, *An introduction to differentiable manifolds.* Imperial College Press, 2003.
2. R.W.R. Darling, *Differential forms and connections.* CUP, 1994.
3. M. Spivak, *Differential Geometry, Volume 2.* Publish or Perish, 1999.
4. F.W. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer-Verlag, 1983.

Additional support

Three or four examples sheets will be provided and four associated examples classes will be given. The fourth class will take place at the start of the Lent Term and will also fulfil a revision function.

Morse Theory (L24)

Jacob Rasmussen

The basic idea of Morse theory is to study how the topology of a smooth manifold M is related to the critical points of a smooth function $f : M \to \mathbf{R}$. In the hands of Lefshetz, Morse, Bott, Smale, Witten, and Floer, this technique has been one of the most productive methods in geometric topology over the course of the last century. It remains vitally important today.

The course will begin with the basics of Morse theory on finite dimensional manifolds, including the Morse lemma, handle decompositions, and the Morse complex. We will then discuss some applications, which will be drawn from the following list:

- The h-cobordism theorem; the high-dimensional Poincare conjecture.
- The topology of loop spaces; Bott periodicity.
- Lagrangian Floer homology; the Arnold conjecture.
- Lefshetz fibrations; the Lefshetz hyperplane theorem.

There is clearly more material in the list above than can be covered in 24 lectures; exactly which topics we discuss will depend on audience background and interest.

Pre-requisites

I will assume the Michaelmas term courses on Differential Geometry and Algebraic Topology. Some knowledge of Lie groups and/or symplectic geometry may be helpful, but is not required.

Literature

1. M. Audin and M. Damian, *Morse Theory and Floer homology* (translated from the French). Springer, 2013.
2. D. McDuff and D. Salamon, *Introduction to Symplectic Topology.* Oxford University Press, 1998.
3. J. Milnor, *Morse Theory.* Princeton University Press, 1963.
4. J. Milnor, *Lectures on the h-cobordism theorem.* Princeton University Press, 1965.

Additional support

There will be four examples sheets and four associated examples classes, as well as a revision class in the Easter Term.

Spectral Geometry (L24)

Dennis Barden

The aim of this course is to give an overview of the work that has blossomed in response to Mark Kac' naive sounding question, first posed in 1966: "Can one hear the shape of a drum?" In other, more general, words can one determine the geometry of a Riemannian manifold from the spectrum, the set of eigenvalues together with their multiplicities, of the Laplace operator. The answer is, unsurprisingly, no: many pairs, and even continuous families, of manifolds have since been constructed that have the same spectrum yet are not isometric. But surprisingly, almost yes: these examples are very special, usually highly symmetric, so that it is still possible that generically (a term that may be defined to suit the context) manifolds are spectrally determined. In fact this has already been shown to be the case in certain contexts and, indeed, Kac' question in its original context, planar domains with smooth boundary, remains open.

Pre-requisites

This subject is very much inter-disciplinary involving (Riemannian) geometry, analysis and topology as well as some algebra and minor forays into other subjects. However the results needed will mostly be stated without proof, so that the level of knowledge required will be that which is sufficient to understand and apply the statements of the theorems, rather than knowing or necessarily understanding their proofs.

Nothing is truly apposite for preliminary reading: full (indeed overfull) notes will be produced during the course, including a long reference list. A look at any item of the literature below will give some feeling for the subject, though none contains nor is contained in the course.

Literature

1. M. Berger, P. Gauduchon and E. Mazet, *Le Spectre d'une Variété Riemannienne.* Lecture Notes in Math. 194, Springer-Verlag, Berlin-Heidelberg- New York, 1971.
2. P. Buser, *Geometry and Spectra of Compact Riemann Surfaces.* Birkhäuser, Boston, 1992
3. Carolyn Gordon, *Survey of Isospectral Manifolds,* in Handbook of Differential Geometry Vol. 1. Elsevier Science, 2000
4. Isaac Chavel, *Eigenvalues in Riemannian Geometry.* Academic Press, 1984.
5. S.Rosenberg, *The Laplacian on a Riemannian Manifold.* L.M.S. Student Texts, 31, Cambridge University Press, 1997.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Complex Manifolds (L24)

Mark Gross

The goal is to help students learn the basic theory of complex manifolds. An outline of the course is as follows.

- Basic concepts of complex manifolds, holomorphic vector bundles, holomorphic tangent and cotangent bundles (for which the corresponding concepts from the real smooth manifolds will be assumed). Canonical line bundles, normal bundle for a submanifold and the adjunction formula.

- Brief description of sheaf cohomology, with deduction of de Rham and Dolbeault cohomology for complex manifolds.
- Hermitian metrics, connections, curvature and Chern classes for complex vector bundles. Case of holomorphic vector bundles.
- Harmonic forms: the Hodge theorem and Serre duality (general results on elliptic operators will be assumed).
- Compact Kähler manifolds. Hodge and Lefschetz decompositions on cohomology, Kodaira-Nakano vanishing, Kodaira embedding theorem.

Pre-requisites

A knowledge of basic Differential Geometry from the Michaelmas term will be essential.

Literature

1. J. P. Demailly, *Complex analytic and differential geometry.* Available as a pdf at

   `https://www-fourier.ujf-grenoble.fr/~demailly/documents.html`

2. P. Griffiths and J. Harris, *Principles of Algebraic Geometry.* Wiley, 1978.
3. D. Huybrechts, *Complex Geometry — an introduction.* Springer, 2004.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Algebraic Surfaces (L24)

Non-Examinable (Graduate Level)

Ian Grojnowski

This course is about the geometry of algebraic surfaces, a subject which began in the 1850s with the discovery (Cayley, Salmon) of the 27 lines on a non-singular cubic.

We wll start with the classical description of del Pezzo surfaces as the blow up of $\mathbf{P}^2$ in at most 8 points, phrasing this more precisely in terms of the moduli of Looijenga pairs (X, E) of the del Pezzo and an anti-canonical divisor.

We will play a little with the geometry, arithmetic, non-commutative geometry and representation theory involved in this, and its generalisation to other surfaces.

There is an exhaustive literature on algebric surfaces, both ancient and contemporary. Part of the reason for the course is to re-examine it from a more modern perspective. However, I am very much not an expert. So though there wil be no prerequisites other than basic algebraic geometry, if you are an expert in any aspect of algebraic surfaces, you should pop in now and then and heckle.

Pre-requisites

Basic algebraic geometry (coherent sheaf cohomology), semisimple Lie algebras or algebraic groups.

Topics in Algebraic Geometry (L16)

Non-Examinable (Graduate Level)

Tyler L. Kelly

We will study the geometry of Calabi-Yau varieties, focussing primarily on the theory of K3 surfaces. K3 Surfaces have a rich theory which crosses through many aspects of algebraic geometry, requiring various tools.

We will try to strike the right balance of geometric intuition with algebraic machinery in the course. The direction of the lectures will have a bias towards mirror symmetry; however, much of the course will be more classical methods.

Topics include polarisations of K3 surfaces, Picard lattices, elliptic fibrations, Kodaira's classification of singular fibers, Hodge theory of Calabi-Yau varieties, periods, and Picard-Fuchs equations. If we have time, we will continue into more modern applications and contexts of this theory.

Note: This course will not meet during the weeks of 25-29 January and 7-11 March.

Pre-requisites

The minimal prerequisite is the Part III course in Algebraic Geometry. Understanding of coherent sheaf cohomology and the Hodge decomposition is useful.

Literature

1. M. Gross, D. Huybrechts, D. Joyce, *Calabi-Yau Manifolds and Related Geometries.* Springer, 2003.
2. D. Huybrechts *Lectures on K3 Surfaces,* 2011. Available at:

   `http://www.math.uni-bonn.de/people/huybrech/K3Global.pdf`

3. R. Laza, M. Schütt, N. Yui (Eds.), *Calabi-Yau Varieties: Arithmetic, Geometry and Physics, Lecture Notes on Concentrated Graduate Courses.* Fields Institute Monographs, Springer, 2015.

Logic

Introduction to Category Theory (M24)

Prof. P.T. Johnstone

Category theory begins with the observation (Eilenberg–Mac Lane 1942) that the collection of all mathematical structures of a given type, together with all the maps between them, is itself an instance of a nontrivial structure which can be studied in its own right. In keeping with this idea, the real objects of study are not so much categories themselves as the maps between them—functors, natural transformations and (perhaps most important of all) adjunctions. Category theory has had considerable success in unifying ideas from different areas of mathematics; it is now an indispensable tool for anyone doing research in topology, abstract algebra, mathematical logic or theoretical computer science (to name just a few areas). This course aims to give a general introduction to the basic grammar of category theory, without any (intentional!) bias in the direction of any particular application. It should therefore be of interest to a large proportion of pure Part III students.

The following topics will be covered in the first three-quarters of the course:

Categories, functors and natural transformations. Examples drawn from different areas of mathematics. Faithful and full functors, equivalence of categories, skeletons. [4 lectures]

Locally small categories. The Yoneda lemma. Structure of set-valued functor categories: generating sets, projective and injective objects. [2 lectures]

Adjunctions. Description in terms of comma categories, and by triangular identities. Uniqueness of adjoints. Reflections and coreflections. [3 lectures]

Limits. Construction of limits from products and equalizers. Preservation and creation of limits. The Adjoint Functor Theorems. [4 lectures]

Monads. The monad induced by an adjunction. The Eilenberg–Moore and Kleisli categories, and their universal properties. Monadic adjunctions; Beck's Theorem. [4 lectures]

The remaining seven lectures will be devoted to topics chosen by the lecturer, probably from among the following:

Filtered colimits. Finitary functors, finitely-presentable objects. Applications to universal algebra.

Regular categories. Embedding theorems. Categories of relations, introduction to allegories.

Abelian categories. Exact sequences, projective resolutions, derived functors. Introduction to homological algebra.

Monoidal categories. Coherence theorems, monoidal closed categories, enriched categories. Weighted limits.

Fibrations. Indexed categories, internal categories, definability. The indexed adjoint functor theorem.

Pre-requisites

There are no specific pre-requisites other than some familiarity with undergraduate-level abstract algebra, although a first course in logic would be helpful. Some of the examples discussed will involve more detailed knowledge of particular topics in algebra or topology, but the aim will be to provide enough examples for everyone to understand at least some of them.

Literature

1. S. Mac Lane *Categories for the Working Mathematician.* Springer 1971 (second edition 1998). Still the best one-volume book on the subject, written by one of its founders.

2. S. Awodey *Category Theory.* Oxford U.P. 2006. A more recent treatment very much in the spirit of Mac Lane's classic (Awodey was Mac Lane's last PhD student), but rather more gently paced.

3. T. Leinster *Basic Category Theory.* Cambridge U.P. 2014. Another gently-paced alternative to Mac Lane: easy to read, but it doesn't cover the whole course.

4. F. Borceux *Handbook of Categorical Algebra.* Cambridge U.P. 1994. Three volumes which together provide the best modern account of everything an educated mathematician should know about categories: volume 1 covers most but not all of the Part III course.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Computability and Logic (M24)

Thomas Forster

Wellfoundedness: structural induction and wellfounded induction. Fixed-point theorems. Primitive recursive functions, General recursive functions. Ackermann's function. Goodstein's function. Finite state machines; pumping lemma, Kleene's theorem, Myhill-Nerode. Turing Machines. Decidable and semidecidable sets. Existence of a Universal Turing machine. Craig's theorem. Word problems. Kleene's T-function. Unsolvability of the Halting Problem. Fixed-point theorem. Rice's theorem. Recursive inseparability and Tennenbaum's theorem. Representation of computable functions by λ-terms using Church numerals, Curry-Howard, Church-Rosser. Trakhtenbrot's theorem. Productive sets. Gödel's incompleteness theorem. WQO theory: Kruskal's theorem. Recursive ordinals. Hierarchies of fast-growing functions. Kleene-Post. Friedberg-Muchnik and the Priority method. Baker-Gill-Solovay.

Pre-requisites

No familiarity with computability is assumed: this is an advanced beginners course for students strong enough to do Part III. To the extent that I am assuming that everybody has done Part II Set Theory and Logic I am making assumptions about their inclinations rather than their mastery of specific material.

Literature

There are numerous good books on this subject. The following is in paperback, and holders of a Cambridge Blue card can acquire it from the CUP bookshop in town for a 15% discount.

1. Cutland, N. Computability, Cambridge University Press

My draught notes for this course are linked from my teaching page

`www.dpmms.cam.ac.uk/~tf/cam_only/partiiicomputability.pdf`

and will be gradually updated as I work on them over the summer.

Topics in Set Theory (M24)

Oren Kolman

This course is a relatively self-contained introduction to independence results in modern set theory and their repercussions in contemporary mathematics. It focuses on the ideas and techniques in the proofs, using forcing, that the Continuum Hypothesis ($2^{\aleph_0} = \aleph_1$) and combinatorial assertions relating to infinite trees can be neither proved nor refuted from the principles of ordinary set theory. Applications in algebra, analysis and topology will be illustrated. We shall treat several of the following topics.

Infinitary combinatorics. Cofinality. Stationary sets. Fodor's lemma. Filters and ideals. Ulam's theorem. Cardinal exponentiation. Beth and Gimel functions. Generalized Continuum Hypothesis. Singular Cardinals Hypothesis. Partial orders and trees: Aronszajn, Suslin, and Kurepa. Prediction principles (diamonds, squares). Martin's Axiom. Hypotheses of Suslin and Kurepa; the tree property and weak compactness.

Axiomatics. The formal axiomatic system of ordinary set theory (ZFC). Models of set theory. Absoluteness. Simple independence results. Transfinite recursion. Ranks. Reflection principles. Constructibility.

Forcing. Generic extensions. The forcing theorems. Examples. Adding reals; collapsing cardinals. Introduction to iterated forcing. Internal forcing axioms. Proper forcing.

Large cardinals. Introduction to large cardinals. Ultrapowers. Scott's theorem. Embeddings of V.

Partition relations and possible cofinality theory. Partition relations. Model-theoretic methods. Ramsey's theorem; Erdős-Rado theorem. Kunen's theorem. Walks on ordinals. Todorcevic's theorem. Introduction to pcf theory.

Pre-requisites

The Part II course *Logic and Set Theory* or its equivalent is essential. It will also suffice to have studied enough of the material in the preliminary reading.

Preliminary Reading

1. Kunen, K. *Set Theory*, Studies in Logic, vol. 34, revised edition, College Publications, London, 2013, pages 1–81.
2. Weaver, N. *Forcing for Mathematicians*, World Scientific, 2014, pages 1–20.

Literature

Basic material

1. † Drake, F. R., Singh, D. *Intermediate Set Theory*, John Wiley, Chichester, 1996.
2. Eklof, P. C., Mekler, A. H. *Almost Free Modules*, rev. ed., North-Holland, Amsterdam, 2002.
3. Halbeisen, L. *Combinatorial Set Theory With a Gentle Introduction to Forcing*, Springer, Berlin, 2012.
4. Kanamori, A. *The Higher Infinite*, 2^{nd} ed., Springer, Berlin, 2009.
5. † Kunen, K. *Set Theory*, Studies in Logic, vol. 34, revised edition, College Publications, London, 2013.
6. Weaver, N. *Forcing for Mathematicians*, World Scientific, 2014.

Advanced topics

7. Burke, M. R., Magidor, M. *Shelah's pcf theory and its applications*, Ann. Pure Appl. Logic 50 (1990), 207–254.
8. Kanamori, A., Foreman, M. *Handbook of Set Theory*, Springer, Berlin, 2012.
9. † Shelah, S. *Proper and Improper Forcing*, 2^{nd} edition, Springer, Berlin, 1998. Chapters 1 and 2.

10. Shelah, S. *Cardinal Arithmetic*, Oxford University Press, New York, 1994.

11. Todorcevic, S. *Combinatorial dichotomies in set theory*, Bull. Symbolic Logic 17 (2011), 1–72.

12. Todorcevic, S. *Notes on Forcing Axioms*, World Scientific, 2014.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. Individual consultations will be offered. There will be a two-hour revision class in the Easter Term.

Topics in Category Theory (L24)

Ignacio López Franco

This course aims to introduce the basic theory of monoidal and braided categories and some of its connections to the theory of Hopf algebras (algebraic structures that include quantum groups and affine algebraic groups). The theory of braided categories, an evolution of the symmetric categories, arose in the eighties more or less at the same time as Drinfel′d work on quasi-triangular Hopf algebras and quantum groups. Since then, many aspects of both theories have developed hand in hand, and in some respects there is a dictionary that allow us to translate constructions and properties from one to the other. This dictionary is sometimes called Tannakian duality or reconstruction, and characterises bialgebras, Hopf algebras and (co)quasi triangular bialgebras in terms of extra structure on their categories of (co)representations: respectively, a monoidal structure, internal homs, a braiding.

Topics to be covered include: coherence theorem for monoidal categories, internal homs and duals, braidings; coalgebras, bialgebras and comodules; duals and antipodes, Hopf algebras. (Co)quasi triangular bialgebras and their (co)modules; Tannakian reconstruction of Hopf algebras. If time allows, integrals in Hopf algebras.

Pre-requisites

The first course on Category Theory of Michaelmas Term, or equivalent knowledge, is essential. Familiarity with coalgebras and comodules, or representations of algebraic groups, may be of help.

Literature

1. S. Mac Lane, *Categories for the working mathematician.* Springer-Verlag, 1998.

2. A. Joyal and R. Street, *Braided tensor categories.* Advances in Mathematics, vol. 102, 1993, pp. 20–79.

3. C. Kassel, *Quantum groups.* Graduate Texts in Mathematics, vol. 155, Springer-Verlag, 1995.

4. Street, Ross, *Quantum groups: a path to current algebra.* Australian Mathematical Society Lecture Series, Vol. 19, Cambridge University Press, 2007.

Additional support

There will be four example sheets. Example classes and a revision class will be provided.

Descriptive Set Theory (M24)

Non-Examinable (Graduate Level)

A.R.D. Mathias

This course constitutes an introduction to the concepts and methods of the set-theoretical side of mathematical logic, and will take as its unifying theme the study of Borel and other definable subsets of Euclidean space and other Polish (aka complete separable metric) spaces, which began in the first decade of the twentieth century.

In the second decade, it was discovered that analytic sets (that is, continuous images of Borel sets) need not be Borel; a landmark result was Souslin's proof in 1916 that a subset of a Polish space is Borel if both it and its complement are analytic. The fascination of this proof is that it involves not just the Polish space but also the set of countable ordinals.

In the third and fourth decades, the emergent subject of recursion theory (aka computability) contributed to the refinement of the theory. A further development, in the 1950s, was the study, given a game of infinite length between two players, of whether one of the players will have a winning strategy, and the discovery that such questions necessarily involve strong set-theoretic axioms.

The course aims to be self-contained, but familiarity with the idea of definition by recursion on a well-founded relation will be helpful. Among topics to be treated are:

Well-orderings, ordinals, axioms and transitive models of Zermelo-Fraenkel set theory

Borel codes and their absoluteness; analytic sets, seen as projections of trees; Souslin's theorem.

Kleene boundedness, norms, scales, uniformisation; von Neumann choice for analytic sets

Lebesgue measure and Baire category; the simplest non-measurable set; the Ramsey and perfect set properties; selective ultrafilters.

The determinacy of open games; proof in ZFC of the determinacy of Borel games; proof, using a large cardinal, of the determinacy of analytic games.

Unprovability of Borel determinacy in ZFC *without* the scheme of replacement.

The Martin measure; unprovability of analytic determinacy in ZFC.

Consequences of the axiom of determinacy.

If time permits, results on Borel and other definable equivalence relations, and on universally Baire sets.

Background reading

1 and 2 are from the early period; 3, though perhaps too advanced, has excellent historical remarks; 4 is rather easier.

1. K.Kuratowski *Topology*
2. J.C.Oxtoby *Measure and Category*
3. Y.N.Moschovakis *Descriptive set theory*
4. A.S.Kechris *Classical descriptive set theory*

WQOs and BQOs (E16)

Non-Examinable (Graduate Level)

Thomas Forster

A WQO ("well-quasi-order") is a quasiorder (preorder) with no infinite antichains and no infinite (strictly) descending chains. WQOs are to be found all over the place: Kruskal's theorem states that the embedding relation between finite trees decorated with elements of a WQO is a WQO. Seymour-Robertson states that finite graphs are WQO by the graph minor relation, and there is a recent analogue for matroids. We shall prove Kruskal (it will also be treated briefly in my Part III *Computability and Logic* course in Michaelmas) but not Seymour-Robertson. The class of all WQOs is not closed under all the operations one might expect and this fact leads one to a natural subclass (of "Better Quasi Orders") of WQOs that is so closed. The definition of BQO is subtle! Laver proved that the class of countable linear order types into which one cannot embed the rationals is BQO under injective embedding, and I plan to present a proof of this fact.

This is a rapidly expanding area and the aim of a twelve lecture course can only be to provide a thorough grounding and some pointers.

Pre-requisites

A background in Logic and Combinatorics will help, but this material should be accessible to any Pure graduate student, Part III and above.

Literature

There is no textbook. (This course is in part a side-effect of the lecturer's endeavour to write one!) A home page for this course will be maintained on

`www.dpmms.cam.ac.uk/~tf/cam_only/partivmaterials.html`

and there are links thence to my draught notes for this course and some Part III essays from earlier years on the theorem of Laver alluded to above.

`http://hdl.handle.net/2027.42/41670`

Number Theory

Algebraic Number Theory (M24)

J. A. Thorne

Algebraic number theory lies at the foundation of much current research in number theory, from Fermat's last theorem to the proof of the Sato–Tate conjecture, and is a beautiful subject in its own right. This will be a second course in algebraic number theory, with an emphasis on local (p-adic) aspects of the theory.

Topics likely to be covered include:

Dedekind domains, localization, and passage to completion. The p-adic numbers.

Galois theory of Dedekind domains and ramification theory.

Artin and abelian L–functions.

Class field theory (review of statements only).

Pre-requisites

Part II Galois Theory and Part IB Groups, Rings and Modules (or equivalent) are essential pre-requisites.

Literature

1. S. Lang, *Algebraic number theory.* Graduate Texts in Mathematics, 110. Springer-Verlag, New York, 1994.

2. H. P. F. Swinnerton-Dyer, *A brief guide to algebraic number theory.* London Mathematical Society Student Texts, 50. Cambridge University Press, Cambridge, 2001.

3. J.-P. Serre, *Local fields.* Graduate Texts in Mathematics, 67. Springer-Verlag, New York-Berlin, 1979.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Probabilistic Number Theory (M24)

Adam Harper

Probabilistic number theory began gradually in the work of authors like Hardy and Ramanujan, Turán, and Erdős and Kac, when they proved local limit theorems and a central limit theorem for the function $\omega(n)$, the number of distinct prime factors of n, by exploiting the "almost independence" of divisibility by different primes. Nowadays probabilistic ideas (sometimes of great sophistication) permeate throughout pure mathematics, providing powerful but sometimes misleading heuristics, and methods for rigorously attacking difficult problems. This course will try to illustrate some areas of number theory where probabilistic ideas have been significant, in the process developing the relevant probabilistic tools.

The course will cover some of the following topics, depending on time and audience preferences.

1. *Additive functions.* Definition of additive functions, and the analogy with sums of independent random variables. The Turán–Kubilius inequality for the variance, and applications. Proof of the Erdős–Kac central limit theorem via the method of moments. The rate of convergence in the Erdős–Kac theorem. The special case $\omega(n)$. Application to Vinogradov-type bilinear sums, and ergodic theory.

2. *Heuristics.* The Möbius function $\mu(n)$, and heuristics for the Riemann Hypothesis. Heuristics for primes in tuples (Hardy–Littlewood) and in short intervals (Cramér, via Borel–Cantelli). Success and failure of these heuristics. The Möbius Randomness conjectures.

3. *The Riemann zeta function on the critical line.* Definition and introduction to the zeta function $\zeta(s)$. Selberg's central limit theorem for $\log |\zeta(1/2 + it)|$. The rise of random matrix theory, and moments of the zeta function. Moments of zeta via Selberg's central limit theorem. Heuristics about extreme values of $\log |\zeta(1/2 + it)|$. The connection with branching random walk.

4. *Other topics* (if time permits). Non-Gaussian approximations and limit theorems. Symmetry types in random matrix theory and the Selberg central limit theorem. Probabilistic construction and deletion arguments (e.g. in additive combinatorics).

Pre-requisites

This course will assume that you have attended a basic course on probability theory, and therefore have some familiarity with things like Markov's inequality and some basic statement of the central limit theorem. It will *not* assume familiarity with any number theory concepts, nor with any more advanced probability theory (in particular there is no need to have attended or enjoyed a measure-theoretic course on probability). The course will have a flavour of estimating complicated objects and handling error terms, which might be familiar from previous courses in analysis or probability.

Literature

1. G. Tenenbaum, *Introduction to analytic and probabilistic number theory.* Cambridge Studies in Advanced Mathematics, vol. 46, 1995.

2. E. C. Titchmarsh. *The Theory of the Riemann Zeta-function.* Second edition, revised by D. R. Heath-Brown. Oxford University Press, 1986

I don't know of any single book that covers all the material in the course. Tenenbaum's book is quite a nice introduction to analytic number theory, with good material on additive functions and also some coverage of the Riemann zeta function (although not to the level of the course). Titchmarsh's book is the classic introduction to the zeta function. The two volumes *Probabilistic Number Theory I* and *Probabilistic Number Theory II* by P. D. T. A. Elliott are monographs rather than textbooks, but give an extensive account of classical probabilistic number theory with many historical notes. Many other books contain some relevant material, for example Iwaniec and Kowalski, *Analytic Number Theory.*

Additional support

I plan to write three examples sheets and run three associated examples classes. There will also be a revision class in the Easter Term.

Elliptic Curves (L24)

Tom Fisher

Elliptic curves are the first non-trivial curves, and it is a remarkable fact that they have continuously been at the centre stage of mathematical research for centuries. This will be an introductory course on the arithmetic of elliptic curves, concentrating on the study of the group of rational points. The following topics will be covered, and possibly others if time is available.

Weierstrass equations and the group law. Methods for putting an elliptic curve in Weierstrass form. Definition of the group law in terms of the chord and tangent process. Associativity via the identification with the Jacobian. Elliptic curves as group varieties.

Isogenies. Definition and examples. The degree of an isogeny is a quadratic form. The invariant differential and separability. The torsion subgroup over an algebraically closed field.

Elliptic curves over finite fields. Hasse's theorem.

Elliptic curves over local fields. Formal groups and their classification over fields of characteristic 0. Minimal models, reduction mod p, and the formal group of an elliptic curve. Singular Weierstrass equations.

Elliptic curves over number fields. The torsion subgroup. The Lutz-Nagell theorem. The weak Mordell-Weil theorem via Kummer theory. Heights. The Mordell-Weil theorem. Galois cohomology and Selmer groups. Descent by 2-isogeny. Numerical examples.

Pre-requisites

Familiarity with the main ideas in the Part II courses *Galois Theory* and *Number Fields* will be assumed. The first few lectures will include a review of the necessary geometric background, but some previous knowledge of algebraic curves (at the level of the first two chapters of [3]) would be very helpful. Later in the course, some basic knowledge of the field of p-adic numbers will be assumed.

Preliminary Reading

1. J.H. Silverman and J. Tate, Rational Points on Elliptic Curves, Springer, 1992.

Literature

2. J.W.S. Cassels, Lectures on Elliptic Curves, CUP, 1991.
3. J.H. Silverman, The Arithmetic of Elliptic Curves, Springer, 1986.

Additional support

There will be four example sheets and four associated examples classes.

Modular Forms (L16)

Prof. A. J. Scholl

Modular Forms are classical objects that appear in many areas of mathematics, from number theory to representation theory and mathematical physics. Most famous is, of course, the role they played in the proof of Fermat's Last Theorem, through the conjecture of Shimura-Taniyama-Weil that elliptic curves are modular. This course will cover the classical theory of modular forms (modular curves over $\mathbf{C}$, Hecke operators, Dirichlet series) and some number theoretic applications.

Pre-requisites

Prerequisites for the course are fairly modest; some knowledge of quadratic fields and of holomorphic functions in one complex variable (including basic concepts from the theory of Riemann surfaces) will be helpful.

Literature

1. J. P. Serre, *A course in Arithmetic*, Graduate Texts in Maths. **7**, Springer, New York, 1973 (Chapter VII is an easy-going introduction to the subject).

2. D. Bump, *Automorphic forms and representations*, Cambridge Studies in Adv. Maths. **55**, CUP, Cambridge, 1997 (Sections 1.1-1.6 of Chapter I are particularly relevant).

3. F. Diamond, J. Shurman, *A First Course in Modular Forms*, Graduate Texts in Maths. **228**, Springer, New York, 2005 (a good reference providing also an introduction to the algebraic theory of modular forms).

4. J. Milne, *Modular Functions and Modular Forms*, Lecture notes from a course, download available at `http://www.jmilne.org/math`.

Additional references for enthusiasts

5. T. Miyake, *Modular Forms*, Springer, Berlin, 1989 (a standard reference for classical theory of modular forms).

6. F. Diamond, J. Im, *Modular forms and modular curves*, in: *Seminar on Fermat's Last Theorem*, CMS Conf. Proc. 17, Amer. Math. Soc., Providence, RI, 1995, 39-133.

7. J. Coates, Shing-Tung Yau, *Elliptic curves, modular forms & Fermat's last theorem- Conference Proceedings*, International Press, Cambridge, MA, 1997 (in particular, the survey article by H. Darmon, F. Diamond, R. Taylor).

8. H. Hida, *Elementary theory of L-functions and Eisenstein series*, London Math. Soc. Student Texts **26**, CUP, Cambridge, 1993 (not so elementary introduction to arithmetic of modular forms).

p-adic Families of Modular Forms (M16)

Non-Examinable (Graduate Level)

Dr. G. Rosso

The aim of the course is to explain how an automorphic form can be p-adically deformed. After recalling the theory of p-adic modular forms, we shall explain the main ideas behind Hida's construction of families for ordinary forms with particular focus on the two possible different approaches (cohomological or coherent).

His constructions have been generalized to many other settings (finite slope families, forms on higher rank groups) and has connection with p-adic L-functions. According to the taste of the audience we shall deal with some of these topics.

Probability

Advanced Probability (M24)

James Norris and Gourab Ray

The aim of the course is to introduce students to advanced topics in modern probability theory. The emphasis is on tools required in the rigorous analysis of stochastic processes, such as Brownian motion, and in applications where probability theory plays an important role.

Review of measure and integration: sigma-algebras, measures, and filtrations; integrals and expectation; convergence theorems; product measures, independence, and Fubini's theorem.
Conditional expectation: Discrete case, Gaussian case, conditional density functions; existence and uniqueness; basic properties.
Martingales: Martingales and submartingales in discrete time; optional stopping; Doob's inequalities, upcrossings, martingale convergence theorems; applications of martingale techniques.
Stochastic processes in continuous time: Kolmogorov's criterion, regularization of paths; martingales in continuous time.
Weak convergence: Definitions and characterizations; convergence in distribution, tightness, Prokhorov's theorem; characteristic functions, Lévy's continuity theorem.
Sums of independent random variables: Strong laws of large numbers; central limit theorem; Cramér's theory of large deviations.
Brownian motion: Wiener's existence theorem, scaling and symmetry properties; martingales associated with Brownian motion, the strong Markov property, hitting times; properties of sample paths, recurrence and transience; Brownian motion and the Dirichlet problem; Donsker's invariance principle.
Poisson random measures: Definitions, compound Poisson processes; infinite divisibility, the Lévy-Khinchin formula, Lévy-Itô decomposition.

Pre-requisites

A basic familiarity with measure theory and the measure-theoretic formulation of probability theory is very helpful. These foundational topics will be reviewed at the beginning of the course, but students unfamiliar with them are expected to consult the literature (for instance, Williams' book) to strengthen their understanding.

Literature

- D. Applebaum, Lévy processes (2nd ed.), Cambridge University Press 2009.
- R. Durrett, Probability: Theory and Examples (4th ed.), CUP 2010.
- O. Kallenberg, Foundations of Modern Probability, Springer-Verlag, 1997.
- D. Williams, Probability with martingales, CUP 1991.

Additional support

Four example sheets will be provided along with supervisions. There will be a revision class in Easter term.

Stochastic Calculus (L24)

Jason Miller

This course will be an introduction to Itô calculus.

- *Brownian motion.* Existence and sample path properties.
- *Stochastic calculus for continuous processes.* Martingales, local martingales, semi-martingales, quadratic variation and cross-variation, Itô's isometry, definition of the stochastic integral, Kunita-Watanabe theorem, and Itô's formula.
- *Applications to Brownian motion and martingales.* Lévy characterization of Brownian motion, Dubins-Schwartz theorem, martingale representation, Girsanov theorem, conformal invariance of planar Brownian motion, and Dirichlet problems.
- *Stochastic differential equations.* Strong and weak solutions, notions of existence and uniqueness, Yamada-Watanabe theorem, strong Markov property, and relation to second order partial differential equations.
- *Stroock–Varadhan theory.* Diffusions, martingale problems, equivalence with SDEs, approximations of diffusions by Markov chains.

Pre-requisites

We will assume knowledge of measure theoretic probability as taught in Part III Advanced Probability. In particular we assume familiarity with discrete-time martingales and Brownian motion.

Literature

1. R. Durrett *Probability: theory and examples.* Cambridge. 2010
2. I. Karatzas and S. Shreve *Brownian Motion and Stochastic Calculus.* Springer. 1998
3. P. Morters and Y. Peres *Brownian Motion.* Cambridge. 2010
4. D. Revuz and M. Yor, *Continuous martingales and Brownian motion.* Springer. 1999
5. L.C. Rogers and D. Williams *Diffusions, Markov Processes, and Martingales.* Cambridge. 2000

Schramm-Loewner Evolutions (L16)

James Norris

Schramm-Loewner Evolution (SLE) is a family of random curves in the plane, indexed by a parameter $\kappa \geq 0$. These non-crossing curves are the fundamental tool used to describe the scaling limits of a host of natural probabilistic processes in two dimensions, such as critical percolation interfaces and random spanning trees. Their introduction by Oded Schramm in 1999 was a milestone of modern probability theory.

The course will focus on the definition and basic properties of SLE. The key ideas are conformal invariance and a certain spatial Markov property, which make it possible to use Itô calculus for the analysis. In particular we will show that, almost surely, for $\kappa \leq 4$ the curves are simple, for $4 \leq \kappa < 8$ they have double points but are non-crossing, and for $\kappa \geq 8$ they are space-filling. We will then explore the properties of the curves for a number of special values of κ (locality, restriction properties) which will allow us to relate the curves to other conformally invariant structures.

The fundamentals of conformal mapping will be needed, though most of this will be developed as required. A basic familiarity with Brownian motion and Itô calculus will be assumed but recalled.

Literature

1. Nathanaël Berestycki and James Norris. Lecture notes on SLE.
   `http://www.statslab.cam.ac.uk/~james/Lectures`
2. Wendelin Werner. *Random planar curves and Schramm-Loewner evolutions*, arXiv:math.PR/0303354, 2003.
3. Gregory F. Lawler. *Conformally Invariant Processes in the Plane*, AMS, 2005.

Additional support

Two examples sheets will be provided and examples classes given. There will be a revision class in Easter Term.

Percolation and Related Topics (L16)

Geoffrey Grimmett

The percolation process is the simplest probabilistic model for a random medium in finite-dimensional space. It has a central role in the general theory of disordered systems arising in the mathematical sciences, and it has strong connections with statistical mechanics. Percolation has a reputation as a source of beautiful mathematical problems that are simple to state but seem to require new techniques for solution, and a number of such problems remain very much alive. Amongst connections of topical importance are the relationships to so-called Schramm–Loewner evolutions (SLE), and to the theory of phase transitions in physics.

The basic theory of percolation will be described in this course, with some emphasis on areas for future development. The fundamental techniques, including correlation and/or concentration inequalities and their ramifications,will be covered. The related topics may include self-avoiding walks, and further models from interacting particle systems, and (if time permits) certain physical models for the ferromagnet such as the Ising and Potts models.

Pre-requisites

There are no essential pre-requisites beyond probability and analysis at undergraduate levels, but a familiarity with the measure-theoretic basis of probability will be helpful.

Literature

The following texts will cover the majority of the course, and are available online.

1. Grimmett, G. R., *Probability on Graphs*, Cambridge University Press, 2010;
   see `http://www.statslab.cam.ac.uk/~grg/books/pgs.html`
2. Grimmett, G. R., *Three theorems in discrete random geometry*, Probability Surveys 8, (2011) 403–411; see `http://arxiv.org/abs/1110.2395`

Additional support

It is expected that there will be two example sheets and two classes.

Statistics

The courses in statistics form a coherent Masters-level course in statistics, covering theoretical statistics, applied statistics and applications. You may take all of them, or a subset of them. Core courses are Modern Statistical Methods and Applied Statistics in the Michaelmas Term.

All statistics courses for examination in Part III assume that you have taken an introductory course in statistics and one in probability, with syllabuses that cover the topics in the Cambridge undergraduate courses Probability in the first year and Statistics in the second year. It is helpful if you have taken more advanced courses, although not essential. For Applied Statistics and other applications courses, it is helpful, but not essential, if you have already had experience of using a software package, such as R or Matlab, to analyse data. The statistics courses assume some mathematical maturity in terms of knowledge of basic linear algebra and analysis. However, they are designed to be taken without a background in measure theory, although some knowledge of measure theory is helpful for Topics in Statistical Theory.

The desirable previous knowledge for tackling the statistics courses in Part III is covered by the following Cambridge undergraduate courses. The syllabuses are available online at

`http://www.maths.cam.ac.uk/undergrad/schedules/`

Year		*Courses*
First	*Essential*	Probability
Second	*Essential*	Statistics
	Helpful for some courses	Markov Chains
Third	*Helpful*	Principles of Statistics
	Helpful	Statistical Modelling
	For additional background	Probability and Measure

If you have not taken the courses marked 'essential'then you should review the relevant material over the vacation. If you have more time, then it would be helpful to review other courses as indicated.

Modern Statistical Methods (M16)

Rajen Shah

The remarkable development of computing power and other technology now allows scientists and businesses to routinely collect datasets of immense size and complexity. Most classical statistical methods were designed for situations with many observations and a few, carefully chosen variables. However, we now often gather data where we have huge numbers of variables, in an attempt to capture as much information as we can about anything which might conceivably have an influence on the phenomenon of interest. This dramatic increase in the number variables makes modern datasets strikingly different, as well-established traditional methods perform either very poorly, or often do not work at all.

Developing methods that are able to extract meaningful information from these large and challenging datasets has recently been an area of intense research in statistics, machine learning and computer science. In this course, we will study some of the methods that have been developed to study such datasets. We aim to cover a selection of the following topics:

- Penalised regression, including Ridge regression, the Lasso and variants;
- Machine learning methods including Boosting, Support vector machines, and the kernel trick;
- Multiple testing, including the False Discovery Rate and the Benjamini–Hochberg procedure;
- Graphical modelling and aspects of causal inference.

Pre-requisites

Basic knowledge of statistics, probability and linear algebra.

Literature

1. T. Hastie, R. Tibshirani and J. Friedman *The Elements of Statistical Learning.* 2nd edition. Springer, 2001.
2. P. Bühlmann, S. van de Geer, *Statistics for High-Dimensional Data.* Springer, 2011.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Applied Statistics (Michaelmas and Lent (24))

Davide Pigoli

This course is split over two terms, with 16 hours (8 lectures and 8 practical classes) in the Michaelmas Term and 8 hours (4 lectures and 4 practical classes) in the Lent Term. It is a practical course aiming to develop skills in analysis and interpretation of data. In the Michael- mas Term, core topics in applied statistics are studied, while in the Lent Term more specialised topics are covered. The statistical methods below will be put into practice using R.

Michaelmas Term

Introduction to R. Exploratory data analysis, graphical summaries.

Linear regression and its assumptions. Relevant diagnostics: residuals, Q-Q plots, leverages, Cook's distances and related methods. Hypothesis tests for linear models, ANOVA, F-tests. Interpretation of interactions.

The essentials of generalised linear modelling. Discrete data analysis: binomial and Poisson regression. Diagnostics, goodness-of-fit and model selection.

Lent term

Some special topics. Previous examples include generalised additive models and longitudinal data analysis.

Pre-requisites

It is assumed that you will have done an introductory statistics course, including: elementary probability theory; maximum likelihood; hypothesis tests (t-tests, χ^2-tests, F-tests); confidence intervals.

Literature

1. Agresti, A. (1990) *Categorical Data Analysis.* Wiley.
2. Dobson, A.J. and Barnett A. (2008) *An Introduction to Generalized Linear Models.* Third edition. Chapman & Hall/CRC.
3. Faraway, J. J. (2014) *Linear models with R.* CRC Press.
4. Faraway, J. J. (2005) *Extending the linear model with R: generalized linear, mixed effects and non-parametric regression models.* CRC press.

Additional support

This course includes practical classes in both the Michaelmas and Lent Terms, where statistical methods are introduced in a practical context and where students carry out analysis of datasets under the guidance of the lecturer. In practical classes, the students have the opportunity to discuss statistical questions with the lecturer. Three examples sheets will be provided and there will be three associated examples classes. Because emphasis in this course is placed on the importance of the clear presentation of statistical analyses, students will also have the opportunity to hand in written reports on two analyses, and these will be marked and feedback given to students. There will be a revision class in the Easter Term.

Biostatistics (M10+L14)

This course consists of two components, Statistics in Medical Practice (10 lectures) and Analysis of Survival Data (14 lectures). Together these make up one 3 unit (24 lecture) course. You must take the two components together for the examination.

Statistics in Medical Practice (M10)

R. Turner, C. Jackson, J. Wason, S. Villar, D. de Angelis, A. Presanis, P. Birrell, S. Seaman

Each lecture will be a self-contained study of a topic in biostatistics, which may include clinical trials, meta-analysis, missing data, multi-state models and infectious disease modelling. The relationship between the medical issue and the appropriate statistical theory will be illustrated.

Pre-requisites

Undergraduate-level statistical theory, including estimation, hypothesis testing and interpretation of findings.

Literature

There are no course books, but relevant medical papers will be made available before some lectures for prior reading. It would be very useful to have some familiarity with media coverage of medical stories involving statistical issues, e.g. from Behind the Headlines on the NHS Choices website: `http://www.nhs.uk/News/Pages/NewsIndex.aspx`. A few books to complement the course material are listed below.

1. Armitage P, Berry G, Matthews JNS. *Statistical Methods in Medical Research.* Wiley-Blackwell, 2001
2. Borenstein M, Hedges L, Higgins JPT, Rothstein HR. *Introduction to Meta-Analysis.* Wiley, 2009
3. Jennison C, Turnbull B. *Group Sequential Methods with Applications to Clinical Trials.* Chapman and Hall, 2000

Additional support

An example class will be given, with question sheets and solutions.

Analysis of Survival Data (L14)

F. P. Treasure

Fundamentals of Survival Analysis:

Characteristics of survival data; censoring. Definition and properties of the survival function, hazard and integrated hazard. Examples.

Review of inference using likelihood. Estimation of survival function and hazard both parametrically and non-parametrically.

Explanatory variables: accelerated life and proportional hazards models. Special case of two groups. Model checking using residuals.

Current Topics in Survival Analysis:

In recent years there have been lectures on: frailty, cure, relative survival, empirical likelihood, counting processes and multiple events.

Pre-requisites

This course assumes assumes that you have attended an undergraduate course in statistics and that you are familiar with hypothesis testing, point and interval estimation, and likelihood methods. Attendance at the Michaelmas term course 'Applied Statistics' would be very helpful, not least for the introduction to the R language.

Literature

1. P. Armitage, J. N. S. Matthews and G. Berry *Statistical Methods in Medical Research* (4th ed.), Oxford: Blackwell (2001) [Chapter on Survival Analysis for preliminary reading].
2. D. R. Cox and D. Oakes *Analysis of Survival Data* London: Chapman and Hall (1984).
3. M. K. B. Parmar and D. Machin *Survival Analysis: A Practical Approach* Chichester: John Wiley (1995)
4. Therneau T.M. and Grambsch P.M. *Modelling Survival Data: Extending the Cox Model* New York: Springer (2000)

Additional support

There will be a two hour revision class based on student-selected examination questions in the Easter Term.

Time Series and Monte Carlo Inference (M12+L12)

This course consists of two components, Time Series, and Monte Carlo Inference, each consisting of 12 lectures. Together these make up one 3 unit (24 lecture) course. You must take the two components together for the examination.

Time Series (M12)

Jean-Marc Freyermuth

This is the first part of the lecture course which also includes Monte Carlo Methods.

A time series is a collection of observations acquired over the time. In these lectures, we model time series data as realizations of some class of stochastic processes. Our main goal is to perform statistical inference on the data generating process in order to choose a suitable model, estimate its parameters and ultimately forecast it. We will essentially study the parametric modeling of linear stochastic processes using the AutoRegressive Moving Average model and some of its variants. Basics of frequency domain analysis of time series will be covered as well. If time allows, the last lecture will be devoted to more recent topics of time-varying ARMA and periodic ARMA models.

Pre-requisites

Basic knowledge of statistics and probability.

Literature

1. P. J. Brockwell and R. A. Davis *Introduction to Time Series and Forecasting.* 2nd edition. Springer Texts in Statistics, 2002.
2. P. J. Brockwell and R. A. Davis, *Time Series: Theory and Methods.* Springer Series in Statistics, 2009. 1993.
3. R.H Shumway and D.S Stoffer, *Time Series Analysis and its Applications: with R Examples.* Springer, 2010. 1993.

Additional support

Two examples sheets will be provided and two associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Monte Carlo Inference (L12)

Kayvan Sadeghi

Monte Carlo methods are concerned with the use of stochastic simulation techniques for statistical inference. These have had an enormous impact on statistical practice, especially Bayesian computation, over the last 20 years, due to the advent of modern computing architectures and programming languages. This course covers the theory underlying some of these methods and illustrates how they can be implemented and applied in practice. The following topics will be covered: Techniques of random variable generation, Monte Carlo integration, Importance Sampling, Markov chain Monte Carlo (MCMC) methods for Bayesian inference, Gibbs sampling, Metropolis-Hastings algorithm, reversible jump MCMC.

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Pre-requisites

You should have attended introductory Probability and Statistics courses. A basic knowledge of Markov chains would be helpful. Prior familiarity with a statistical programming package such as R or MATLAB would also be useful.

Literature

1. P. J. E. Gentle, *Random Number Generation and Monte Carlo Methods, (Second Edition).* Springer, 2003.
2. B. D. Ripley, *Stochastic Simulation. Wiley, 1987.*
3. *W.D. Gamerman and H. F. Lopes,* Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference, (Second Edition). Chapman and Hall, 2006.
4. C.P. Robert and G. Casella, *Monte Carlo Statistical Methods, (Second Edition). Springer, 2004.*
5. *5. C. P. Robert and G. Casella,* Introducing Monte Carlo Methods with R. Springer, 2010.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Statistics for Stochastic Processes (L16)

Jakob Söhl

This course gives an introduction to inference for stochastic processes. Stochastic processes are widely used for modelling in many fields and are especially popular in finance. Different classes of processes are introduced and studied such as diffusions, Lévy processes and Itô semimartingales. Observations schemes include high-frequency and low-frequency observations. Estimation procedures focus on nonparametric methods for the volatility, the drift, the invariant density and the jump measure. The convergence rates of the estimators are discussed.

Pre-requisites

This course assumes knowledge of probability theory as covered by the lecture Advanced Probability in Cambridge, for example of the topic of martingales, the construction of Brownian motion and its properties. Necessary background on stochastic calculus will be provided during the course. For a throughout treatment of stochastic calculus it is advisable to attend the lecture Stochastic Calculus and Applications that is offered in lent term as well. A familiarity with statistical concepts can be useful but is not a necessary prerequisite for the course.

Literature

1. Y.A. Kutoyants, *Statistical Inference for Ergodic Diffusion Processes.* Springer, 2004.
2. J. Söhl, *Statistics for Stochastic Processes.* Lecture notes available online, 2015.

Additional support

Two examples sheets will be provided and two associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Topics in Statistical Theory (L16)

Quentin Berthet

The objective of this course is give an introduction to topics in modern statistical theory. Important problems in high-dimensional and nonparametric statistics will be covered, as well as some techniques used to solve them. Emphasis will be placed on theoretical results.

- Estimation in high dimension: deviation bounds, structural assumptions (e.g. sparsity), links with convex geometry, spectral methods.

- Minimax theory: notion of information-theoretic lower bounds, distance and divergence between distributions, optimal rates.

- Nonparametric statistics: density estimation, regression.

Pre-requisites

A good background in probability theory, as well as elements of linear algebra and functional analysis. A preliminary course in mathematical statistics can be helpful, but it is not necessary.

Literature

No book will be explicitly followed, but some of the material is covered in

A. Tsybakov, *Introduction to nonparametric estimation*, Springer 2009

Additional support

Three example sheets will be provided and associated examples classes will be given. There will be a revision class in the Easter Term.

Concentration Inequalities (E16)

Non-Examinable (Graduate Level)

Stéphane Boucheron

Concentration inequalities for functions of independent random variables is an area of probability theory that has witnessed a great revolution in the last few decades, and has applications in a wide variety of areas such as machine learning, statistics, discrete mathematics, and high-dimensional geometry. Roughly speaking, if a function of many independent random variables does not depend too much on any of the variables then it is concentrated in the sense that with high probability, it is close to its expected value. This course offers a host of inequalities to illustrate this rich theory.

It describes the interplay between the probabilistic structure (independence) and a variety of tools ranging from functional inequalities to transportation arguments to information theory. Applications to the study of empirical processes, random projections, random matrix theory, and threshold phenomena are also presented.

Pre-requisites

We shall assume notions of probability theory.

Literature

1. Boucheron, S., Lugosi, G., & Massart, P. (2013). *Concentration inequalities: A nonasymptotic theory of independence.* OUP Oxford.

2. Chatterjee, S. (2014). *Superconcentration and related topics.* Springer.

3. Garling, D. J. (2007). *Inequalities: a journey into linear analysis.* Cambridge University Press.

4. Ledoux, M. (2005). *The concentration of measure phenomenon (No. 89).* American Mathematical Soc.

5. Raginsky, M., Sason, I. (2014). *Concentration of Measure Inequalities in Information Theory, Communications, and Coding.* Now Publishers Inc..

6. Tropp, Joel A. *An introduction to matrix concentration inequalities.* arXiv preprint arXiv:1501.01571 (2015).

`http://www.lpma-paris.fr/~boucheron`

Operational Research and Mathematical Finance

Advanced Financial Models (M24)

M.R. Tehranchi

This course is an introduction to financial mathematics, with a focus on the pricing and hedging of contingent claims. It complements the material in Advanced Probability and Stochastic Calculus & Applications.

- *Discrete time models.* Filtrations and martingales. Arbitrage, martingale deflators and equivalent martingale measures. Attainable claims and market completeness. European and American claims. Optimal stopping.
- *Brownian motion and stochastic calculus.* Brief survey of stochastic integration. Girsanov's theorem. Itô's formula. Martingale representation theorem.
- *Continuous time models.* Admissible strategies. Black–Scholes model. The implied volatility surface. Pricing and hedging via partial differential equations. Dupire's formula. Stochastic volatility models.
- *Interest rate models.* Short rates, forward rates and bond prices. Markovian short rate models. The Heath–Jarrow–Morton drift condition.

Pre-requisites

Familiarity with measure-theoretic probability will be assumed.

Literature

1. M. Baxter & A. Rennie. *Financial calculus: an introduction to derivative pricing.* Cambridge University Press, 1996
2. M. Musiela and M. Rutkowski. *Martingale Methods in Financial Modelling.* Springer, 2006
3. D. Kennedy.*Stochasic Financial models.* Chapman & Hall, 2010
4. D. Lamberton & B. Lapeyre.*Introduction to stochastic calculus applied to finance.* Chapman & Hall, 1996
5. S. Shreve. *Stochastic Calculus for Finance: Vol. 1 and 2.* Springer-Finance, 2005

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Mathematics of Operational Research (M24)

Richard Weber

The course covers a selection of mathematical tools and models for operational research:

- Lagrangian sufficiency theorem. Lagrange duality. Supporting hyperplane theorem. Sufficient conditions for convexity of the optimal value function. Fundamentals of linear programming. Linear program duality. Shadow prices. Complementary slackness. [2]

- Simplex algorithm. Two-phase method. Dual simplex algorithm. Gomory's cutting plane method. [3]
- Complexity of algorithms. NP-completeness. Exponential complexity of the simplex algorithm. Polynomial time algorithms for linear programming. [2]
- Network simplex algorithm. Transportation and assignment problems, Ford-Fulkerson algorithm, max-flow/min-cut theorem. Shortest paths, Bellman-Ford algorithm, Dijkstra's algorithm. Minimum spanning trees, Prim's algorithm. MAX CUT, semidefinite programming, interior point methods. [5]
- Branch and bound. Dakin's method. Exact, approximate, and heuristic methods for the travelling salesman problem. [3]
- Cooperative and non-cooperative games. Two-player zero-sum games. Existence and computation of Nash equilibria, Lemke-Howson algorithm. Bargaining. Coalitional games, core, nucleolus, Shapley value. Mechanism design, Arrow's theorem, Gibbard-Satterthwaite theorem, VCG mechanisms. Auctions, revenue equivalence, optimal auctions. [9]

Pre-requisites

The course is accessible to a candidate with mathematical maturity who has no previous experience of operational research; however it is expected that most candidates will already have had exposure to some of the topics listed above.

Literature

1. M.S. Bazaraa, J.J. Jarvis and H.D. Sherali: Linear Programming and Network Flows, Wiley (1988).
2. D. Bertsimas, J.N. Tsitsiklis. Introduction to Linear Optimization. Athena Scientific (1997).
3. N. Nisan, T. Roughgarden, E. Tardos, V. Vazirani. Algorithmic Game Theory. Cambridge University Press (2007).
4. M. Osborne, A. Rubinstein: A Course in Game Theory. MIT Press (1994).

Additional support

Three examples sheets will be provided and three associated examples classes will be given.

Stochastic Networks (M24)

Frank Kelly

Communication networks underpin our modern world, and provide fascinating and challenging examples of large-scale stochastic systems. This course uses stochastic models to shed light on important issues in the design and control of communication networks.

Randomness arises in communication systems at many levels: for example, the initiation and termination times of calls in a telephone network, or the statistical structure of the arrival streams of packets at routers in the Internet. How can routing, flow control and connection acceptance algorithms be designed to work well in uncertain and random environments? And can we design these algorithms using simple local rules so that they produce coherent and purposeful behaviour at the macroscopic level?

The first two parts of the course will describe a variety of classical models that can be used to help understand the performance of large-scale stochastic networks. Queueing and loss networks will be studied, as well as random access schemes and the concept of an effective bandwidth. Parallels will be drawn with models from physics, and with models of traffic in road networks.

The third part of the course will more recently developed models of packet traffic and of congestion control algorithms in the Internet. This is an area of some practical importance, with network operators, hardware and software vendors, and regulators actively seeking ways of delivering new services reliably and effectively. The complex interplay between end-systems and the network has attracted the attention of economists as well as mathematicians and engineers.

We describe enough of the technological background to communication networks to motivate our models, but no more. Some of the ideas described in the book are finding application in financial, energy, and economic networks as computing and communication technologies transform these areas. But communication networks currently provide the richest and best developed area of application within which to present a connected account of the ideas.

Pre-requisites

Mathematics that will be assumed to be known before the start of the course: Part IB Optimization and Markov Chains. Familiarity with Part II Applied Probability would be useful, but is not assumed.

Preliminary Reading

A feeling for some of the ideas of the course can be taken from

The mathematics of traffic in networks. In *Princeton Companion to Mathematics* (Edited by Timothy Gowers; June Barrow-Green and Imre Leader, associate editors) Princeton University Press, 2008. 862-870.

Literature

Reference 3 is the course text.

1. B. Hajek *Communication Network Analysis.*
2. F.P. Kelly *Reversibility and Stochastic Networks.* Cambridge University Press, 2011.
3. F. Kelly and E. Yudovina *Stochastic Networks.* Cambridge University Press, 2014.
4. R. Srikant and L. Ying *Communication Networks: An Optimization, Control and Stochastic Networks Perspective.* Cambridge University Press, 2013.

Additional support

Examples sheets will be provided and associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Particle Physics, Quantum Fields and Strings

The courses on Symmetries, Fields and Particles, Quantum Field Theory, Advanced Quantum Field Theory and The Standard Model are intended to provide a linked course covering High Energy Physics. The remaining courses extend these in various directions. Knowledge of Quantum Field Theory is essential for most of the other courses. The Standard Model course assumes knowledge of the course Symmetries, Fields and Particles.

Desirable previous knowledge

Basic quantum mechanics, wave functions, amplitudes and probabilities. Quantisation in terms of commutation relations between coordinates q and corresponding momenta p. Schrödinger and Heisenberg pictures. Dirac bra and ket formalism.

Harmonic oscillator, its solution using creation and annihilation operators.

Angular momentum operators and their commutation relations. Determination of possible states $|jm\rangle$ from the basic algebra. Idea of spin as well as orbital angular momentum. Two body systems. Clebsch-Gordan coefficients for decomposition of products of angular momentum states.

Perturbation theory, degenerate case and to second order. Time dependent perturbations, 'Golden Rule' for decay rates. Cross sections, scattering amplitudes in quantum mechanics, partial wave decomposition.

Lagrangian formulation of dynamics. Normal modes. Familiarity with Lorentz transformations and use of 4-vectors in special relativity, 4-momentum p^μ for a particle and energy-momentum conservation in 4-vector form. Relativistic formulation of electrodynamics using $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and Lagrangian density $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$.

Basic knowledge of δ-functions (including in 3 dimensions) and Fourier transforms. Basic properties of groups and the idea of a matrix representation. Permutation group.

The desirable previous knowledge needed to tackle the Particle Physics, Quantum Fields and Strings courses is covered by the following Cambridge undergraduate courses. Students starting Part III from outside might like to peruse the syllabuses on the WWW at

`http://www.maths.cam.ac.uk/undergrad/schedules/`

Year		*Courses*
Second	*Essential*:	Quantum Mechanics, Methods, Complex Methods.
	Helpful:	Electromagnetism.
Third	*Essential*:	Principles of Quantum Mechanics, Classical Dynamics.
	Very helpful:	Applications of Quantum Mechanics, Statistical Physics, Electrodynamics.

If you have not taken the courses equivalent to those denoted 'essential', then you should review the relevant material over the vacation.

Quantum Field Theory (M24)

Malcolm J. Perry

Quantum Field Theory is the language in which all of modern physics is formulated. It represents the marriage of quantum mechanics with special relativity and provides the mathematical framework in which to describe the interactions of elementary particles.

This first Quantum Field Theory course introduces the basic types of fields which play an important role in high energy physics: scalar, spinor (Dirac), and vector (gauge) fields. The relativistic invariance and symmetry properties of these fields are discussed using Lagrangian language and Noether's theorem.

The quantisation of the basic non-interacting free fields is firstly developed using the Hamiltonian and canonical methods in terms of operators which create and annihilate particles and anti-particles. The associated Fock space of quantum physical states is explained together with ideas about how particles propagate in spacetime and their statistics. How these feilds interact with a classical electromagnertic field is described.

Next, we introduce the path integral which is an alternative way of describing quantum fields. The paht integral is fundamental in introducing interaction into quantum field theory. Interactions are described using perturbative theory and Feynman diagrams. This is first illustrated for theories with a purely scalar field interaction, and then for a couplings between scalar fields and fermions. Finally Quantum Electrodynamics, the theory of interacting photons, electrons and positrons, is introduced and elementary scattering processes are computed.

Finally, the idea of loops in Feynman diagrams are explored and the queston of the consequent infinities looked at. Ways of dealing with the infinities will be expolored in the Advanced Quantum Field Theory course which follows on directly from this one.

Pre-requisites

You will need to be comfortable with the Lagrangian and Hamiltonian formulations of classical mechanics and with special relativity. You will also need to have taken an advanced course on quantum mechanics.

Literature

1. M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1996).
2. A. Zee, *Quantum Field Theory in a Nutshell*, Princeton University Press, (2010)
3. M. Srednicki, *Quantum Field Theory*, Cambridge University Press, (2007). (a free prelminary version is available here `http://web.physics.ucsb.edu/~mark/ms-qft-DRAFT.pdf`
4. M. Schwartz, *Quantum Field Theory and the Standard Model*, Cambridge University Press (2014).

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.

Symmetries, Fields and Particles (M24)

N. Dorey

This course introduces the various types of elementary particle – quarks, leptons, gauge and Higgs particles – and the various symmetry groups useful for understanding their properties. Some symmetries in particle physics are exact, and some only approximate. The most important symmetry groups are specific Lie groups, including SU(2), SU(3), the Lorentz and Poincaré groups.

Basic Lie group theory will be covered, including Lie algebras and the relation between Lie algebras and Lie groups. The representation theory of SU(2) (closely related to quantum mechanical angular momentum theory) will be extended to give the theory of SU(3) representations. Hadrons, the particles containing quarks and antiquarks, are classified by representations of SU(3) because of the approximate flavour symmetry among quarks.

The Standard Model of particles is a gauge theory, a quantum field theory with an exact, locally acting Lie group symmetry. The structure of gauge theory Lagrangians will be introduced, and also the Higgs mechanism for (spontaneous) symmetry breaking and mass generation.

The course ends with a discussion of the Lorentz and Poincaré groups, and how their representations are used to classify momentum and spin states of relativistic particles.

The course is designed to be taken in conjunction with the Quantum Field Theory course, and as a preliminary to the Standard Model course, although it is formally independent of these.

Pre-requisites

Basic finite group theory, including subgroups and orbits. Special relativity and quantum theory, including orbital angular momentum theory and Pauli spin matrices. Basic ideas about manifolds, including coordinates, dimension, tangent spaces.

It will be useful to have an outline knowledge of elementary particles, as in several books, e.g. D.H. Perkins, Introduction to High Energy Physics, 4th ed., Cambridge University Press, 2000.

Literature

1. G. Costa and G. Fogli, *Symmetries and Group Theory in Particle Physics, Lecture Notes in Physics 823.* Springer, 2012.
2. H.F. Jones, *Representations and Physics.* 2nd edition. Taylor and Francis, 1998.
3. H. Georgi, *Lie Algebras in Particle Physics.* Westview Press, 1999.
4. J. Fuchs and C. Schweigert, *Lie Algebras and Representations.* Cambridge University Press, 2003.

Additional support

A set of course notes is available on the Part III Examples and Lecture Notes webpage. Four examples sheets will be provided and four associated examples classes in moderate-sized groups will be given by graduate students.

Statistical Field Theory (M16)

M B Wingate

[Below is the course desciption from previous years. This year will follow a similar structure, but I may decide to make some changes as I write my version of the lectures.]

This course is an introduction to the renormalization group, the basis for a modern understanding of field theory, and the construction of effective field theories. The discussion is concerned with statistical systems including their relationship with quantum field theory in its Euclidean formulation.

The phenomenology of phase transitions is reviewed, leading to the introduction of the theory of critical phenomena. Landau-Ginsburg theory and mean field theory are presented and applied to the Ising model. The classification of phase transitions and their relationship with critical points is presented, and the renormalization group is introduced first in the context of the soluble 1D Ising model and then in general. The renormalization group is used for calculating properties of systems near a phase transition, for example in the Ising and Gaussian models, and the concepts of critical exponents, anomalous dimensions, and scaling are discussed.

The idea of the continuum limit for models controlled by a critical point and the relationship with continuum quantum field theory is elucidated.

Perturbation theory is introduced for the scalar field model with interactions and some example calculations are presented.

Pre-requisites

Background knowledge of Statistical Mechanics at an undergraduate level is essential. This course complements the Quantum Field Theory and Advanced Quantum Field Theory courses.

Literature

1. J M Yeomans, *Statistical Mechanics of Phase Transitions*, Clarendon Press (1992).
2. J J Binney, N J Dowrick, A J Fisher, and M E J Newman, *The Theory of Critical Phenomena*, Oxford University Press (1992).
3. M Le Bellac, *Quantum and Statistical Field Theory*, Oxford University Press (1991).
4. M Kardar, *Statistical Physics of Fields*, Cambridge University Press (2007).
5. D Amit and V Martín-Mayor, *Field Theory, the Renormalization Group, and Critical Phenomena*, 3rd edition, World Scientific (2005).
6. C Itzykson and J-M Drouffe, *Statistical Field Theory*, Vols. 1-2, Cambridge University Press (1991).
7. L D Landau and E M Lifshitz, Statistical Physics, Pergamon Press (1996).

Additional support

Three examples sheets will be provided and three associated examples classes will be given.

Advanced Quantum Field Theory (L24)

DB Skinner

Quantum Field Theory (QFT) provides the most profound description of Nature we currently possess. As well as being the basic theoretical framework for describing elementary particles and their interactions (excluding gravity), QFT also plays a major role in areas of physics and mathematics as diverse as string theory, condensed matter physics, topology and geometry, astrophysics and cosmology.

This course builds on the Michaelmas Quantum Field Theory course, using techniques of path integrals and functional methods to study quantum gauge theories. Gauge Theories are a generalisation of electrodynamics and form the backbone of the Standard Model - our best theory encompassing all particle physics. In a gauge theory, fields have an infinitely redundant description; we can transform the fields by a different element of a Lie Group at every point in space-time and yet still describe the same physics. Quantising a gauge theory requires us to eliminate this infinite redundancy. In the path integral approach, this is done using tools such as ghost fields and BRST symmetry. We discuss the construction of gauge theories and their most important observables, Wilson Loops. Time permitting, we will explore the possibility that a classical symmetry may be broken by quantum effects. Such anomalies have many important consequences, from constraints on interactions between matter and gauge fields, to the ability to actually render a QFT inconsistent.

A further major component of the course is to study Renormalization. Wilsons picture of Renormalisation is one of the deepest insights into QFT – it explains why we can do physics at all! The essential point is that the physics we see depends on the scale at which we look. In QFT, this dependence is governed by evolution along the Renormalisation Group (RG) flow. The course explores renormalisation systematically, from the use of dimensional regularisation in perturbative loop integrals, to the difficulties inherent in trying to construct a quantum field theory of gravity. We discuss the various possible behaviours of a QFT under RG flow, showing in particular that the coupling constant of a non-Abelian gauge theory can effectively become small at high energies. Known as asymptotic freedom, this phenomenon revolutionised our understanding of the strong interactions. We introduce the notion of an Effective Field Theory that describes the low energy limit of a more fundamental theory and helps parametrise possible departures from this low energy approximation. From a modern perspective, the Standard Model itself appears to be but an effective field theory.

Time permitting, we may also discuss various modern topics in QFT, such as dualities, localization and topological QFTs,

Pre-requisites

Knowledge of the Michaelmas term Quantum Field Theory course will be assumed. Familiarity with the course Symmetries, Fields and Particles would be very helpful.

Preliminary Reading

1. Zee, A., *Quantum Field Theory in a Nutshell*, 2^{nd} edition, PUP (2010).

Literature

1. Srednicki, M., *Quantum Field Theory*, CUP (2007).
2. Weinberg, S., *The Quantum Theory of Fields*, vols. 1 & 2, CUP (1996).
3. Banks, T. *Modern Quantum Field Theory: A Concise Introduction*, CUP (2008).
4. Peskin, M. and Schroeder, D., *An Introduction to Quantum Field Theory*, Perseus Books (1995).

Additional support

There will be four problem sheets handed out during the course. Classes for each of these sheets will be arranged during Lent Term. There will also be a general revision class during Easter Term.

Standard Model (L24)

C.E. Thomas

The Standard Model of particle physics is, by far, the most successful application of quantum field theory (QFT). At the time of writing, it accurately describes all experimental measurements involving strong, weak, and electromagnetic interactions. The course aims to demonstrate how this model, a QFT with gauge group $SU(3) \times SU(2) \times U(1)$ and fermion fields for the leptons and quarks, is realised in nature. It is intended to complement the more general Advanced QFT course.

We begin by defining the Standard Model in terms of its local (gauge) and global symmetries and its elementary particle content (spin-half leptons and quarks, and spin-one gauge bosons). The parity P, charge-conjugation C and time-reversal T transformation properties of the theory are investigated. These need not be symmetries manifest in nature; e.g. only left-handed particles feel the weak force and so it violates parity symmetry. We show how CP violation becomes possible when there are three generations of particles and describe its consequences.

Ideas of spontaneous symmetry breaking are applied to discuss the Higgs Mechanism and why the weakness of the weak force is due to the spontaneous breaking of the $SU(2) \times U(1)$ gauge symmetry. Recent measurements of what appear to be Higgs boson decays will be presented.

We show how to obtain cross sections and decay rates from the matrix element squared of a process. These can be computed for various scattering and decay processes in the electroweak sector using perturbation theory because the couplings are small. We touch upon the topic of neutrino masses and oscillations, an important window to physics beyond the Standard Model.

The strong interaction is described by quantum chromodynamics (QCD), the non-abelian gauge theory of the (unbroken) $SU(3)$ gauge symmetry. At low energies quarks are confined and form bound states called hadrons. The coupling constant decreases as the energy scale increases, to the point where perturbation theory can be used. As an example we consider electron-positron annihilation to final state hadrons at high energies. Time permitting, we will discuss nonperturbative approaches to QCD. For example, the framework of effective field theories can be used to make progress in the limits of very small and very large quark masses.

Both very high-energy experiments and very precise experiments are currently striving to observe effects that cannot be described by the Standard Model alone. If time permits, we comment on how the Standard

Model is treated as an effective field theory to accommodate (so far hypothetical) effects beyond the Standard Model.

Pre-requisites

It is necessary to have attended the Quantum Field Theory and the Symmetries, Fields and Particles courses, or to be familiar with the material covered in them. It would be advantageous to attend the Advanced QFT course during the same term as this course, or to study renormalisation and non-abelian gauge fixing.

Reading to complement course material

1. M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory*, Addison-Wesley (1995).
2. H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin/Cummings (1984).
3. T-P. Cheng and L-F. Li, *Gauge Theory of Elementary Particle Physics*, Oxford University Press (1984).
4. I.J.R. Aitchison and A.J.G. Hey, *Gauge Theories in Particle Physics*, IoP Publishing (2012) (two volumes or earlier 1989 edition in one volume).
5. F. Halzen and A.D. Martin, *Quarks and Leptons: An Introductory Course in Modern Particle Physics*, John Wiley and Sons (1984).
6. J.F. Donoghue, E. Golowich and B.R. Holstein, *Dynamics of the Standard Model*, Cambridge University Press (1992).

Additional support

Four examples sheets will be provided and four associated examples classes will be given.

String Theory (L24)

Paul K. Townsend

String Theory supposes that elementary particles are excitations of a string, which could be open (with two endpoints) or closed. Closed strings have a massless spin-2 particle in their spectrum, which suggests that String Theory is a theory of quantum gravity. Open strings yield analogous generalisations of gauge theory, so a theory of open and closed strings is potentially one that can unify gravity with the forces of the standard model of particle physics. This course will introduce the strings of string theory as constrained Hamiltonian systems, focusing initially on the Nambu-Goto (NG) string, and using extensively the insights provided by a similar investigation of the relativistic point particle. It will be seen that the NG string is a gauge theory of the (infinite-dimensional) 2D conformal group.

Various methods of quantisation of the NG string, including light-cone gauge, and "old covariant" (and possibly BRST); this will reveal that there is a critical space-time dimension (26) and that the ground state is a tachyon. A study of the possible boundary conditions on open strings will suggest an interpretation in terms of branes. Superstring theory will be introduced, in the RNS formalism, as a "square-root" of the NG string. The light-cone gauge will be used to show that the the critical dimension is 10. It will be explained briefly why superstring theories are tachyon-free and why there are five of them.

The path integral formulation of QM will be explained, and why you don't need fields to do QFT. The generalisation to strings will lead to ideas of conformal field theory, a computation of the Virasoro-Shapiro amplitude for the scattering of closed-string tachyons of the NG string, and a discussion of some general features of string perturbation theory. This will include a look at the "one-loop" quantum corrections and why there are no UV divergences. Other topics that may be discussed are T-duality and how the five superstring theories are unified by "M-Theory".

Pre-requisites

This course assumes you know the basics of (i) Special Relativity and (ii) Quantum Mechanics. Complete typed course notes will be provided. The course structure is rather different from anything that can be found in text books or on-line reviews but the following are useful for general background and/or specific topics covered in the course:

Literature

1. Green, Schwarz and Witten, *Superstring Theory: Vol. 1:Introduction* CUP 1987.
2. Brink and Henneaux, *Principles Of String Theory*, Plenum 1988.
3. Lüst and Theisen, *Lecture Notes in Physics: Superstring Theory*, Springer 1989.
4. Gleb Arutyanov, *Lectures on String Theory*, e-booksdirectory.com
5. David Tong, *String Theory*, arXiv:0908.0333

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will also be a weekly office hour during the Lent term for questions about the lectures.

Supersymmetry (L16)

F. Quevedo

This course provides an introduction to the use of supersymmetry in quantum field theory. Supersymmetry combines commuting and anti-commuting dynamical variables and relates fermions and bosons.

Firstly, a physical motivation for supersymmetry is provided. The supersymmetry algebra and representations are then introduced, followed by superfields and superspace. 4-dimensional supersymmetric Lagrangians are then discussed, along with the basics of supersymmetry breaking. The minimal supersymmetric standard model will be introduced. If time allows a short discussion of supersymmetry in higher dimensions will be briefly discussed.

Three examples sheets and examples classes will complement the course.

Pre-requisites

It is necessary to have attended the Quantum Field Theory and the Symmetries in Particle Physics courses, or be familiar with the material covered in them.

Preliminary Reading

1. The first chapters of `http://arxiv.org/abs/hep-ph/0505105`

Literature

For more advanced topics later in the course, it will helpful to have a knowledge of renormalisation, as provided by the Advanced Quantum Field Theory course. It may also be helpful (but not essential) to be familiar with the structure of The Standard Model in order to understand the final lecture on the minimal supersymmetric standard model.

Beware: most of the supersymmetry references contain errors in minus signs, aside (as far as I know) Wess and Bagger.

1. Course lecture notes from last year:

   http://www.damtp.cam.ac.uk/user/examples/3P7.pdf

2. Videos of a very similar lecture course: follow the links from

   http://users.hepforge.org/~allanach/teaching.html

3. Supersymmetric Gauge Field Theory and String Theory, Bailin and Love, IoP Publishing (1994) has nice explanations of the physics. An erratum can be found at

   http://www.phys.susx.ac.uk/~mpfg9/susyerta.htm

4. Introduction to supersymmetry, J.D. Lykken, hep-th/9612114. This introduction is good for extended supersymmetry and more formal aspects.

5. Supersymmetry and Supergravity, Wess and Bagger, Princeton University Press (1992). Note that this terse and more mathematical book has the opposite sign of metric to the course.

6. A supersymmetry primer, S.P. Martin, hep-ph/9709256 is good and detailed for phenomenological aspects, although with the opposite sign metric to the course.

Classical and Quantum Solitons (E16)

N. S. Manton

Solitons are solutions of classical field equations with particle-like properties. They are localised in space, have finite energy and are stable against decay into radiation. The stability usually has a topological explanation. After quantisation, they give rise to new particle states in the underlying quantum field theory that are not seen in perturbation theory. We will focus mainly on kink solitons in one space dimension, and on Skyrmions in three dimensions. Solitons in gauge theories will also be mentioned.

Pre-requisites

This course assumes you have taken Quantum Field Theory and Symmetries, Fields and Particles. The small amount of topology that is needed will be developed during the course.

Literature

1. N. Manton and P. Sutcliffe, *Topological Solitons.* C.U.P., 2004 (Chapters 1,3,4,5,9).

2. R. Rajaraman, *Solitons and Instantons.* North-Holland, 1987.

3. A. Vilenkin and E.P.S. Shellard, *Cosmic Strings and other Topological Defects.* C.U.P., 1994 (Chapter 3).

Additional support

Two examples sheets will be provided and two associated examples classes will be given.

Relativity and Gravitation

These courses provide a thorough introduction to General Relativity and Cosmology. The Michaelmas term courses introduce these subjects, which are then developed in more detail in the Lent term courses on Black Holes and Advanced Cosmology. Applications of Differential Geometry to Physics explains how many physical theories can be formulated elegantly using the language of differential geometry. Non-examinable courses explore more advanced topics.

Desirable previous knowledge

Suffix notation, vector and tensor analysis. Variational principle and Lagrangian formulation of dynamics. Familiarity with Lorentz transformations and use of 4-vectors in special relativity, 4-momentum p^μ for a particle and energy-momentum conservation in 4-vector form. Relativistic formulation of electrodynamics using $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and Lagrangian density $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$.

Knowledge of basic mathematical methods, including Fourier transforms, normal modes, and δ-function (including 3-dimensions). Basic quantum mechanics, wave functions, amplitudes and probabilities. Familiarity with aspects of statistical physics and thermodynamics, including notions of thermal equilibrium, entropy, black body radiation, and Fermi-Dirac, Bose-Einstein and Boltzmann distributions.

The desirable previous knowledge needed to tackle the Relativity and Gravitation courses is covered by the following Cambridge undergraduate courses. Students starting Part III from outside might like to peruse the syllabuses on the WWW at

`http://www.maths.cam.ac.uk/undergrad/schedules/`

Year		*Courses*
First	*Essential*:	Vectors & Matrices, Diff. Eq., Vector Calculus, Dynamics & Relativity.
Second	*Essential*:	Methods, Quantum Mechanics, Variational Principles.
	Helpful:	Electromagnetism, Geometry, Complex Methods.
Third	*Essential*:	Classical Dynamics.
	Very helpful:	General Relativity, Stat. Phys., Electrodynamics, Cosmology.
	Helpful:	Further Complex Methods, Asymptotic methods.

If you have not taken the courses equivalent to those denoted 'essential', then you should review the relevant material over the vacation.

General Relativity (M24)

Ulrich Sperhake

General Relativity is the theory of space-time and gravitation proposed by Einstein in 1915. It remains at the centre of theoretical physics research, with applications ranging from astrophysics to string theory. This course will introduce the theory using a modern, geometric, approach.

Course website: `http://www.damtp.cam.ac.uk/user/us248/Lectures/lectures.html`

Pre-requisites

This course will be self-contained, so previous knowledge of General Relativity is not essential. However, many students have already taken an introductory course in General Relativity (e.g. the Part II course). If you have not studied GR before, then it is strongly recommended that you study an introductory book (e.g. Hartle or Schutz) before attending this course. Certain topics usually covered in a first course, e.g. the solar system tests of GR, will not be covered in this course.

Familiarity with Newtonian Gravity and special relativity is essential. Knowledge of the relativistic formulation of electrodynamics is desirable. Familiarity with finite-dimensional vector spaces, the calculus of functions $f : R^m \to R^n$, and the Euler-Lagrange equations will be assumed.

Preliminary Reading

1. J. B. Hartle,*An introduction to Einstein's General Relativity.* Addison-Wesley, 2003.
2. B. Schutz,*A First Course in General Relativity.* Cambridge University Press, 2009.

Literature

1. R. M. Wald,*General Relativity.* University of Chicago Press, 1984.
2. S. M. Carroll,*Spacetime and Geometry: An Introduction to General Relativity.* Addison-Wesley, 2004.
3. J. M. Stewart,*Advanced General Relativity.* Cambridge University Press, 1993.
4. L. Ryder,*Introduction to General Relativity.* Cambridge University Press, 2009.
5. E. Gourgoulhon,*3+1 Formalism and Bases of Numerical Relativity.*

   `http://arxiv.org/abs/gr-qc/0703035` .

Chapter 1 of John Stewart's book gives a concise overview of differential geometry which also guides this part of the course. Carroll's and Ryder's books are very readable introductions. Gourgoulhon's notes provide a comprehensive overview of the space-time split of general relativity. Wald's book discusses many advanced topics; very suitable for obtaining comprehensive treatment on isolated topics.

Additional support

Three examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Course website: `http://www.damtp.cam.ac.uk/user/us248/Lectures/lectures.html`

Cosmology (M24)

James Fergusson, David Marsh

This course covers the last 13.8 billion years of the evolution of your universe, from the initial inflationary quantum perturbations to the creation of galaxies we observe today. The course will follow the following format

1. Geometry and Dynamics
2. Inflation
3. Cosmological Perturbation Theory
4. Structure Formation
5. Thermal History
6. Initial Conditions from Inflation

Pre-requisites

This course is taught in a self contained manner so could be attempted by any sufficiently keen part III student but some basic knowledge of Relativity, Quantum Mechanics and Statistical Mechanics will likely be quite helpful.

Literature

1. Dodelson, *Modern Cosmology*
2. Kolb and Turner, *The Early Universe*
3. Weinberg, *Cosmology*

Additional support

Four examples sheets will be provided and four associated examples classes will be given.

Black Holes (L24)

Harvey Reall

A black hole is a region of space-time that is causally disconnected from the rest of the Universe. The study of black holes reveals many surprising and beautiful properties, and has profound consequences for quantum theory. The following topics will be discussed:

1. Upper mass limit for relativistic stars. Schwarzschild black hole. Gravitational collapse.
2. The initial value problem, strong cosmic censorship.
3. Causal structure, null geodesic congruences, Penrose singularity theorem.
4. Penrose diagrams, asymptotic flatness, weak cosmic censorship.
5. Reissner-Nordstrom and Kerr black holes.
6. Energy, angular momentum and charge in curved spacetime.
7. The laws of black hole mechanics. The analogy with laws of thermodynamics.
8. Quantum field theory in curved spacetime. The Hawking effect and its implications.

Pre-requisites

Familiarity with the Michaelmas term courses *General Relativity* and *Quantum Field Theory* is essential.

Literature

1. H. S. Reall, *Part 3 Black Holes*: lecture notes available at www.damtp.cam.ac.uk/user/hsr1000
2. R.M. Wald, *General relativity*, University of Chicago Press, 1984.
3. S.W. Hawking and G.F.R. Ellis, *The large scale structure of space-time*, Cambridge University Press, 1973.
4. V.P. Frolov and I.D. Novikov, *Black holes physics*, Kluwer, 1998.
5. N.D. Birrell and P.C.W. Davies, *Quantum fields in curved space*, Cambridge University Press, 1982.
6. R.M. Wald, *Quantum field theory in curved spacetime and black hole thermodynamics*, University of Chicago Press, 1994.

Additional support

Four examples sheets will be distributed during the course. Four examples classes will be held to discuss these. A revision class will be held in the Easter term.

Advanced Cosmology (L24)

Paul Shellard and Anthony Challinor

This course will take forward at much greater depth some of the topics in modern cosmology covered in the Michaelmas Term course. The prediction from fundamental theory for the statistical properties of the primordial perturbations remains the key area of confrontation with cosmological observations, both from large-scale structure and the cosmic microwave background (CMB). This course will develop the mathematical tools and physical understanding necessary for research in this very active area.

Cosmological perturbation theory

- The $3 + 1$ formalism and the Einstein equations
- Linearised Einstein equations for an expanding universe
- Review of density perturbation theory, transfer functions etc.
- Statistics of random fields

Cosmic microwave background

- Relativistic kinetic theory
- The Boltzmann equation
- The CMB temperature power spectrum
- Photon scattering and diffusion
- Primordial gravitational waves and the CMB
- CMB Polarization

Topical issues: Inflationary theory and non-Gaussianity

- "In-in" formalism and higher-order correlation functions
- Non-Gaussianities from alternative inflationary models
- Observational non-Gaussianity: CMB and large-scale structure

Pre-requisites

Material from the Michaelmas term *Cosmology* is essential. Familiarity with introductory Quantum Field Theory is recommended.

Literature

Textbooks

1. Dodelson, S., *Modern Cosmology*, Elsevier (2003).
2. Mukhanov, V., *Physical Foundation of Cosmology*, Cambridge (2005).
3. Weinberg, S., *Cosmology*, Oxford University Press (2008).
4. Misner, C.W., Thorne, K.S., and Wheeler, J.A., *Gravitation*, Freeman (1973).
5. Durrer, R., *The Cosmic Microwave Background*, Cambridge (2008).

Useful references

1. Bardeen, J.M., *Cosmological Perturbations From Quantum Fluctuations To Large Scale Structure*, DOE/ER/40423-01-C8 Lectures given at 2nd Guo Shou-jing Summer School on Particle Physics and Cosmology, Nanjing, China, Jul 1988. (Available on request.)
2. Mukhanov, V.F., Feldman, H.A., and Brandenberger, R.H., *Theory of cosmological perturbations*, Physics Reports, 215, 203 (1992).
3. Ma, C., and Bertschinger, E., *Cosmological Perturbation Theory in Synchronous and Conformal Newtonian Gauges*, Astrophysical Journal, 455, 7 (1995) [astro-ph/9506072].
4. Hu, W. and White, M., *CMB anisotropies: Total angular momentum method*, Physical Review D, 56, 596 (1997) [astro-ph/9702170].
5. Hu, W. and White, M., *A CMB polarization primer*, New Astronomy, 2, 323 (1997) [astro-ph/97006147].
6. Maldacena, J., *Non-gaussian features of primordial fluctuations in single field inflationary models*, Journal of High Energy Physics, 5, 13 (2003).
7. Chen, X., *Primordial Non-Gaussianities from Inflation Models* [arxiv:1002.1416].
8. Wang, Yi., *Inflation, Cosmic Perturbations and Non-Gaussianities* , arXiv:1303.1523 (Conference Lecture Notes).
9. Ligouri, M., Sefusatti, E., Fergusson, J.R., and Shellard, E.P.S., *Primordial Non-Gaussianity and Bispectrum Measurements in the Cosmic Microwave Background and Large-Scale Structure*, Advances in Astronomy, 2010, 73 (2010) [arxiv:1001.4707]

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Applications of Differential Geometry to Physics. (L16)

Maciej Dunajski

This is a course designed to develop the Differential Geometry required to follow modern developments in Theoretical Physics. The following topics will be discussed.

- Differential Forms and Vector Fields.
    1. One parameter groups of transformations.
    2. Vector fields and Lie brackets.

3. Exterior algebra.
4. Hodge Duality.

- Geometry of Lie Groups.

1. Group actions on manifolds.
2. Homogeneous spaces and Kaluza Klein theories.
3. Metrics on Lie Groups.

- Fibre bundles and instantons.

1. Principal bundles and vector bundles.
2. Connection and Curvature.
3. Twistor space.

Pre-requisites

Basic General Relativity (Part II level) or some introductory Differential Geometry course (e.g. Part II differential geometry) is essential. Part III General Relativity is desirable.

Literature

1. `http://www.damtp.cam.ac.uk/research/gr/members/gibbons/gwgPartIII_DGeometry2011-1.pdf`
2. Flanders, H. Differential Forms. Dover
3. Dubrovin, B., Novikov, S. and Fomenko, A. Modern Geometry. Springer
4. Eguchi, T., Gilkey, P. and Hanson. A. J. Physics Reports 66 (1980) 213-393
5. Arnold. V. Mathematical Methods of Classical Mechanics. Springer.
6. Dunajski. M. Solitons, Instantons and Twistors. OUP.

Additional support

Two examples sheets will be provided and two associated examples classes will be given.

Spinor Techniques in General Relativity (L24)

Non-Examinable (Graduate Level)

Irena Borzym (12 Lectures) and Peter O'Donnell (12 Lectures)

Spinor structures and techniques are an essential part of modern mathematical physics. This course provides a gentle introduction to spinor methods which are illustrated with reference to a simple 2-spinor formalism in four dimensions. Apart from their role in the description of fermions, spinors also often provide useful geometric insights and consequent algebraic simplifications of some calculations which are cumbersome in terms of spacetime tensors.

The first half of the course will include an introduction to spinors illustrated by 2-spinors. Topics covered will include the conformal group on Minkowski space and a discussion of conformal compactifications, geometry of scri, other simple simple geometric applications of spinor techniques, zero rest mass field equations, Petrov classification, the Plucker embedding and a comparison with Euclidean spacetime. More specific references will be provided during the course and there will be worked examples and handouts provided during the lectures.

The second half of the course will include: Newman-Penrose (NP) spin coefficient formalism, NP field equations, NP quantities under Lorentz transformations, Geroch-Held-Penrose (GHP) formalism, modified GHP formalism, Goldberg-Sachs theorem, Lanczos potential theory, Introduction to twistors. There will be no problem sets.

Pre-requisites

The Part 3 general relativity course is a prerequisite.

No prior knowledge of spinors will be assumed.

Literature

Introductory material.

1. L. P. Hughston and K. P. Tod, *Introduction to General Relativity.* Freeman, 1990.
2. C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation.* Freeman, 1973.

Best Course Reference Text for Lectures 1 to 12.

J.M. Stewart, *Advanced General Relativity.* CUP, 1993.

Best Course Reference Text for Lectures 13 to 24.

P O'Donnell, *Introduction to 2-spinors in general relativity.* World Scientific, 2003.

Reading to complement course material.

1. Penrose and Rindler, *Spinors and Spacetime Volume 1.* Cambridge Monographs on Mathematical Physics, 1987.
2. S. Ward and Raymond O. Wells, *Twistor Geometry and Field theory.* Cambridge Monographs on Mathematical Physics, 1991 .
3. Robert J. Baston, Michael G. Eastwood, *The Penrose Transform.* Clarendon Press, 1989.
4. S. A Huggett and P. Tod, *Introduction to Twistor Theory.* World Scientific, 2003.
5. R.M. Wald, *General Relativity.* World Chicago UP, 1984.
6. S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Spacetime.* CUP, 1973.

Overdetermined PDEs (E8)

Non-Examinable (Graduate Level)

Maciej Dunajski

The course will cover the Frobenius Theorem, involutivity, the Cartan Kahler theory, and a geometric take on the method of characteristics. There will be examples from projective and Riemannian geometry, as well as mathematical physics.

Pre-requisites

A basic course in either Differential Geometry, Geometry of Surfaces, or General Relativity.

Literature

1. Bryant R. L., Chern S. S., Gardner R. B., Goldschmidt H. L. Griffiths P. A., (1991) Exterior differential systems, Mathematical Sciences Research Institute Publications, **18**, Springer-Verlag, New York.

2. Ivey, T. A. Landsberg, J. M. (2003) Cartan for Beginners: Differential Geometry via Moving Frames and Exterior Differential Systems, AMS.

3. Dunajski, M. (2009) Solitons, Instantons Twistors. Oxford Graduate Texts in Mathematics, Oxford University Press.

4. Bryant, R. (2014) Notes on exterior differential system `arXiv:1405.3116`.

Astrophysics

Introduction to Astrophysics courses

These courses provide a broad introduction to research in theoretical astrophysics; they are taken by students of both Part III Mathematics and Part III Astrophysics. The courses are mostly self-contained, building on knowledge that is common to undergraduate programmes in theoretical physics and applied mathematics. For specific pre-requisites please see the individual course descriptions.

Astrophysical Fluid Dynamics (M24)

Gordon Ogilvie

Fluid dynamics is involved in a very wide range of astrophysical phenomena, such as the formation and internal dynamics of stars and giant planets, the workings of jets and accretion discs around stars and black holes, and the dynamics of the expanding Universe. Effects that can be important in astrophysical fluids include compressibility, self-gravitation and the dynamical influence of the magnetic field that is 'frozen in' to a highly conducting plasma.

The basic models introduced and applied in this course are Newtonian gas dynamics and magnetohydrodynamics (MHD) for an ideal compressible fluid. The mathematical structure of the governing equations and the associated conservation laws will be explored in some detail because of their importance for both analytical and numerical methods of solution, as well as for physical interpretation. Steady solutions with spherical or axial symmetry reveal the physics of winds and jets from stars and discs. The linearized equations determine the oscillation modes of astrophysical bodies, as well as determining their stability and their response to tidal forcing. The aim of the course is to provide familiarity with the basic phenomena and techniques that are of general relevance to astrophysics. Wherever possible the emphasis will be on simple examples, physical interpretation and application of the results in astrophysical contexts.

Provisional synopsis

- Overview of astrophysical fluid dynamics and its applications.
- Equations of ideal gas dynamics and MHD, including compressibility, thermodynamic relations and self-gravitation.
- Physical interpretation of ideal MHD, with examples of basic phenomena.
- Conservation laws, symmetries and hyperbolic structure. Stress tensor and virial theorem.
- Linear waves in homogeneous media. Nonlinear waves, shocks and other discontinuities.
- Spherically symmetric steady flows: stellar winds and accretion.
- Axisymmetric rotating magnetized flows: astrophysical jets.
- Stellar oscillations. Introduction to asteroseismology and astrophysical tides.
- Local dispersion relation. Internal waves and instabilities in stratified rotating astrophysical bodies.

Pre-requisites

This course is suitable for both astrophysicists and fluid dynamicists. An elementary knowledge of fluid dynamics, thermodynamics and electromagnetism will be assumed.

Literature

1. Choudhuri, A. R. (1998). *The Physics of Fluids and Plasmas.* Cambridge University Press.
2. Landau, L. D., & Lifshitz, E. M. (1987). *Fluid Mechanics*, 2nd ed. Butterworth–Heinemann.
3. Pringle, J. E., & King, A. R. (2007). *Astrophysical Flows.* Cambridge University Press. Available as an e-book from

   `http://ebooks.cambridge.org`

4. Shu, F. H. (1992). *The Physics of Astrophysics*, vol. 2: *Gas Dynamics.* University Science Books.
5. Thompson, M. J. (2006). *An Introduction to Astrophysical Fluid Dynamics.* Imperial College Press.

Additional support

Four example sheets will be provided and four associated classes will be given by the lecturer. It is anticipated that extended notes supporting the lecture course will be available in electronic form. There will be a revision class in Easter Term.

Extrasolar Planets: Atmospheres and Interiors (M24)

Nikku Madhusudhan

The field of extrasolar planets (or 'exoplanets') is one of the most dynamic frontiers of modern astronomy. Exoplanets are planets orbiting stars beyond the solar system. Thousands of exoplanets are now known with a wide range of sizes, temperatures, and orbital parameters, covering all the categories of planets in the solar system (gas giants, ice giants, and rocky planets) and more. The field is now moving into a new era of Exoplanet Characterization, which involves understanding the atmospheres, interiors, and formation mechanisms of exoplanets, and ultimately finding potential biosignatures in the atmospheres of rocky exoplanets. These efforts are aided by both high-precision spectroscopic observations as well as detailed theoretical models of exoplanets.

The present course will cover the theory and observations of exoplanetary atmospheres and interiors. Topics in theory will include (1) physicochemical processes in exoplanetary atmospheres (e.g. radiative transfer, energy transport, temperature profiles and stratospheres, equilibrium/non-equilibrium chemistry, atmospheric dynamics, clouds/hazes, etc) (2) models of exoplanetary atmospheres and observable spectra (1-D and 3-D self-consistent models, as well as parametric models and retrieval techniques) (3) exoplanetary interiors (equations of state, mass-radius relations, and internal structures of giant planets, super-Earths, and rocky exoplanets), and (4) relating atmospheres and interiors to planet formation. Topics in observations will cover observing techniques and state-of-the-art instruments used to observe exoplanetary atmospheres of all kinds. The latest observational constraints on all the above-mentioned theoretical aspects will be discussed. The course will also include a discussion on detecting biosignatures in rocky exoplanets, the relevant theoretical constructs and expected observational prospects with future facilities.

Pre-requisites

The course material should be accessible to students in physics or mathematics at the masters and doctoral level, and to astronomers and applied mathematicians in general. Knowledge of basic radiative transfer and chemistry is preferable but not necessary. The course is self contained and basic concepts will be introduced for completeness.

Literature

1. Chapters on exoplanetary atmospheres and interiors in the book Protostars and Planets VI, University of Arizona Press (2014), eds. H. Beuther, R. Klessen, C. Dullemond, Th. Henning. Most of these chapters are available publicly on the astro-ph arXiv.
2. Seager, S., Exoplanet Atmospheres: Physical Processes, Princeton Series in Astrophysics (2010).
3. Exoplanets, University of Arizona Press (2011), ed. S. Seager.
4. de Pater, I. and Lissauer J., Planetary Sciences, Cambridge University Press (2010).

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Planetary System Dynamics (M24)

Mark Wyatt

This course will cover the principles of celestial mechanics and their application to the Solar System and to extrasolar planetary systems. These principles have been developed over the centuries since the time of Newton, but this field continues to be invigorated by ongoing observational discoveries in the Solar System, such as the reservoir of comets in the Kuiper belt, and by the rapidly growing inventory of(well over 1000) extrasolar planets and debris discs that are providing new applications of these principles and the emergence of a new set of dynamical phenomena. The course will consider gravitational interactions between components of all sizes in planetary systems (i.e., planets, asteroids, comets and dust) as well as the effects of collisions and other perturbing forces. The resulting theory has numerous applications that will be elaborated in the course,including the growth of planets in the protoplanetary disc, the dynamical interaction between planets and how their orbits evolve, the sculpting of debris discs by interactions with planets and the destruction of those discs in collisions, and the evolution of circumplanetary ring and satellite systems.

Specific topics to be covered include:

1. Planetary system architecture: overview of Solar System and extrasolar systems, detectability, planet formation
2. Two-body problem: equation of motion, orbital elements, barycentric motion, Kepler's equation, perturbed orbits
3. Small body forces: stellar radiation, optical properties, radiation pressure, Poynting-Robertson drag, planetocentric orbits, stellar wind drag, Yarkovsky forces, gas drag, motion in protoplanetary disc, minimum mass solar nebula, settling, radial drift
4. Three-body problem: restricted equations of motion, Jacobi integral, Lagrange equilibrium points, stability, tadpole and horseshoe orbits
5. Close approaches: hyperbolic orbits, gravity assist, patched conics, escape velocity, gravitational focussing, dynamical friction, Tisserand parameter, cometary dynamics, Galactic tide
6. Collisions: accretion, coagulation equation, runaway and oligarchic growth, isolation mass, viscous stirring, collisional damping, fragmentation and collisional cascade, size distributions, collision rates, steady state, long term evolution, effect of radiation forces
7. Disturbing function: elliptic expansions, expansion using Legendre polynomials and Laplace coefficients, Lagrange's planetary equations, classification of arguments

8. Secular perturbations: Laplace coefficients, Laplace-Lagrange theory, test particles, secular resonances, Kozai cycles, hierarchical systems

9. Resonant perturbations: geometry of resonance, physics of resonance, pendulum model, libration width, resonant encounters and trapping, evolution in resonance, asymmetric libration, resonance overlap

Pre-requisites

This course is self-contained.

Literature

1. Murray C. D. and Dermott S. F.,*Solar System Dynamics.* Cambridge University Press,1999.
2. Armitage P. J., *Astrophysics of Planet Formation.* Cambridge University Press, 2010.
3. de Pater I. and Lissauer J. J., *Planetary Sciences.* Cambridge University Press, 2010.
4. Valtonen M. and Karttunen H., *The Three-Body Problem.* Cambridge University Press, 2006.
5. Seager S., *Exoplanets.* University of Arizona Press, 2011.
6. Perryman M., *The Exoplanet Handbook.* Cambridge University Press, 2011.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Structure and Evolution of Stars (M24)

A.N.Żytkow

Our attempts at gaining insight into the structure and evolution of stars rely on a mathematical description of the physical processes which determine the nature of stars. Such a mathematical description naturally follows the laws of conservation of mass, momentum and energy. The basic equations for spherical stars will be derived and boundary conditions described. These basic equations have to be supplemented by a number of appropriately chosen equations describing the methods of energy transport, the equation of state, the physics of opacity and nuclear reactions, all of which will be discussed. Some familiarity with the principles of hydrodynamics, thermodynamics, quantum mechanics, atomic and nuclear physics will be assumed.

Approximate solutions of the equations will be shown; polytropic gas spheres, homology principles, the virial theorem will be presented. The evolution of a star will be discussed, starting from the main-sequence, following the stages in which various nuclear fuels are exhausted and leading to the final outcome as white dwarfs, neutron stars or black holes.

The only way in which we may test stellar structure and evolution theory is through comparison of the theoretical results to observations. Throughout the course, reference will be made to the observational properties of the stars, with particular reference to the Hertzsprung-Russell diagram, the mass-luminosity law and spectroscopic information.

Pre-requisites

At least a basic understanding of hydrodynamics, electromagnetic theory, thermodynamics, quantum mechanics, atomic and nuclear physics although a detailed knowledge of all of these is not expected.

Preliminary Reading

1. Shu, F. *The Physical Universe*, W. H. Freeman University Science Books, 1991.
2. Phillips, A. *The Physics of Stars*, Wiley, 1999.

Literature

1. Kippenhahn, R. and Weigert, A. *Stellar Structure and Evolution, Second Edition*, Springer-Verlag, 2012.
2. Iben, I. *Stellar Evolution Physics, Vol. 1 and 2.* Cambridge University Press, 2013.
3. Prialnik, D. *An Introduction to the Theory of Stellar Structure and Stellar Evolution*, CUP, 2000.
4. Padmanabhan, T. *Theoretical Astrophysics, Volume II: Stars and Stellar Systems*, CUP, 2001.

Additional support

There will be four example sheets each of which will be discussed during an examples class. There will be a one-hour revision class in the Easter Term.

Magnetohydrodynamics (M16)

Prof. M.R.E. Proctor

Magnetohydrodynamics is the study of the interaction between magnetic fields and conducting fluids. Two main effects are of interest. Firstly moving conducting fluid can generate electric currents from magnetic fields through Faraday induction, and this leads to changes in the magnetic field. For sufficiently vigorous flows magnetic fields can be self-excited ('fluid dynamos'), and this process is responsible for the generation of magnetic fields in the Earth, Sun and other astrophysical bodies. Quite recently fluid dynamos have been demonstrated in the laboratory. While induction is a linear process, nonlinearity is induced since magnetic fields exert forces on the fluid, and these are proportional to the square of the field strength. This interaction leads to new types of wave motion ('Alfven waves') in conducting magnetised fluids, and has large scale effects of for example the statistics of fully developed turbulence and the morphology of sunspots. The course will treat both the basic theory and a number of applications as time permits. The theory will be developed in a classical rather than relativistic framework.

Pre-requisites

Knowledge of fluid dynamics and basic electrodynamics would be an advantage.

Literature

1. Moffatt, H.K. Generation of magnetic fields in conducting fluids. C.U.P. (out of print)
2. Priest, E.R. Solar magnetohyrodynamics. Kluwer.
3. Dormy and Soward, eds. Mathematical Aspects of Natural Dynamos. CRC Press
4. S.Chandrasekhar. Hydrodynamics and Hydromagnetic Stability. Dover.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

The Origin and Evolution of Galaxies (M16)

Martin Haehnelt

Galaxies are a fundamental building block of our Universe. The course will give an account of the physics of the formation of galaxies and their central supermassive black holes in the context of the standard paradigm for the formation of structure in the Universe.

Specific topics to be covered include the following:

- Observed properties of galaxies
- Cosmological framework and basic physical processes
- The interplay of galaxies and the intergalactic medium from which they form
- Numerical Methods for modeling galaxy formation
- Collapse of dark matter haloes and the inflow/outflow of baryons
- The hierarchical build-up of galaxies
- The origin and evolution of the central supermassive black holes in galaxies
- Towards understanding the origin of the Hubble sequence of galaxies

Pre-requisites

The course is aimed at astronomers/astrophysicists (including beginning graduate students). It should be also suitable for interested physicists and applied mathematicians. The course is self-contained, but students who have previously attended introductory courses in General Relativity and/or Cosmology will have an easier start.

Literature

1. Mo, H., van den Bosch, F., White, S., *Galaxy Formation and Evolution*, 2010, Cambridge University Press.
2. Sparke, L., Gallagher, J.S., *Galaxies in the Universe*, 2nd ed., 2007, Cambridge University Press.
3. Schneider, P., *Extragalactic Astronomy and Cosmology: An Introduction*, 2006, Springer.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a revision class in the Easter Term.

Galactic Astronomy and Dynamics (L24)

Wyn Evans

Astrophysics provides many examples of complex dynamical systems. This course covers the mathematical tools to describe Galaxies as well as reviewing their observational properties. The behaviour of these systems is controlled by Newton's laws of motion and Newton's law of gravity. Galaxies are dynamically very young, a typical star like the Sun having orbited only thirty or so times around the galaxy. The motions of stars in Galaxies are described using classical statistical mechanics, since the number of stars is so great. The study of large assemblies of stars interacting via long-range forces provides many unusual examples of cooperative phenomena, such as bars and spiral structure. The interplay between astrophysical dynamics and modern cosmology is also important – much of the evidence for dark matter is dynamical in origin.

1. Observational overview. Stellar populations in galaxies, galaxy morphology and classification. Dust and gas in galaxies. Scaling Laws.
2. Theory of the gravitational potential. Poisson's equation. Spherical, spheroidal and disk-like systems.
3. Regular and chaotic orbits, the epicyclic approximation, surfaces of section, integrals of motion, action-angle coordinates, adiabatic invariance.
4. Collisionless stellar dynamics, the Boltzmann equation, the Jeans Theorem, the Jeans equations, equilibrium models, astrophysical applications.
5. Collisional dynamics, the Fokker-Planck equation, dynamical friction.
6. Globular cluster evolution, evaporation and ejection, the gravothermal catastrophe, the effect of hard and soft binaries.
7. Galactic stability, the Jeans length, theories of spiral structure, the role of resonances.
8. The Milky Way Galaxy, the Local Group. Disk, bar, bulge and halo of the Milky Way

Pre-requisites

This course is suitable for applied mathematicians and astrophysicists. Although the course is self-contained, familiarity with Lagrangian & Hamiltonian mechanics and mathematical methods would be useful.

Preliminary Reading

1. Harwit M., 1982 *Cosmic Discovery: The Search, Scope and Heritage of Astronomy*, Basic Books
2. Elmegreen D.M., 1997 *Galaxies and Galactic Structure*, Prentice Hall
3. Sparke L., Gallagher J., 2007 *Galaxies in the Universe*, Cambridge University Press

Literature

1. Bertin G., 2000, *The Dynamics of Galaxies*, Cambridge University Press
2. Binney J., Tremaine S., 2007, *Galactic Dynamics*, Princeton University Press
3. Heggie D., Hut P. 2003, *The Million Body Problem*, Cambridge University Press
4. Murray C, Dermott S., 1999, *Solar System Dynamics*, Cambridge University Press

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will also be a two-hour revision class in the Easter Term.

Dynamics of Astrophysical Discs (L16)

Henrik Latter

Discs of matter in orbital motion around a massive central body occur in numerous situations in astrophysics. For example, Saturn's rings consist of trillions of metre-sized iceballs that undergo gentle collisions as they orbit the planet and behave collectively like a (non-Newtonian) fluid. Protostellar or protoplanetary discs are the dusty gaseous nebulae that surround young stars for their first few million years; they accommodate the angular momentum of the collapsing cloud from which the star forms, and are the sites of planet formation. Plasma accretion discs are found around black holes in interacting binary star systems and in the centres of active galaxies; they reveal the properties of the compact central objects and produce some of the most luminous sources in the Universe. These diverse systems have much in common dynamically.

The theoretical study of astrophysical discs combines aspects of orbital dynamics and continuum mechanics (fluid dynamics or magnetohydrodynamics). The evolution of an accretion disc is governed by the conservation of mass and angular momentum and is regulated by the efficiency of angular momentum transport. An astrophysical disc is a rotating shear flow whose local behaviour can be analysed in a convenient model known as the shearing sheet. Various instabilities can occur and give rise to sustained angular momentum transport. The resonant gravitational interaction of a planet or other satellite with the disc within which it orbits generates waves that carry angular momentum and energy. This process leads to orbital evolution of the satellite and is one of the factors shaping the observed distribution of extrasolar planets.

Provisional synopsis:

- Occurrence of discs in various astronomical systems, basic physical and observational properties.
- Orbital dynamics, characteristic frequencies, precession, elementary mechanics of accretion.
- Viscous evolution of an accretion disc.
- Vertical disc structure, thin-disc approximations, thermal instability in cataclysmic variables.
- The shearing sheet, symmetries, shearing waves.
- Incompressible dynamics: hydrodynamic stability, vortices and dust dynamics in protoplanetary disks.
- Compressible dynamics: density waves, gravitational instability and 'gravitoturbulence' in planetary rings and protoplanetary discs.
- Satellite-disc interaction, impulse approximation, gap opening by embedded planets.
- Magnetorotational instability, 'dead zones' in protoplanetary discs.

Pre-requisites

Newtonian mechanics and basic fluid dynamics. Some knowledge of magnetohydrodynamics is helpful for the magnetorotational instability.

Literature

1. Frank, J., King, A. & Raine, D. (2002), Accretion Power in Astrophysics, 3rd edn, CUP.
2. Pringle, J. E. (1981), Annu. Rev. Astron. Astrophys. 19, 137.

Additional support

Three examples sheets will be provided and three associated examples classes will be given.

Binary Stars (L16)

Christopher Tout

A binary star is a gravitationally bound system of two component stars. Such systems are common in our Galaxy and a substantial fraction interact in ways that can significantly alter the evolution of the individual stellar components. Many of the interaction processes lend themselves to useful mathematical modelling when coupled with an understanding of the evolution of single stars.

In this course we begin by exploring the observable properties of binary stars and recall the basic dynamical properties of orbits by way of introduction. This is followed by an analysis of tides, which represent the simplest way in which the two stars can interact. From there we consider the extreme case in which tides become strong enough that mass can flow from one star to the other. We investigate the stability of such mass transfer and its effects on the orbital elements and the evolution of the individual stars. As a prototypical example we examine Algol-like systems in some detail. Mass transfer leads to the concept of stellar rejuvenation and blue stragglers. As a second example we look at the Cataclysmic Variables in which the accreting component is a white dwarf. These introduce us to novae and dwarf novae as well as a need for angular momentum loss by gravitational radiation or magnetic braking. Their formation requires an understanding of significant orbital shrinkage in what is known as common envelope evolution. Finally we apply what we have learnt to a number of exotic binary stars, such as progenitors of type Ia supernovae, X-ray binaries and millisecond pulsars.

Pre-requisites

The Michaelmas term course on Structure and Evolution of Stars is very useful but not absolutely essential. Knowledge of elementary Dynamics and Fluids will be assumed.

Literature

1. Pringle J. E. and Wade R. A., *Interacting Binary Stars.* CUP.

Reading to complement course material

1. Eggleton P. P., *Evolutionary Processes in Binary and Multiple Stars.* CUP.

Additional support

Three examples sheets will be provided and three associated two-hour classes will be given. There will be a two-hour revision class in the Easter Term.

Quantum Computation, Information and Foundations

Quantum Information Theory (M24)

William Matthews

Quantum Information Theory (QIT) is an exciting, young field which lies at the intersection of Mathematics, Physics and Computer Science. It was born out of Classical Information Theory, which is the mathematical theory of acquisition, storage, transmission and processing of information. QIT is the study of how these tasks can be accomplished, using quantum-mechanical systems. The underlying quantum mechanics leads to some distinctively new features which have no classical analogues. These new features can be exploited, not only to improve the performance of certain information-processing tasks, but also to accomplish tasks which are impossible or intractable in the classical realm.

This is an introductory course on QIT, which should serve to pave the way for more advanced topics in this field.

The course will start by introducing a mathematical framework, based on the postulates of quantum mechanics and widely used in the study of quantum information theory, in which we can describe the time evolution of open systems (quantum operations) and very general forms of measurement (instruments and POVMs). Along the way we will prove results establishing the non-locality of quantum mechanics (Bell's theorem), the fact that quantum information cannot be perfectly copied (the "no-cloning" theorem), and fundamental limits on how well different states of a quantum system can be distinguished by measurements.

Building on this we will develop some of the major results of classical and quantum information theory, which concern data compression and the reliable transmission of information over noisy communication channels. Key mathematical ideas introduced in the process will be the classical and quantum notions of entropy and information, channel capacities, as well as random coding arguments. We will also look at the remarkable "dense coding" and "teleportation" protocols, which make use of the strange phenomenon of entanglement to accomplish tasks that would otherwise be impossible, and look at various ways of classifying and quantifying entangled states.

Pre-requisites

Knowledge of basic quantum mechanics will be assumed. However, an additional lecture can be arranged for students who do not have the necessary background in quantum mechanics. Elementary knowledge of Probability Theory, Vector Spaces and Linear Algebra will be useful.

Literature

The following books and lecture notes provide some interesting and relevant introductory reading material.

1. M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information; Cambridge University Press, 2000.
2. M. M. Wilde, From Classical to Quantum Shannon Theory, CUP; `http://arxiv.org/abs/1106.1445`.
3. J. Preskill, Chapter 5 of his lecture notes: Lecture notes on Quantum Information Theory `http://www.theory.caltech.edu/~preskill/ph229/#lecture`

Additional support

Course Instructor: Felix Leditzky

There will be four examples sheets (distributed in class) and four associated examples classes. The last examples class will be in Lent term. The course instructor will be Felix Leditzky.

Quantum Computation (M16)

Richard Jozsa

Quantum mechanical processes can be exploited to provide new modes of information processing that are beyond the capabilities of any classical computer. This leads to remarkable new kinds of algorithms (so-called quantum algorithms) that can offer a dramatically increased efficiency for the execution of some computational tasks. Notable examples include integer factorisation (and consequent efficient breaking of commonly used public key crypto systems) and database searching. In addition to such potential practical benefits, the study of quantum computation has great theoretical interest, combining concepts from computational complexity theory and quantum physics to provide striking fundamental insights into the nature of both disciplines.

The course will cover the following topics:

Notion of qubits, quantum logic gates, circuit model of quantum computation. Basic notions of quantum computational complexity, oracles, query complexity.

The quantum Fourier transform. Exposition of fundamental quantum algorithms including the Deutsch-Jozsa algorithm, Shors factoring algorithm, Grovers searching algorithm.

A selection from the following further topics (or others):
(i) Quantum teleportation and the measurement-based model of quantum computation;
(ii) Lower bounds on quantum query complexity;
(iii) Applications of phase estimation in quantum algorithms;
(iv) Quantum simulation for local hamiltonians.

Pre-requisites

It is desirable to have familiarity with the basic formalism of quantum mechanics especially in the simple context of finite dimensional state spaces (state vectors, composite systems, unitary matrices, Born rule for quantum measurements). Revision notes will be provided giving a summary of the necessary material including an exercise sheet covering notations and relevant calculational techniques of linear algebra. It would be desirable for you to look through this material at (or slightly before) the start of the course. Any encounter with basic ideas of classical theoretical computer science (complexity theory) would be helpful but is not essential.

Literature

1. Nielsen, M. and Chuang, I., *Quantum Computation and Quantum Information.* CUP, 2000.
2. Kaye, P., Laflamme, R. and Mosca, M. *An Introduction to Quantum Computing.* OUP, 2007.
3. John Preskill *Lecture Notes on Quantum Information Theory*, available at

   `http://www.theory.caltech.edu/people/preskill/ph219/`

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Philosophy of Physics

The courses in Philosophy of Physics are open to all students doing Part III, but are formally listed as graduate courses. This means there is no exam at the end of May for any such course; but a Part III student can get credit for them by doing their submitted Part III essay in association with one of the courses. More generally, the Philosophy of Physics courses are intended as a refreshing and reflective companion to the other Part III courses, especially the courses in theoretical physics.

Philosophical Aspects of Quantum Field Theory (M8)

Non-Examinable (Part III Level)

J. Butterfield and A. Caulton

Quantum field theory has for many decades been the framework for several basic and outstandingly successful physical theories. Nowadays, it is being addressed by philosophy of physics (which has traditionally concentrated on conceptual questions raised by non-relativistic quantum mechanics and relativity). This course will introduce this literature. More specifically, we will address the following topics: particle vs. field, including second vs. field quantization; localisation; and the algebraic approach to quantum field theory.

Pre-requisites

There are no formal prerequisites. Previous familiarity with the quantum field theory, such as provided by the Part III courses, will be helpful.

Preliminary Reading

This list of introductory reading is approximately in order of increasing difficulty.

1. S. Weinberg (1997), 'What is Quantum Field Theory, and What Did We Think It Is?'. Available online at: http://arxiv.org/abs/hep-th/9702027; and in Cao ed.

2. D. Wallace (2006), 'In defense of naiveté: The conceptual status of Lagrangian quantum field theory', *Synthese*, **151** (1):33-80, 2006. Available online at: http://arxiv.org/pdf/quant-ph/0112148v1

3. R. Clifton and H. Halvorson (2001), 'Are Rindler quanta real? Inequivalent particle concepts in quantum field theory', *British Journal for Philosophy of Science*, **52**, pp 417-470. Sections 1, 2.1, 2.2, 3.1, 3.2. Available online at: http://arxiv.org/abs/quant-ph/0008030

Literature

This list of readings to complement course material is approximately in order of increasing difficulty.

1. D. Wallace (2001), 'Emergence of particles from bosonic quantum field theory'. Available online at: http://arxiv.org/abs/quant-ph/0112149

2. T. Cao, (ed.) *The Conceptual Foundations of Quantum Field Theory.* Cambridge University Press, 1999.

3. L. Ruetsche, *Interpreting Quantum Theories.* Oxford University Press, 2011.

4. W. Greiner. *Relativistic Quantum Mechanics.* 2nd edition. Springer 1997.

5. W. Greiner and J. Reinhardt. *Field Quantization.* Springer 1996.

6. R. Haag. *Local Quantum Physics: fields, particles, algebras.* Springer 1992.

7. A. Duncan, *The Conceptual Framework of Quantum Field Theory.* Oxford University Press, 2012.

8. Boyd, J.P., *The Devil's invention: asymptotic, superasymptotic and hyperasymptotic series*, Acta Applicandae, **56**, 1-98 (1999). Also available at

   `http://hdl.handle.net/2027.42/41670`

Additional support

A Part III essay will be offered in conjunction with this course.

Space-time in Light of Particle Physics (L8)

Non-Examinable (Part III Level)

J. Brian Pitts

Pre-requisites

Imagine a world in which gravitation theory and particle physics have always mixed freely. These lectures will sketch such a world by taking a particle physics-flavoured look at gravitational theory, including General Relativity and some (perhaps) serious competition. Wigner's taxonomy in terms of mass and spin provides a starting place. How do we knonw that gravity is a tensor and not a scalar or vector? How (if at all) do we know that gravity is massless? What can we say about gravity and space-time if gravity is not presumed to be exceptional? Could one plausibly arrive at Einstein's equations? Do Einstein's principles, if not assumed, reappear as theorems? What does one make of conservation laws and Noether's theorems? How do spinors fit in, especially given nonlinear group realizations? We will also glance at Einstein's process of discovery, which historians have noticed involved a physical strategy involving an analogy to electromagnetism and attention to conservation laws, in addition to the now-famous principles. Such ideas were later applied independently by particle physics in derivations of Einstein's equations as describing a self-interacting massless spin-2 field. The mathematics used will be relativistic classical field theory. An Essay will be associated with this course.

Literature

1. Preskill-Thorne foreword to the *Feynman Lectures on Gravitation*, 1995.

2. D. Giulini, 'What Is (Not) Wrong with Scalar Gravity?', *Studies in History and Philosophy of Modern Physics*, 39:154-180, 2008, gr-qc/0611100v2.

3. P. G. O. Freund and Y. Nambu, 'Scalar fields coupled to the trace of the energy-momentum tensor', *Physical Review*, 174:1741–1743, 1968.

4. S. N. Gupta, 'Gravitation and Electromagnetism', *Physical Review*, 96:1683–1685, 1954.

5. V. I. Ogievetsky and I. V. Polubarinov, 'Interacting Field of Spin 2 and the Einstein Equations', *Annals of Physics*, 35:167–208, 1965.

6. S. Deser, 'Supergravity: A Unique Self-Interacting Theory', in P. L. García, A. Pérez-Rendón, and J. M. Souriau, eds., *Differential Geometrical Methods in Mathematical Physics*, volume 836 of *Lecture Notes in Mathematics*, pages 432–439, 1980.

7. Arkady I. Vainshtein, 'To the Problem of Nonvanishing Gravitation Mass', *Physics Letters B*, 39:393–394, 1972.

8. David G. Boulware and Stanley Deser, 'Can Gravitation Have a Finite Range?', *Physical Review D*, 6:3368–3382, 1972.

9. S. F. Hassan and R. A. Rosen, 'On Non-linear Actions for Massive Gravity', *Journal of High Energy Physics*, 1107(009), 2011, arXiv:1103.6055v3 [hep-th].

10. J. Renn and T. Sauer, 'Heuristics and Mathematical Representation in Einstein's Search for a Gravitational Field Equation', in H. Goenner, J. Renn, J. Ritter and T. Sauer, *The Expanding Worlds of General Relativity*, pp. 87-125, Einstein Studies, volume 7, Birkhäuser, Boston, 1999.

Applied and Computational Analysis

Applied and computational analysis (ACA) is concerned with mathematical tools of broad applicability, e.g. ordinary and partial differential equations, nonlinear dynamical systems, integrable systems, numerical analysis, approximation theory, inverse problems and image analysis. While the approach is mathematical, the ultimate destination of these tools is to applications. This tension between the pure and the applied is at the core of different ACA themes.

Set-valued Analysis and Optimisation (M16)

Tuomo Valkonen

Modern approaches to image processing, machine learning, and various big data applications, almost invariably involve the solution of non-smooth optimisation problems. Already at the start, in the characterisation of optimal solutions to these problems, and the development of numerical methods, we run into the most fundamental concept of set-valued analysis: the convex subdifferential. For the understanding of the stability and sensitivity of solutions under perturbations of data and model parameters, we need to delve further into the differentiation of general set-valued functionsa fascinating concept faced with many challenges. In this course, we will take a look at the central analytical results of this area, along with developing some practical numerical methods with an eye to image processing and data science.

The course will cover at least:

1. Minima of non-smooth functions—subdifferentials—convex analysis
2. Methods for convex minimisation—Moreau–Yosida regularisation
3. Sensitivity analysis—Lipschitz properties of set-valued mappings
4. Graphical derivatives and coderivatives—the Mordukhovich criterion

Pre-requisites

Knowledge of undergraduate calculus and linear algebra is required, as well as elementary (A-level) geometry. A basic course in optimisation theory is recommended, however not necessary.

Literature

1. J.-B. Hiriart-Urruty, C. Lemarchal, *Convex Analysis and Minimization Algorithms I–II*, Springer-Verlag, 1993.
2. R. T. Rockafellar, R. J.-B. Wets, *Variational Analysis*, Springer, 1998.

Additional support

Three examples sheets will be provided and three associated examples classes will be given.

Inverse Problems (M16)

Martin Benning

Solving an inverse problem is the task of computing an unknown physical quantity that is related to given, indirect measurements via a forward model. Inverse problems appear in a vast majority of applications, including imaging (Computed Tomography (CT), Positron Emission Tomography (PET), Magnetic Resonance Imaging (MRI), Electron Tomography (ET), microscopic imaging, geophysical imaging), signal-and image-processing, computer vision, machine learning and (big) data analysis in general, and many more.

Inverting a forward model however is not straightforward in most relevant applications, for two basic reasons: either a (unique) inverse model simply does not exist, or existing inverse models heavily amplify small measurement errors. In this course we are going to address the mathematical aspects of inverse problems, and discuss the concept of regularisation for finding stable approximations of the inverse of a specific forward model.

Pre-requisites

This course assumes basic knowledge in analysis and linear algebra, as well as their numerical counterparts. In addition to that, basic programming skills in MATLAB are required.

Additional knowledge in partial differential equations, functional analysis, variational calculus, image processing or (convex) optimisation is beneficial, but not mandatory.

Literature

1. Engl, Heinz Werner, Martin Hanke, and Andreas Neubauer. Regularization of inverse problems. Vol. 375. Springer Science & Business Media, 1996.

2. Natterer, Frank and Frank Wübbeling. Mathematical Methods in Image Reconstruction. Vol. 73, SIAM, 2001.

3. Martin Burger, Inverse Problems. Lecture notes winter 2007/2008.

   `http://wwwmath.uni-muenster.de/num/Vorlesungen/IP_WS07/skript.pdf`

Additional support

The amount of example sheets and associated example classes will be announced at the beginning of the lecture.

Distribution Theory and Applications (M16)

A.C.L. Ashton

This course will give an introduction to the theory of distributions and its application to the study of linear PDEs. We aim to make mathematical sense of objects like the Dirac delta function and find out how to meaningfully take the Fourier transform of a polynomial. The course will focus on the use of distributions, rather than the functional-analytic foundations of the theory.

First we will cover the basic definitions for distributions and related spaces of test functions. Then we will look at operations such as differentiation, translation, convolution and the Fourier transform. We will look at the Sobolev spaces $H^s(\mathbf{R}^n)$ and $H^s_{\text{loc}}(X)$ and their description in terms of the Fourier transform of tempered distributions. Time permitting, the material that follows will address questions such as

- What does a generic distribution look like?
- Why are solutions to Laplace's equation always infinitely differentiable?
- Which functions are the Fourier transform of a distribution?

i.e. structure theorems, elliptic regularity, Paley-Wiener-Schwartz. In the final part of the course we will study Hörmander's oscillatory integrals.

Pre-requisites

Elementary concepts from undergraduate real analysis. Some knowledge of complex analysis would be advantageous (e.g. the level of IB Complex Analysis/Methods). No knowledge of functional analysis is assumed.

Preliminary Reading

1. Friedlander & Joshi, *Introduction to the Theory of Distributions.* Cambridge University Press, 1998.
2. Lighthill, *Introduction to Fourier Analysis and Generalised Functions.* Cambridge University Press, 1958.
3. Folland, *Introduction to Partial Differential Equations.* Princeton University Press, 1995.

Literature

1. Hörmander, *The Analysis of Partial Differential Operators: Vol I.* Springer Verlag, 1985.
2. Reed & Simon, *Methods of Modern Mathematical Physics: Vol I-II.* Academic Press, 1979.
3. Tréves, *Linear Partial Differential Equations with Constant Coefficients.* Routledge, 1966.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. Model solutions will be made available.

Boundary Value Problems for Linear PDEs (L16)

Athanassios S. Fokas

Recent developments in the area of the so-called *integrable nonlinear* Partial Differential Equations (PDEs) have led to the emergence of a new method for solving boundary value problems, which is usually referred to as the *Unified Transform* (UT).

The UT will be implemented to:

(a) Linear evolution PDEs in one spatial variable formulated either on the half-line or on a finite interval. Examples include the heat equation and the Stokes equation (linearised version of the KdV).

(b) Linear elliptic PDEs in two spatial variables formulated in the interior of a convex polygon. Examples include the Laplace, the modified Helmholtz, and the Helmholtz equations.

For the above problems, in addition to presenting integral representations of the solution, simple numerical techniques for the effective computation of the solution will also be introduced.

Pre-requisites

The course only requires some elementary knowledge of complex analysis.

Literature

1. A.S. Fokas, *A unified method for boundary value problems.* 1st edition. SIAM, 2008.

Additional support

Three examples sheets will be provided and three associated examples classes will be given.

Compressed Sensing and Sampling Theory (L16)

Non-Examinable (Graduate Level)

Anders Hansen

This is a graduate course on sampling theory and compressed sensing for use in signal processing and medical imaging. Compressed sensing is a theory of randomisation, sparsity and non-linear optimisation techniques that breaks traditional barriers in sampling theory. Since its introduction in 2004 the field has exploded and is rapidly growing and changing. Thus, we will take the word contemporary quite literally and emphasise the latest developments, however, no previous knowledge of the field is assumed. Although the main focus will be on compressed sensing, it will be presented in the general framework of sampling theory. The course will also present related areas of sampling theory such as generalised sampling.

The course will be fairly self contained, and applications will be emphasised (in particular, signal processing, Magnetic Resonance Imaging (MRI) and X-ray Tomography). The lectures will cover the most up to date research, and although this is a Part III course, it is also aimed at Phd students and post docs who are interested in using compressed sensing and generalised sampling in their research. Students from other disciplines than mathematics are encouraged to participate.

Pre-requisites

Sampling theory and compressed sensing require a mix of mathematical tools from approximation theory, harmonic analysis, linear algebra, functional analysis, optimisation and probability theory. The course will contain discussions of both finite-dimensional and infinite-dimensional/analog signal models and thus linear algebra, Fourier analysis and functional analysis (at least basic Hilbert space theory) are important. The course will be self-contained, but students are encouraged to refresh their memories on properties of the Fourier transform as well as basic Hilbert space theory. Some basic knowledge of wavelets is useful as well as basic probability.

Preliminary Reading

For a quick and dense review of basic Fourier analysis and functional analysis chapters 5 and 8 of "Real Analysis" (Folland) are good choices. For an introductory exposition to Hilbert space theory one may use "An Introduction to Hilbert Space" (Young). And for a review of wavelets see chapters 1 and 2 of "A First Course on Wavelets" (Hernandez, Weiss). The course will cover some of the chapters of "Compressed Sensing" (Eldar, Kutyniok), so to get a feeling about the topic one may consult chapter 1 as a start.

1. Eldar, Y and Kutyniok, G., Compressed Sensing, CUP
2. Folland, G. B., Real Analysis, Wiley.
3. Hernandez, E. and Weiss, G., A First Course on Wavelets, CRC
4. Young, N., An Introduction to Hilbert Space, CUP

Literature

The following reading list complements the course material.

1. Adcock, B and Hansen, A., Stable reconstructions in Hilbert spaces and the resolution of the Gibbs phenomenon, Appl. Comp. Harm. Anal., 32 (2012)

2. Candès, E. and Romberg, J. and Tao, T., Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information, IEEE Trans. Inform. Theory 52 (2006)
3. Donoho, D., Compressed sensing, IEEE Trans. Inform. Theory 52 (2006)
4. Körner, T. W., Fourier Analysis, CUP
5. Reed, M. and Simon, B., Functional Analysis, Elsevier

Additional support

As this is a non-examinable course there will be no examples classes, however, there will be several computer tutorials where practical implementations and real world examples will be discussed. There will also occasionally be lectures given by people from other groups outside of mathematics using compressed sensing in practice.

Homogenization of PDEs (E16)

Non-Examinable (Graduate Level)

Harsha Hutridurga

This course aims to introduce the theory of Homogenization. Partial Differential Equations with highly oscillating coefficients arise in the study of many physical phenomena (composite materials, porous media flows, rarefied gas dynamics, turbulence etc.). Homogenization, loosely speaking, replaces the PDE with highly oscillating coefficients by an equivalent PDE which "on average" behaves like the original heterogeneous PDE. All along this course, emphasis shall be given on the study of PDEs with periodically oscillating coefficients. We shall be studying the homogenization of the following PDEs:

1. Diffusion Equation: to study the conductivity of mixtures.
2. Stokes' Equation: to derive the celebrated 'Darcy's Law' in porous media.
3. Convection-Diffusion Equation: to derive the expression for 'Taylor Dispersion'.
4. Linear Boltzmann Equation: to study the interaction of the monokinetic particles with the background medium.
5. Euler Equations (incompressible): to derive the k-ε model for turbulence.

A formal method of 'Asymptotic Expansions' will be introduced in the beginning of the course followed by the more rigorous methods like the 'Energy Method' and the notion of 'Two scale Convergence'. This course shall also address the handicap of the standard Finite Element Method (FEM) to arrive at a numerical solution to PDEs with highly oscillating coefficients. As an application of the theory of Homogenization, this handicap of FEM shall be overcome by the introduction of Multiscale Finite Element Method (MFEM).

Pre-requisites

1. Some basic notions of PDEs (C. Mouhot's 'Analysis of PDE' might be useful).
2. Some compactness results from Functional Analysis (shall be recalled during the course).
3. Some basic notions of FEM (A. Iserles's 'Numerical Solution of DEs' might be useful).

Literature

1. A. Bensoussan, J.L. Lions, G.C. Papanicolaou, *Asymptotic analysis for periodic structures*, North-Holland, Amsterdam, 1978.

2. D. Cioranescu, P. Donato, *An introduction to homogenization*, Oxford lecture series in mathematics and its applications 17, Oxford University Press, New York, 1999.

3. G. Allaire, *Homogenization and two-scale convergence*, SIAM J. Math. Anal., Vol 23, No.6, pp.1482-1518, (1992).

4. T.Y. Hou, X-H. Wu, Z. Cai, *Convergence of a multiscale finite element method for elliptic problems with rapidly oscillating coefficients*, Math. Comp., Vol 68, No.227, pp.913-943, (1999).

Continuum Mechanics

The four courses in the Michaelmas Term are intended to provide a broad educational background for any student preparing to start a PhD in fluid dynamics. The courses in the Lent Term are more specialized and in some cases (see the course descriptions) build on the Michaelmas Term material.

Desirable previous knowledge

For all the fluid dynamics courses, previous attendance at an introductory course in fluid dynamics will be assumed. In practice, familiarity with the continuum assumption, the material derivative, the stress tensor and the Navier-Stokes equation will be assumed, as will basic ideas concerning incompressible, inviscid fluid mechanics (e.g. Bernoulli's Theorem, vorticity, potential flow). Some knowledge of basic viscous flow, such as Stokes flow, lubrication theory and elementary boundary-layer theory, is highly desirable. Previous attendance at a course on wave theory covering concepts such as wave energy and group velocity, is desirable for some courses. No previous knowledge of solid mechanics, Earth Sciences, or biology is required.

In summary, knowledge of Chapters 1-8 of 'Elementary Fluid Dynamics' (D.J. Acheson, Oxford), plus Chapter 3 of 'Waves in Fluids' (J. Lighthill, Cambridge)(which deals with dispersive waves) would give a student an excellent grounding.

Familiarity with basic vector calculus (including Cartesian tensors), differential equations, complex variable techniques (e.g. Fourier Transforms) and techniques for solution of elementary PDEs, such as Laplace's equation, Poisson's equation, the diffusion equation and the simple wave equation, will be assumed. Knowledge of elementary asymptotic techniques would be helpful.

A Cambridge student taking continuum courses in Part III would be expected to have attended the following undergraduate courses

Year	*Courses*
First	Differential Equations, Dynamics and Relativity, Vector Calculus, Vectors & Matrices.
Second	Methods, Complex Methods, Fluid Dynamics.
Third	Fluid Dynamics, Waves, Asymptotic Methods.

Students starting Part III from outside Cambridge might like to peruse the syllabuses for the above courses on WWW with URL:

`http://www.maths.cam.ac.uk/undergrad/schedules/`

Slow Viscous Flow (M24)

J.R. Lister

In many flows of natural interest or technological importance, the inertia of the fluid is negligible. This may be due to the small scale of the motion, as in the swimming of micro-organisms and the settling of fine sediments, or due to the high viscosity of the fluid, as in the processing of glass and the convection of the Earth's mantle.

The course will begin by presenting the fundamental principles governing flows of negligible inertia. A number of elegant results and representations of general solutions will be derived for such flows. The motion of rigid particles in a viscous fluid will then be discussed. Many important phenomena arise from the deformation of free boundaries between immiscible liquids under applied or surface-tension forcing. The flows generated by variations in surface tension due to a temperature gradient or contamination by surfactants will be analysed in the context of the translation and deformation of drops and bubbles and in the context of thin films. The small cross-stream lengthscale of thin films renders their inertia negligible and allows them to be analysed by lubrication or extensional-flow approximations. Problems such as the fall of a thread of honey from a spoon and the subsequent spread of the pool of honey will be analysed in this way. Inertia is also negligible in flows through porous media such as the extraction of oil from sandstone reservoirs, movement of groundwater through soil or the migration of melt through a partially molten mush. Some basic flows in porous media may be discussed.

The course aims to examine a broad range of slow viscous flows and the mathematical methods used to analyse them. The course is thus generally suitable for students of fluid mechanics, and provides background for applied research in geological, biological or rheological fluid mechanics.

Pre-requisites

As described above in the introduction to courses in Continuum Mechanics. Familiarity with basic vector calculus including Cartesian tensors and the summation convention is particularly useful for the first half of the course.

Preliminary Reading

1. D.J. Acheson. *Elementary Fluid Dynamics.* OUP (1990). Chapter 7
2. G.K. Batchelor. *An Introduction to Fluid Dynamics.* CUP (1970). pp.216–255.
3. L.G. Leal. *Laminar flow and convective transport processes.* Butterworth (1992). Chapters 4 & 5.

Literature

1. J. Happel & H. Brenner. *Low Reynolds Number Hydrodynamics.* Kluwer (1965).
2. S. Kim & J. Karrila. *Microhydrodynamics: Principles and Selected Applications.* (1993)
3. C. Pozrikidis. *Boundary Integral and Singularity Methods for Linearized Viscous Flow.* CUP (1992).
4. O.M. Phillips. *Flow and Reactions in Permeable Rocks.* CUP (1991).

Additional support

Four two-hour examples classes will be given by the lecturer to cover the four examples sheets. There will be a further revision class in the Easter Term.

Fluid Dynamics of the Environment (M24)

S.B. Dalziel, A.W. Woods and N.M. Vriend

Understanding and predicting the impact of human activity on the environment is a critical challenge in our time. The fluid dynamics of oceans and atmospheres plays a vital role in regulating many aspects of our Earth and our direct environment. This course introduces the basic fluid dynamics necessary to build mathematical models of the environment in which we live, and focuses on problems which occur over sufficiently small time and length scales to be largely unaffected by the earth's rotation.

The course begins by considering the governing equations of fluid flow in the presence of (typically small) density variations. Internal gravity waves can occur in the case of density variations in a fluid, since these variations provide a restoring force. The course highlights some of the rich and surprising dynamics of these waves. In particular, internal gravity waves radiate energy vertically as well as horizontally, and their interaction with boundaries can focus this energy and cause mixing far from where the energy was input.

Density variations within fluids can also drive the flow and the course will consider two important and related classes where the flow is either tall and thin or long and shallow. Both classes allow substantial simplification of the governing equations by integrating them over the smaller dimension. First, a relatively localised source can drive the rise of a turbulent 'plume' of buoyant fluid. Volcanic eruption clouds, accidental releases of pollution and the natural ventilation of buildings are just three examples of such flows. Second, when there are lateral gradients in fluid density interacting with horizontal or sloping boundaries, turbulent 'density or gravity currents can develop. Similarly, for a stratified ambient fluid, an 'intrusion can develop

The buoyancy driving these flows may be due to differences in temperature or composition (e.g. salt or water vapour concentration), or due to the presence of a second phase such as particles or bubbles. Examples of particle-laden flows include snow avalanches, turbidity currents and pyroclastic flows. Particle-fluid and particle-particle interactions introduce a new range of interesting features. Particle suspension and deposition are important in a broad range of phenomena such as dune building and sand transport.

Pre-requisites

Undergraduate fluid dynamics is desirable.

Literature

1. B. R. Sutherland, Internal gravity waves, Cambridge University Press, 2010.
2. J. S. Turner, Buoyancy Effects in Fluids, Cambridge University Press, 1979.
3. J. Pedlosky, Geophysical Fluid Dynamics, Springer, 1987.

Additional support

In addition to the lectures, four examples sheets will be provided and four associated examples classes will run in parallel to the course. There will be a revision class in the Easter Term.

Hydrodynamic Stability (M24)

Colm-cille Caulfield

Developing an understanding by which "small" perturbations grow, saturate and modify fluid flows is central to addressing many challenges of interest in fluid mechanics. Furthermore, many applied mathematical tools of much broader relevance have been developed to solve hydrodynamic stability problems, and hydrodynamic stability theory remains an exceptionally active area of research, with several exciting new developments being reported over the last few years.

In this course, an overview of some of these recent developments will be presented. After a brief introduction to the general concepts of flow instability, presenting a range of examples, the major content of this course will be focussed on the broad class of flow instabilities where velocity "shear" and fluid inertia play key dynamical roles. Such flows, typically characterised by sufficiently"high" Reynolds number Ud/ν, where U and d are characteristic velocity and length scales of the flow, and ν is the kinematic viscosity of the fluid, are central to modelling flows in the environment and industry. They typically demonstrate the key role played by the redistribution of vorticity within the flow, and such vortical flow instabilities often trigger the complex, yet hugely important process of "transition to turbulence".

A hierarchy of mathematical approaches will be discussed to address a range of "stability" problems, from more traditional concepts of "linear" infinitesimal normal mode perturbation energy growth on laminar parallel shear flows to transient, inherently nonlinear perturbation growth of general measures of perturbation magnitude over finite time horizons where flow geometry and/or fluid properties play a dominant role. The course will also discuss in detail physical interpretations of the various flow instabilities considered, as well as the industrial and environmental application of the results of the presented mathematical analyses.

Pre-requisites

Undergraduate fluid mechanics, linear algebra, complex analysis and asymptotic methods.

Literature

1. F. Charru *Hydrodynamic Instabilities* CUP 2011.
2. P. G. Drazin & W. H. Reid *Hydrodynamic Stability* 2nd edition. CUP 2004.
3. P. J. Schmid & D. S. Henningson, *Stability and transition in shear flows.* Springer, 2001.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a two-hour revision class in the Easter Term.

Fluid Dynamics of the Solid Earth (M16)

Jerome A. Neufeld

The dynamic evolution of the solid Earth is governed by a rich variety of physical processes occurring on a wide range of length and time scales. The Earth's core is formed by the solidification of a mixture of molten iron and various lighter elements, a process which drives predominantly compositional convection in the liquid outer core, thus producing the geodynamo responsible for the Earth's magnetic field. At very much longer time scales, radiogenic heating of the solid mantle drives solid-state convection resulting in plume-like features possibly responsible for features such as the Hawaiian sea mounts. Nearer the surface, convection drives the motion of brittle plates which are responsible for the Earth's topography as can be felt and imaged through the seismic record. Upwelling mantle material also drives partial melting of mantle rocks resulting in compaction, and ultimately in the propagation of viscous melt through the elastic crust. On the Earth's surface, and at very much faster rates, the same physical processes of viscous and elastic deformation coupled to phase changes govern the evolution of the Earth's cryosphere, from the solidification of sea ice to the flow of glacial ice.

This course will use the wealth of observations of the solid Earth to motivate mathematical models of physical processes that play key roles in many other environmental and industrial processes. Mathematical topics will include the onset and scaling of convection, the coupling of fluid motions with changes of phase at a boundary, the thermodynamic and mechanical evolution of multicomponent or multiphase systems, the coupling of fluid flow and elastic flexure or deformation, and the flow of fluids through porous materials.

Pre-requisites

A basic understanding of viscous fluid dynamics. Mathematical methods, particularly the solution of ordinary and partial differential equations.

Literature

1. M.G. Worster. *Solidification of Fluids.* In Perspectives in Fluid Dynamics: a Collective Introduction to Current Research. Edited by G.K. Batchelor, H.K. Moffatt and M.G. Worster. pp. 393–446. CUP (2000)
2. H.E. Huppert. *Geological fluid mechanics.* In Perspectives in Fluid Dynamics: a Collective Introduction to Current Research. Edited by G.K. Batchelor, H.K. Moffatt and M.G. Worster. pp. 393–446. CUP (2000)
3. D.L. Turcotte, G. Schubert. Geodynamics, second edition. CUP (2002)

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.

Fluid Dynamics of Climate (L24)

P.F. Linden J.R. Taylor

Understanding and predicting the Earth's climate is one of the great scientific challenges of our times. Fluid motion in the ocean and atmosphere plays a vital role in regulating the Earth's climate, helping to make the planet hospitable for life. However, the dynamical complexity of this motion and the wide range of space and time scales involved, makes predicting the climate system a very difficult endeavour.

This course provides an introduction to the basic fluid dynamics necessary to build mathematical models of the environment in which we live, focusing on the behaviour large-scale of stratified and rotating flows. The course begins by considering deals with flows where the timescale for the motion is not short compared with a day and the Earth's rotation plays an important role. The additional timescale introduced by the Earth's rotation modifies the dynamics in a profound way for both homogeneous and density stratified flows. The Coriolis force (a fictitious force arising from our use of a frame of reference rotating with the planet) causes a moving parcel of fluid to experience a force directed to its right in the Northern hemisphere (or its left in the Southern hemisphere), introducing a rich wealth of new dynamics. We then examine large-scale dynamics of the atmosphere and the oceans including the phenomena commonly referred to as Rossby waves, eddies, baroclinic instability, ocean gyres and the thermohaline circulation. These processes play a central role in the lateral transport of heat and other tracers.

Pre-requisites

Undergraduate fluid dynamics

Literature

1. A.E. Gill, Atmosphere-Ocean Dynamics. Academic Press (1982).
2. Marshall, J. and R.A. Plumb. Atmosphere, Ocean, and Climate Dynamics. Academic Press. 2008.
3. Pedlosky, J. Geophysical Fluid Dynamics. Springer. (1987).
4. J.S. Turner, Buoyancy Effects in Fluids, Cambridge University Press (1979).

Active Biological Fluids (L24)

Dr Eric Lauga

Fluid mechanics plays a crucial role in a number of biological processes, from the largest of animals to the smallest of cells. In this course, we will give an overview of the hydrodynamic phenomena associated with biological life at the cellular scale, from the fluid mechanics of individual microorganisms and their appendages to the modelling of collective, complex, cell dynamics. We will combine physical description, scaling analysis, and detailed calculations in order to present a wide overview of the subject, and appeal to students in applied mathematics, physics, and theoretical biology. The course will introduce the fluid dynamics and soft matter mechanics relevant to the locomotion of individual cells. Drawing examples from a variety of organisms, we will aim at providing a precise mathematical description of how cells actuate and exploit surrounding fluids in order to self-propel, how they interact with their chemical and mechanical environment, and how populations of cells dynamically influence each other. At the end of the course, students will be equipped to carry out independent research in biological physics and fluid dynamics relevant to the cellular world.

Pre-requisites

Undergraduate fluid dynamics, vector calculus and mathematical methods. Attendance to Part III "Slow Viscous Flows" is required.

Literature

1. Lighthill (1975) Mathematical Biofluiddynamics, SIAM.
2. Purcell (1977) Life at low Reynolds number. *American Journal of Physics*, **45**, 3-11.
3. Childress (1981) Mechanics of Flying and Swimming, Cambridge University Press.
4. Yates (1986) How microorganisms move through water. *American Scientist* **74**, 358-365.
5. Vogel (1996) Life in Moving Fluids, Princeton University Press.
6. Berg (2000) Motile Behavior of Bacteria. *Physics Today*, **53**, 24.
7. Bray (2000) Cell Movements, Garland.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a two-hour revision class in the Easter Term.

Direct and Inverse Scattering of Waves (L16)

Orsola Rath Spivack

The study of wave scattering is concerned with how the propagation of waves is affected by objects, and has a variety of applications in many fields, from environmental science to seismology, medicine, telecommunications, materials science, military applications, and many others. If we know the nature of the objects and we want to find how an incident wave is scattered, we call this a 'direct scattering problem' and practical applications will include for example underwater sound propagation, light transmission through the atmosphere, or the effect of noise in built-up areas. If we measure and know the scattered field produced by an incident wave, but we do not know the nature of the objects that have scattered it, we call this an 'inverse scattering problem' and applications will include for example non-destructive testing of materials, remote sensing with radar or lidar, or medical imaging.

This course will provide the basic theory of wave propagation and scattering and an overview of the main mathematical methods and approximations, with particular emphasis on inhomogeneous and random media, and on the regularisation of inverse scattering problems. Only time-harmonic waves will be normally considered.

Topics covered will include:

1. Boundary value problems and the integral form of the wave equation.
2. The parabolic equation and Born and Rytov approximations for the scattering problem.
3. Scattering by randomly rough surfaces and propagation in inhomogeneous media.
4. Ill-posedness of the inverse scattering problem, and the Moore-Penrose generalised inverse.
5. Regularisation methods and methods for solving some inverse scattering problems.
6. Time reversal and focusing in inhomogeneous media.

This course sits as a bridge between more applied Continuum Mechanics courses on waves and fluid dynamics, and more pure Applied and Computational Analysis course. Students considering this course might also like to consider courses on inverse problems and imaging.

Pre-requisites

This course assumes basic knowledge of ODEs and PDEs, and of Fourier transforms. Some familiarity with linear algebra and with basic concepts in functional analysis is helpful, though by no means necessary.

Preliminary Reading

1. C.W. Groetsch, *Inverse Problems in the Mathematical Sciences.* Braunschweig 1993
2. L.D. Landau and E.M. Lifschitz, *Fluid Dynamics.* Pergamon [Chapter 8]

Literature

1. D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory.* Springer, 1992.
2. D.G. Crighton et al, *Modern Methods in Analytical Acoustics.* Springer, 1992.
3. H.W. Engl, M. Hanke and A. Neubauer, *Regularization of inverse problems.* Kluwer, 2000.
4. A. Ishimaru, *Wave Propagation and Scattering in Random Media.* Academic Press, 1978.
5. A. Kirsch, *An introduction to the mathematical theory of inverse problems.* Springer, 1996.
6. B. Uscinski, *The elements of wave propagation in random media.* McGraw-Hill, 1977.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a two-hour revision class in the Easter Term. The course will be supported by a Moodle site.

Perturbation Methods (L16)

S.S. Pegler & S.J. Cowley

This course will deal with the asymptotic solution to problems in applied mathematics in general when some parameter or coordinate in the problem assumes large or small values. Many problems of physical interest are covered by such asymptotic limits. The methods developed have significance, not only in revealing the underlying structure of the solution, but in many cases providing accurate predictions when the parameter or coordinate has only moderately large or small values.

A number of some of the most useful mathematical tools for finding approximate solutions to equations will be covered, and a range of physical applications will be provided. Specifically, the course will start with a brief review of classical asymptotic methods for the evaluation of integrals, but most of the lectures will be devoted to singular perturbation problems (including the methods of multiple scales and matched asymptotic expansions, and so-called 'exponential asymptotics'), for which straightforward asymptotic methods fail in one of a number of characteristic ways.

More details of the material are as follows, with approximate numbers of lectures in brackets:

- *Methods for Approximating Integrals.* This section will start with a brief review of asymptotic series. This will be followed by various methods for approximating integrals including the 'divide & conquer' strategy, Laplace's method, stationary phase and steepest descents. This will be followed by a discussion of Stokes lines and an introduction to 'asymptotics beyond all orders' in which exponentially small corrections are extracted from the tails of asymptotic series. [6]

- *Multiple Scales.* This method is generally used to study problems in which small effects accumulate over large times or distances to produce significant changes (the 'WKBJLG' method can be viewed as a special case). It is a systematic method, capable of extension in many ways, and includes such ideas as those of 'averaging' and 'time scale distortion' in a natural way. A number of applications will be studied including ray tracing and turning points (e.g. sound or light propagation in an inhomogeneous medium). [4]
- *Matched Asymptotic Expansions.* This method is applicable, broadly speaking, to problems in which regions of rapid variation occur, and where there is a drastic change in the structure of the problem when the limiting operation is performed. Boundary-layer theory in fluid mechanics was the subject in which the method was first developed, but it has since been greatly extended and applied to many fields. Further examples will be given of asymptotics beyond all orders. This section will include a brief introduction to the summation of [divergent] series, e.g. covering Cesàro, Euler and Borel sums, Padé approximants, continued fractions, Shanks' transformations, Richardson extrapolation, and Domb-Sykes plots. [6]

Pre-requisites

Although many of the techniques and ideas originate from fluid mechanics and classical wave theory, no specific knowledge of these fields will be assumed. The only pre-requisites are familiarity with techniques from the theory of complex variables, such as residue calculus and Fourier transforms, and an ability to solve straightforward differential equations and partial differential equations and evaluate simple integrals.

Literature

Introductory Reading

1. Bender, C.M. & Orszag, S., *Advanced Mathematical Methods for Scientists and Engineers*, McGraw-Hill (1978). *This is probably the most comprehensive textbook, but that means that some selective reading is advisable. Note that Bender & Orszag refer to Stokes lines as anti-Stokes lines, and vice versa.*
2. Hinch, E.J., *Perturbation Methods*, Cambridge University Press (1991). *This is the book of the course; some view it as somewhat terse.*
3. Van Dyke, M.D., *Perturbation Methods in Fluid Mechanics*, Parabolic Press, Stanford (1975). *This is the original book on perturbation methods; somewhat dated, but still a useful read.*

Reading to Complement Course Material

1. Berry, M.V., *Waves near Stokes lines*, Proc. R. Soc. Lond. A, **427**, 265–280 (1990).
2. Boyd, J.P., *The Devil's invention: asymptotic, superasymptotic and hyperasymptotic series*, Acta Applicandae, **56**, 1-98 (1999). Also available at

   `http://hdl.handle.net/2027.42/41670` and
   `http://link.springer.com/content/pdf/10.1023/A:1006145903624.pdf`

3. Kevorkian, J. & Cole, J.D., *Perturbation Methods in Applied Mathematics*, Springer (1981).

Additional support

In addition to the lectures, three examples sheets will be provided and three associated 2-hour examples classes will run in parallel to the course. There will be a 2-hour revision class in the Easter Term.

Biological Physics (M24)

Non-Examinable (Part III Level)

Dr Pietro Cicuta, Dr Eileen Nugent

This course explores the physical principles of life at the cellular and molecular level. It examines how these principles shape the behaviour of cells enabling them to sense and react to their environment as they grow and divide. The course aims to demonstrate how apparently diverse aspects of living systems are actually underpinned by the physics of complex systems. Modelling based on physical principles complements the experimental investigations of biologists and can help reveal the essential principles of life.

The course begins with an overview of quantitative cell biology including primer lectures on cell biology for physics students. This is followed by an examination of life from a physicist's perspective and an introduction to the thermal and statistical physics of living systems with examples ranging from ion channel gating to cooperative binding.

The next part of the course looks at how statistical mechanics can be used to study gene regulation focussing on how cells make transcriptional decisions, genetic switches and oscillating genetic networks. We introduce the basic framework of dynamical systems.

We then turn to the study of the cell as a crowded and disordered environment and building on both stat mech and soft matter concepts we explore the effects of this environment on physical models of cellular processes. This leads on to lectures on the cytoskeletal assembly and molecular motors with an emphasis on statistical approaches to modeling dynamical processes within cells.

The final part of the course includes lectures on neural transport including biophysical models of vision, hearing and information processing in neural networks.

The course includes guest lectures on genetic/proteomic networks and an outlook on the most active research areas of biological physics.

Pre-requisites

A working knowledge of Part II Thermal and Statistical Physics is required, and familiarity with Part II Soft Condensed Matter is beneficial.

Literature

1. *Physical Biology of the cell (2nd Edition)*, Phillips, Kondev, Theriot Garcia
2. *Biological Physics*, Freeman Press, Philip Nelson
3. *Physical Models of Living Systems*, Freeman Press, Philip Nelson
4. *Models of Life*, CUP (available online through http://www.lib.cam.ac.uk/), Sneppen
5. *An Introduction to Systems Biology*, Chapman and Hall, Alon
6. *Molecular Biology of the Cell*, Garland Science, Alberts et al (cell biology reference textbook)

Demonstrations in Fluid Mechanics. (L8)

Non-Examinable (Part III Level)

Dr. S.B. Dalziel, Dr. J.A. Neufeld

While the equations governing most fluid flows are well known, they are often very difficult to solve. To make progress it is therefore necessary to introduce various simplifications and assumptions about the nature of the flow and thus derive a simpler set of equations. For this process to be meaningful, it is essential that the relevant physics of the flow is maintained in the simplified equations. Deriving such equations requires a combination of mathematical analysis and physical insight. Laboratory experiments play a role in providing physical insight into the flow and in providing both qualitative and quantitative data against which theoretical and numerical models may be tested.

The purpose of this demonstration course is to help develop an intuitive 'feeling' for fluid flows, how they relate to simplified mathematical models, and how they may best be used to increase our understanding of a flow. Limitations of experimental data will also be encountered and discussed.

The demonstrations will include a range of flows currently being studied in a range of research projects in addition to classical experiments illustrating some of the flows studied in lectures. The demonstrations are likely to include

- instability of jets, shear layers and boundary layers;
- gravity waves, capillary waves internal waves and inertial waves;
- thermal convection, double-diffusive convection, thermals and plumes;
- gravity currents, intrusions and hydraulic flows;
- vortices, vortex rings and turbulence;
- bubbles, droplets and multiphase flows;
- sedimentation and resuspension;
- avalanches and granular flows;
- porous media and carbon sequestration;
- ventilation and industrial flows;
- rotationally dominated flows;
- non-Newtonian and low Reynolds' number flows;
- image processing techniques and methods of flow visualisation.

It should be noted that students attending this course are not required to undertake laboratory work on their own account.

Pre-requisites

Undergraduate Fluid Dynamics.

Literature

1. M. Van Dyke. An Album of Fluid Motion. Parabolic Press.
2. G. M. Homsy, H. Aref, K. S. Breuer, S. Hochgreb, J. R. Koseff, B. R. Munson, K. G. Powell, C. R. Robertson, S. T. Thoroddsen. Multimedia Fluid Mechanics (Multilingual Version CD-ROM). CUP.
3. M. Samimy, K. Breuer, P. Steen, & L. G. Leal. A Gallery of Fluid Motion. CUP.