Applied and Computational Analysis

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What is ACA?

Applied and Computational Analysis (ACA) at DAMTP spans a wide range of themes in

- partial differential equations
- numerical analysis
- dynamical and integrable systems
- approximation theory
- inverse problems
- mathematics of information (compressed sensing, image analysis)

Philosophy: investigation of a broad range of mathematical tools—analytic and computational—which are applicable across many applications.
ACA courses 2015/16

Michaelmas
- Set-Valued Analysis and Optimisation (16)
- Inverse problems (16)
- Distribution Theory and Applications (16)

Lent
- Boundary value problems for linear PDEs (16)
- Compressed Sensing and Sampling Theory (16)

Easter
- Homogenization of PDEs (16)
Modern approaches to image processing, machine learning, and various big data applications, almost invariably involve the solution of non-smooth optimisation problems.

- Minima of non-smooth functions – subdifferentials – convex analysis
- Methods for convex minimisation – Moreau–Yosida regularisation
- Sensitivity analysis – Lipschitz properties of set-valued mappings
- Graphical derivatives and coderivatives – the Mordukhovich criterion
Inverse problems (M16)

\[ f = Au + n \]

Dr Martin Benning

**Problem**: Computing unknown physical quantity from indirect measurements from forward model

**Applications**: imaging (CT, MRI), signal- and image-processing, computer vision, machine learning and (big) data analysis, . . . .

**Mathematical challenges**: ill-posedness, uniqueness and stability (amplification of small measurement errors)

**Course addresses** the mathematical aspects of inverse problems, concept of regularisation for stable inversion.
Distribution Theory and Applications (M16)

$\Delta u = \delta_{x_0}$

Dr Anthony Ashton

Introduction to the theory of distributions and its application to the study of linear PDEs. Address questions such as

- What is a distribution?
- What does a generic distribution look like?
- What are Sobolev spaces $H^s(\mathbb{R}^n)$?
- Why are solutions to Laplace’s equation always infinitely differentiable?
- Which functions are the Fourier transform of a distribution?
Boundary value problems for linear PDEs (L16)

\[ iu_t + u_{xx} - 2\sigma|u|^2u = 0 \]
\[ \sigma = \pm 1 \]
\[ u \in \mathbb{C}. \]

Prof. Thanasis Fokas

**About**: Unified Transform (UT) for the solution of integrable PDEs: method based on “synthesis” as opposed to separation of variables ⇒ analytical solution of several non-separable and non-self adjoint BVPs ⇒ numerical techniques for solving linear elliptic PDEs.

- Integral representations of the solution for heat equation and the Stokes equation, the Helmholtz equations
- Simple Numerical techniques for the effective computation of the solution
Compressed Sensing and Sampling Theory (L16)

Non-examinable!

\[ \min_u \|u\|_1 + \|SFu - f\|_2^2 \]

Dr Anders Hansen

About: sampling theory and compressed sensing for use in signal processing and medical imaging. Compressed sensing is a theory of randomisation, sparsity and non-linear optimisation techniques that breaks traditional barriers in sampling theory.

- Sampling theory
- Compressed sensing
- Applications: signal processing, Magnetic Resonance Imaging (MRI) and X-ray Tomography
Homogenization of PDEs (E16)

Non-examinable!

\[-\nabla \cdot (A_\epsilon(x) \nabla u_\epsilon) = f \quad \text{in } \Omega \]
\[u_\epsilon = 0 \quad \text{on } \partial \Omega\]

Dr Harsha Hutridurga

About: study of PDEs with highly oscillating coefficients, e.g.

- Diffusion Equation: conductivity of mixtures.
- Stokes Equation: Darcy’s Law in porous media.
- Linear Boltzmann Equation: interaction of monokinetic particles with background medium.
- Euler Equations (incompressible): \( k - \epsilon \) model for turbulence.

Homogenization: replace oscillatory PDE by PDE which on average behaves like the original one but is much simpler to solve. Methods: Asymptotic Expansions, Energy Method and the notion of Two scale Convergence. Towards numerical solution: Multiscale FEM.
Related courses

Analysis courses
- Analysis of PDEs
- Functional Analysis
- Elliptic PDEs
- Nonlinear waves
- Optimal transportation
- Function Spaces
- Geometric Analysis
- Many particle systems

ACA courses
- Set-Valued Analysis
- Compressed Sensing
- Inverse Problems
- BVPs PDEs
- Distributions
- Homogenization

Stats & Prob courses
- Modern Statistics
- Monte-Carlo Inference
- Topics in Statistics

Continuum mechanics courses
- Perturbation methods
- Scattering
for all those considering PhD at the Cambridge Centre for Analysis (CCA)

The CCA offers an enhanced 4-year PhD course which covers all aspects of mathematical analysis, including:

Geometric analysis, Partial differential equations, Probability, High dimensional and non-parametric statistics, Inverse problems, Image analysis, Approximation theory, Compressed sensing

The course emphasises a breadth of knowledge, alongside teamwork, communication and a strong sense of community.

Please contact Tessa Blackman cca@maths.cam.ac.uk for advice on the best choice of Part III courses to support an application.