THE MATHEMATICAL TRIPOS 2015–2016

CONTENTS

This booklet contains the schedule, or syllabus specification, for each course of the undergraduate Tripos together with information about the examinations. It is updated every year. Suggestions and corrections should be e-mailed to undergrad-office@maths.cam.ac.uk.

SCHEDULES

Syllabus

The schedule for each lecture course is a list of topics that define the course. The schedule is agreed by the Faculty Board. Some schedules contain topics that are 'starred' (listed between asterisks); all the topics must be covered by the lecturer but examiners can only set questions on unstarred topics.

The numbers which appear in brackets at the end of subsections or paragraphs in these schedules indicate the approximate number of lectures likely to be devoted to that subsection or paragraph. Lecturers decide upon the amount of time they think appropriate to spend on each topic, and also on the order in which they present topics. Some topics in Part IA and Part IB courses have to be introduced in a certain order so as to tie in with other courses.

Recommended books

A list of books is given after each schedule. Books marked with † are particularly well suited to the course. Some of the books are out of print; these are retained on the list because they should be available in college libraries (as should all the books on the list) and may be found in second-hand bookshops. There may well be many other suitable books not listed; it is usually worth browsing college libraries.

In most cases, the contents of the book will not be exactly the same as the content of the schedule, and different styles suit different people. Hence you are advised to consult library copies in the first instance to decide which, if any, would be of benefit to you. Up-to-date prices, and the availability of hard- and soft-back versions, can most conveniently be checked online.

STUDY SKILLS

The Faculty produces a booklet *Study Skills in Mathematics* which is distributed to all first year students and can be obtained in pdf format from `www.maths.cam.ac.uk/undergrad/studyskills`.

There is also a booklet, *Supervision in Mathematics*, that gives guidance to supervisors obtainable from `www.maths.cam.ac.uk/facultyoffice/supervisorsguide/` which may also be of interest to students.

Aims and objectives

The **aims** of the Faculty for Parts IA, IB and II of the Mathematical Tripos are:

- to provide a challenging course in mathematics and its applications for a range of students that includes some of the best in the country;
- to provide a course that is suitable both for students aiming to pursue research and for students going into other careers;
- to provide an integrated system of teaching which can be tailored to the needs of individual students;
- to develop in students the capacity for learning and for clear logical thinking, and the ability to solve unseen problems;
- to continue to attract and select students of outstanding quality;
- to produce the high calibre graduates in mathematics sought by employers in universities, the professions and the public services.
- to provide an intellectually stimulating environment in which students have the opportunity to develop their skills and enthusiasms to their full potential;
- to maintain the position of Cambridge as a leading centre, nationally and internationally, for teaching and research in mathematics.

The **objectives** of Parts IA, IB and II of the Mathematical Tripos are as follows:

After completing Part IA, students should have:

- made the transition in learning style and pace from school mathematics to university mathematics;
- been introduced to basic concepts in higher mathematics and their applications, including (i) the notions of proof, rigour and axiomatic development, (ii) the generalisation of familiar mathematics to unfamiliar contexts, (iii) the application of mathematics to problems outside mathematics;
- laid the foundations, in terms of knowledge and understanding, of tools, facts and techniques, to proceed to Part IB.

After completing Part IB, students should have:

- covered material from a range of pure mathematics, statistics and operations research, applied mathematics, theoretical physics and computational mathematics, and studied some of this material in depth;
- acquired a sufficiently broad and deep mathematical knowledge and understanding to enable them both to make an informed choice of courses in Part II and also to study these courses.

After completing Part II, students should have:

- developed the capacity for (i) solving both abstract and concrete unseen problems, (ii) presenting a concise and logical argument, and (iii) (in most cases) using standard software to tackle mathematical problems;
- studied advanced material in the mathematical sciences, some of it in depth.

EXAMINATIONS

Overview

There are three examinations for the undergraduate Mathematical Tripos: Parts IA, IB and II. They are normally taken in consecutive years. This page contains information that is common to all three examinations. Information that is specific to individual examinations is given later in this booklet in the *General Arrangements* sections for the appropriate part of the Tripos.

The form of each examination (number of papers, numbers of questions on each lecture course, distribution of questions in the papers and in the sections of each paper, number of questions which may be attempted) is determined by the Faculty Board. The main structure has to be agreed by University committees and is published as a Regulation in the Statutes and Ordinances of the University of Cambridge (`http://www.admin.cam.ac.uk/univ/so`). (Any significant change to the format is announced in the *Reporter* as a Form and Conduct notice.) The actual questions and marking schemes, and precise borderlines (following general classing criteria agreed by the Faculty Board — see below) are determined by the examiners.

The examiners for each part of the Tripos are appointed by the General Board of the University. The internal examiners are normally teaching staff of the two mathematics departments and they are joined by one or more external examiners from other universities (one for Part IA, two for Part IB and three for Part II).

For all three parts of the Tripos, the examiners are collectively responsible for the examination questions, though for Part II the questions are proposed by the individual lecturers. All questions have to be signed off by the relevant lecturer; no question can be used unless the lecturer agrees that it is fair and appropriate to the course he or she lectured.

Form of the examination

The examination for each part of the Tripos consists of four written papers and candidates take all four. For Parts IB and II, candidates may in addition submit Computational Projects. Each written paper has two sections: Section I contains questions that are intended to be accessible to any student who has studied the material conscientiously. They should not contain any significant 'problem' element. Section II questions are intended to be more challenging

Calculators are not allowed in any paper of the Mathematical Tripos; questions will be set in such a way as not to require the use of calculators. The rules for the use of calculators in the Physics paper of the Mathematics-with-Physics option of Part IA are set out in the regulations for the Natural Sciences Tripos.

Formula booklets are not permitted, but candidates will not be required to quote elaborate formulae from memory.

Past papers

Past Tripos papers for the last 10 or more years can be found on the Faculty web site `http://www.maths.cam.ac.uk/undergrad/pastpapers/`. Solutions and mark schemes are not available except in rough draft form for supervisors.

Marking conventions

On the written papers of the Mathematical Tripos, Section I questions are marked out of 10 and Section II questions are marked out of 20. In addition to a numerical mark, extra credit in the form of a quality mark may be awarded for each question depending on the completeness and quality of each answer. For a Section I question, a *beta* quality mark is awarded for a mark of 7 or more. For a Section II question, an *alpha* quality mark is awarded for a mark of 15 or more, and a *beta* quality mark is awarded for a mark between 10 and 14, inclusive.

The marks available on the Computational Projects courses are described later in this booklet in the introductions to Parts IB and II, and in more detail in the Computational Projects Manuals, which are available at `http://www.maths.cam.ac.uk/undergrad/catam/`

On the written papers, there are restrictions on the number of questions that may be attempted, indicated by a rubric of the form '*You may attempt at most N questions in Section I/II*'. The Faculty policy is that examiners mark all attempts, even if the number of these exceeds that specified in the rubric, and the candidate is assessed on the best attempts consistent with the rubric. This policy is intended to deal with candidates who accidently violate the rubric: it is clearly not in candidates' best interests to spend time tackling more questions than is permitted by the rubric.

Examinations are 'single-marked', but safety checks are made on all scripts to ensure that all work is marked and that all marks are correctly added and transcribed. Scripts are identified only by candidate number until the final class list has been drawn up. In drawing up the class list, examiners make decisions based only on the work they see: no account is taken of the candidates' personal situation or of supervision reports. Circumstances which seriously hindered your preparation for the examination, such as illness, may be considered at a later stage by the University's Application Committee. But it is very important that you inform your college tutor of the full circumstances at the earliest possible moment so that supporting evidence can be provided.

All appeals must be made through official channels, usually via your college tutor (either via your college tutor or directly to the Registrary if you wish to appeal against the mark you were given). For further information, consult your tutor or the exams section of the CUSU student advice service website `http://www.studentadvice.cam.ac.uk`. Examiners must not be approached either by candidates or their directors of studies as this might jeopardise any formal appeal.

Data Protection Act

To meet the University's obligations under the Data Protection Act (1998), the Faculty deals with data relating to individuals and their examination marks as follows:

- Marks for individual questions and Computational Projects are released routinely after the examinations.
- Scripts and Computational Projects submissions are kept, in line with the University policy, for six months following the examinations (in case of appeals). Scripts are then destroyed; and Computational Projects are anonymised and stored in a form that allows comparison (using anti-plagiarism software) with current projects.
- Neither the Data Protection Act nor the Freedom of Information Act entitle candidates to have access to their scripts. However, data appearing on individual examination scripts are available on application to the University Data Protection Officer and on payment of a fee. Such data would consist of little more than ticks, crosses, underlines, and mark subtotals and totals.

Classification Criteria

As a result of each examination, each candidate is placed in one of the following categories: first class, upper second class (2.1), lower second class (2.2), third class, fail or 'other'. 'Other' here includes, for example, candidates who were ill for all or part of the examination.

In the *exceptionally unlikely event* of your being placed in the fail category, you should contact your tutor or director of studies at once: if you wish to continue to study at Cambridge an appeal (based, for example, on medical evidence) must be made to the Council of the University. There are no 're-sits' in the usual sense; in exceptional circumstances the regulations permit you to re-take a Tripos examination the following year.

The examiners place the candidates into the different classes; they do not rank the candidates within the classes. The primary classification criteria for each borderline, which are determined by the Faculty Board, are as follows:

First / upper second	$30\alpha + 5\beta + m$	
Upper second / lower second	$15\alpha + 5\beta + m$	
Lower second / third	$15\alpha + 5\beta + m$	
Third/ fail	$15\alpha + 5\beta + m$	in Part IB and Part II;
	$2\alpha + \beta$ together with m	in Part IA.

Here, m denotes the number of marks and α and β denote the numbers of quality marks. Other factors besides marks and quality marks may be taken into account.

At the third/fail borderline, examiners may consider if most of the marks have been obtained on only one or two courses.

The Faculty Board recommends that no distinction should be made between marks obtained on the Computational Projects courses in Parts IB and II and marks obtained on the written papers.

The Faculty Board recommends approximate percentages of candidates for each class: 30% firsts; 70–75% upper seconds and above; 90–95% lower seconds and above; and 5–10% thirds and below. These percentages should exclude candidates who did not sit the examination.

The Faculty Board expects that the classification criteria described above should result in classes that can be characterized as follows (after allowing for the possibility that in Parts IB and II stronger performance on the Computational Projects may compensate for weaker performance on the written papers or vice versa):

First Class

Candidates placed in the first class will have demonstrated a good command and secure understanding of examinable material. They will have presented standard arguments accurately, showed skill in applying their knowledge, and generally will have produced substantially correct solutions to a significant number of more challenging questions.

Upper Second Class

Candidates placed in the upper second class will have demonstrated good knowledge and understanding of examinable material. They will have presented standard arguments accurately and will have shown some ability to apply their knowledge to solve problems. A fair number of their answers to both straightforward and more challenging questions will have been substantially correct.

Lower Second Class

Candidates placed in the lower second class will have demonstrated knowledge but sometimes imperfect understanding of examinable material. They will have been aware of relevant mathematical issues, but their presentation of standard arguments will sometimes have been fragmentary or imperfect. They will have produced substantially correct solutions to some straightforward questions, but will have had limited success at tackling more challenging problems.

Third Class

Candidates placed in the third class will have demonstrated some knowledge but little understanding of the examinable material. They will have made reasonable attempts at a small number of questions, but will have lacked the skills to complete many of them.

Examiners' reports

For each part of the Tripos, the examiners (internal and external) write a joint report. In addition, the external examiners each submit a report addressed to the Vice-Chancellor. The reports of the external examiners are scrutinised by the General Board of the University's Education Committee.

All the reports, the examination statistics (number of attempts per question, etc), student feedback on the examinations and lecture courses (via the end of year questionnaire and paper questionnaires), and other relevant material are considered by the Faculty Teaching Committee at the start of the Michaelmas term. The Teaching Committee includes two student representatives, and may include other students (for example, previous members of the Teaching Committee and student representatives of the Faculty Board). The Teaching Committee compiles a lengthy report including various recommendations for the Faculty Board to consider at its second meeting in the Michaelmas term. This report also forms the basis of the Faculty Board's response to the reports of the external examiners.

MISCELLANEOUS MATTERS

Numbers of supervisions

Directors of Studies will arrange supervisions for each course as they think appropriate. Lecturers will hand out examples sheets which supervisors may use if they wish. According to Faculty Board guidelines, the number of examples sheets for 24-lecture, 16-lecture and 12-lecture courses should be 4, 3 and 2, respectively.

Transcripts

In order to conform to government guidelines on examinations, the Faculty is obliged to produce, for use in transcripts, data that will allow you to determine roughly your position within each class. The examiners officially do no more than place each candidate in a class, but the Faculty authorises a percentage mark to be given, via CamSIS, to each candidate for each examination. The percentage mark is obtained by piecewise linear scaling of merit marks within each class. The 1/2.1, 2.1/2.2, 2.2/3 and 3/fail boundaries are mapped to 69.5%, 59.5%, 49.5% and 39.5% respectively and the mark of the 5th ranked candidate is mapped to 95%. If, after linear mapping of the first class, the percentage mark of any candidate is greater than 100, it is reduced to 100%. The percentage of each candidate is then rounded appropriately to integer values.

The merit mark mentioned above, denoted by M, is defined in terms of raw mark m, number of alphas, α, and number of betas, β, by

$$M = \begin{cases} 30\alpha + 5\beta + m - 120 & \text{for candidates in the first class, or in the upper second class with } \alpha \geq 8, \\ 15\alpha + 5\beta + m & \text{otherwise} \end{cases}$$

Faculty Committees

The Faculty has two committees which deal with matters relating to the undergraduate Tripos: the Teaching Committee and the Curriculum Committee. Both have student representatives.

The role of the Teaching Committee is mainly to monitor feedback (questionnaires, examiners' reports, etc) and make recommendations to the Faculty Board on the basis of this feedback. It also formulates policy recommendations at the request of the Faculty Board.

The Curriculum Committee is responsible for recommending (to the Faculty Board) changes to the undergraduate Tripos and to the schedules for individual lecture courses.

Student representatives

There are three student representatives, two undergraduate and one graduate, on the Faculty Board, and two on each of the Teaching Committee and the Curriculum Committee. They are normally elected (in the case of the Faculty Board representatives) or appointed in November of each year. Their role is to advise the committees on the student point of view, to collect opinion from and liaise with the student body. They operate a website: `http://www.maths.cam.ac.uk/studentreps` and their email address is `student.reps@maths.cam.ac.uk`.

Feedback

Constructive feedback of all sorts and from all sources is welcomed by everyone concerned in providing courses for the Mathematical Tripos.

There are many different feedback routes.

- Each lecturer hands out a paper questionnaire towards the end of the course.
- There are brief web-based questionnaires after roughly six lectures of each course.
- Students are sent a combined online questionnaire at the end of each year.
- Students (or supervisors) can e-mail `feedback@maths.cam.ac.uk` at any time. Such e-mails are read by the Chairman of the Teaching Committee and forwarded in anonymised form to the appropriate person (a lecturer, for example). Students will receive a rapid response.
- If a student wishes to be entirely anonymous and does not need a reply, the web-based comment form at `www.maths.cam.ac.uk/feedback.html` can be used (clickable from the Faculty web site).
- Feedback on college-provided teaching (supervisions, classes) can be given to Directors of Studies or Tutors at any time.

The questionnaires are particularly important in shaping the future of the Tripos and the Faculty Board urges all students to respond.

Part IA

GENERAL ARRANGEMENTS

Structure of Part IA

There are two options:

(*a*) Pure and Applied Mathematics;
(*b*) Mathematics with Physics.

Option (*a*) is intended primarily for students who expect to continue to Part IB of the Mathematical Tripos, while Option (*b*) is intended primarily for those who are undecided about whether they will continue to Part IB of the Mathematical Tripos or change to Part IB of the Natural Sciences Tripos (Physics option).

For Option (*b*), two of the lecture courses (Numbers and Sets, and Dynamics and Relativity) are replaced by the complete Physics course from Part IA of the Natural Sciences Tripos; Numbers and Sets because it is the least relevant to students taking this option, and Dynamics and Relativity because much of this material is covered in the Natural Sciences Tripos anyway. Students wishing to examine the schedules for the physics courses should consult the documentation supplied by the Physics department, for example on `http://www.phy.cam.ac.uk/teaching/`.

Examinations

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

All candidates for Part IA of the Mathematical Tripos take four papers, as follows:
Candidates taking Option (*a*) (Pure and Applied Mathematics) will take Papers 1, 2, 3 and 4 of the Mathematical Tripos (Part IA).
Candidates taking Option (*b*) (Mathematics with Physics) take Papers 1, 2 and 3 of the Mathematical Tripos (Part IA) and the Physics paper of the Natural Sciences Tripos (Part IA); they must also submit practical notebooks.

For Mathematics-with-Physics candidates, the marks for the Physics paper are scaled to bring them in line with Paper 4. This is done as follows. The Physics scripts of the Mathematics-with-Physics candidates are marked by the Natural Sciences examiners for Part IA Physics, and a mark for each candidate is given to the Mathematics examiners. Class boundaries for the Physics paper are determined such that the percentages in each class (1, 2.1, 2.2, 3) of all candidates on the Physics paper (including those from NST and CST) are 25, 35, 30, 10 (which are the guidelines across Natural Sciences). All candidates for Paper 4 (ranked by merit mark on that paper) are assigned nominally to classes so that the percentages in each class are 30, 40, 20, 10 (which is the Faculty Board rough guideline for the initial proportion in each class prior to the final overall classification). Piecewise linear mapping of the Physics marks in each Physics class to the Mathematics merit marks in each nominal Mathematics class is used to provide a merit mark for each Mathematics-with-Physics candidate. The merit mark is then broken down into marks, alphas and betas by comparison (for each candidate) with the break down for Papers 1, 2 and 3.

Examination Papers

Papers 1, 2, 3 and 4 of Part IA of the Mathematical Tripos are each divided into two Sections. There are four questions in Section I and eight questions in Section II. Candidates may attempt all the questions in Section I and at most five questions from Section II, of which no more than three may be on the same lecture course.

Each section of each of Papers 1–4 is divided equally between two courses as follows:

Paper 1:	Vectors and Matrices, Analysis I
Paper 2:	Differential Equations, Probability
Paper 3:	Groups, Vector Calculus
Paper 4:	Numbers and Sets, Dynamics and Relativity.

Approximate class boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines.

For convenience, we define M_1 and M_2 by

$$M_1 = 30\alpha + 5\beta + m - 120, \qquad M_2 = 15\alpha + 5\beta + m.$$

M_1 is related to the primary classification criterion for the first class and M_2 is related to the primary classification criterion for the upper and lower second classes.

The second column of each table shows a sufficient criterion for each class. The third and fourth columns shows M_1 (for the first class) or M_2 (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline. The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

Part IA 2014			
Class	**Sufficient condition**	**Borderline candidates**	
1	$M_1 > 625$	605/335,12, 6	608/358,11, 8
2.1	$M_2 > 438$	420/260, 7,11	422/287, 6, 9
2.2	$M_2 > 328$	328/213, 5, 8	334/209, 5,10
3	$2\alpha + \beta > 11$	247/167, 3, 7	273/183, 5, 3

Part IA 2015			
Class	**Sufficient condition**	**Borderline candidates**	
1	$M_1 > 610$	603/323,12, 8	603/343,11,10
2.1	$M_2 > 396$	387/252, 7, 6	397/262, 6, 9
2.2	$M_2 > 311$	310/195, 5, 8	308/223, 1,14
3	$2\alpha + \beta > 10$	256/176, 3, 7	254/199, 1, 8

GROUPS

24 lectures, Michaelmas term

Examples of groups
Axioms for groups. Examples from geometry: symmetry groups of regular polygons, cube, tetrahedron. Permutations on a set; the symmetric group. Subgroups and homomorphisms. Symmetry groups as subgroups of general permutation groups. The Möbius group; cross-ratios, preservation of circles, the point at infinity. Conjugation. Fixed points of Möbius maps and iteration. [4]

Lagrange's theorem
Cosets. Lagrange's theorem. Groups of small order (up to order 8). Quaternions. Fermat-Euler theorem from the group-theoretic point of view. [5]

Group actions
Group actions; orbits and stabilizers. Orbit-stabilizer theorem. Cayley's theorem (every group is isomorphic to a subgroup of a permutation group). Conjugacy classes. Cauchy's theorem. [4]

Quotient groups
Normal subgroups, quotient groups and the isomorphism theorem. [4]

Matrix groups
The general and special linear groups; relation with the Möbius group. The orthogonal and special orthogonal groups. Proof (in $\mathbb{R}^3$) that every element of the orthogonal group is the product of reflections and every rotation in $\mathbb{R}^3$ has an axis. Basis change as an example of conjugation. [3]

Permutations
Permutations, cycles and transpositions. The sign of a permutation. Conjugacy in S_n and in A_n. Simple groups; simplicity of A_5. [4]

Appropriate books

M.A. Armstrong *Groups and Symmetry.* Springer–Verlag 1988
† Alan F Beardon *Algebra and Geometry.* CUP 2005
R.P. Burn *Groups, a Path to Geometry.* Cambridge University Press 1987
J.A. Green *Sets and Groups: a first course in Algebra.* Chapman and Hall/CRC 1988
W. Lederman *Introduction to Group Theory.* Longman 1976
Nathan Carter *Visual Group Theory.* Mathematical Association of America Textbooks

VECTORS AND MATRICES

24 lectures, Michaelmas term

Complex numbers
Review of complex numbers, including complex conjugate, inverse, modulus, argument and Argand diagram. Informal treatment of complex logarithm, *n-th* roots and complex powers. de Moivre's theorem. [2]

Vectors
Review of elementary algebra of vectors in $\mathbb{R}^3$, including scalar product. Brief discussion of vectors in $\mathbb{R}^n$ and $\mathbb{C}^n$; scalar product and the Cauchy–Schwarz inequality. Concepts of linear span, linear independence, subspaces, basis and dimension.

Suffix notation: including summation convention, δ_{ij} and ϵ_{ijk}. Vector product and triple product: definition and geometrical interpretation. Solution of linear vector equations. Applications of vectors to geometry, including equations of lines, planes and spheres. [5]

Matrices
Elementary algebra of 3×3 matrices, including determinants. Extension to $n \times n$ complex matrices. Trace, determinant, non-singular matrices and inverses. Matrices as linear transformations; examples of geometrical actions including rotations, reflections, dilations, shears; kernel and image. [4]

Simultaneous linear equations: matrix formulation; existence and uniqueness of solutions, geometric interpretation; Gaussian elimination. [3]

Symmetric, anti-symmetric, orthogonal, hermitian and unitary matrices. Decomposition of a general matrix into isotropic, symmetric trace-free and antisymmetric parts. [1]

Eigenvalues and Eigenvectors
Eigenvalues and eigenvectors; geometric significance. [2]

Proof that eigenvalues of hermitian matrix are real, and that distinct eigenvalues give an orthogonal basis of eigenvectors. The effect of a general change of basis (similarity transformations). Diagonalization of general matrices: sufficient conditions; examples of matrices that cannot be diagonalized. Canonical forms for 2×2 matrices. [5]

Discussion of quadratic forms, including change of basis. Classification of conics, cartesian and polar forms. [1]

Rotation matrices and Lorentz transformations as transformation groups. [1]

Appropriate books

Alan F Beardon *Algebra and Geometry.* CUP 2005
Gilbert Strang *Linear Algebra and Its Applications.* Thomson Brooks/Cole, 2006
Richard Kaye and Robert Wilson *Linear Algebra.* Oxford science publications, 1998
D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* Nelson Thornes 1992
E. Sernesi *Linear Algebra: A Geometric Approach.* CRC Press 1993
James J. Callahan *The Geometry of Spacetime: An Introduction to Special and General Relativity.* Springer 2000

NUMBERS AND SETS — 24 lectures, Michaelmas term

[Note that this course is omitted from Option (b) of Part IA.]

Introduction to number systems and logic
Overview of the natural numbers, integers, real numbers, rational and irrational numbers, algebraic and transcendental numbers. Brief discussion of complex numbers; statement of the Fundamental Theorem of Algebra.

Ideas of axiomatic systems and proof within mathematics; the need for proof; the role of counter-examples in mathematics. Elementary logic; implication and negation; examples of negation of compound statements. Proof by contradiction. [2]

Sets, relations and functions
Union, intersection and equality of sets. Indicator (characteristic) functions; their use in establishing set identities. Functions; injections, surjections and bijections. Relations, and equivalence relations. Counting the combinations or permutations of a set. The Inclusion-Exclusion Principle. [4]

The integers
The natural numbers: mathematical induction and the well-ordering principle. Examples, including the Binomial Theorem. [2]

Elementary number theory
Prime numbers: existence and uniqueness of prime factorisation into primes; highest common factors and least common multiples. Euclid's proof of the infinity of primes. Euclid's algorithm. Solution in integers of ax+by = c.

Modular arithmetic (congruences). Units modulo n. Chinese Remainder Theorem. Wilson's Theorem; the Fermat-Euler Theorem. Public key cryptography and the RSA algorithm. [8]

The real numbers
Least upper bounds; simple examples. Least upper bound axiom. Sequences and series; convergence of bounded monotonic sequences. Irrationality of $\sqrt{2}$ and e. Decimal expansions. Construction of a transcendental number. [4]

Countability and uncountability
Definitions of finite, infinite, countable and uncountable sets. A countable union of countable sets is countable. Uncountability of $\mathbb{R}$. Non-existence of a bijection from a set to its power set. Indirect proof of existence of transcendental numbers. [4]

Appropriate books

R.B.J.T. Allenby *Numbers and Proofs.* Butterworth-Heinemann 1997
R.P. Burn *Numbers and Functions: steps into analysis.* Cambridge University Press 2000
H. Davenport *The Higher Arithmetic.* Cambridge University Press 1999
A.G. Hamilton *Numbers, sets and axioms: the apparatus of mathematics.* Cambridge University Press 1983
C. Schumacher *Chapter Zero: Fundamental Notions of Abstract Mathematics.* Addison-Wesley 2001
I. Stewart and D. Tall *The Foundations of Mathematics.* Oxford University Press 1977

DIFFERENTIAL EQUATIONS — 24 lectures, Michaelmas term

Basic calculus
Informal treatment of differentiation as a limit, the chain rule, Leibnitz's rule, Taylor series, informal treatment of O and o notation and l'Hôpital's rule; integration as an area, fundamental theorem of calculus, integration by substitution and parts. [3]

Informal treatment of partial derivatives, geometrical interpretation, statement (only) of symmetry of mixed partial derivatives, chain rule, implicit differentiation. Informal treatment of differentials, including exact differentials. Differentiation of an integral with respect to a parameter. [2]

First-order linear differential equations
Equations with constant coefficients: exponential growth, comparison with discrete equations, series solution; modelling examples including radioactive decay.

Equations with non-constant coefficients: solution by integrating factor. [2]

Nonlinear first-order equations
Separable equations. Exact equations. Sketching solution trajectories. Equilibrium solutions, stability by perturbation; examples, including logistic equation and chemical kinetics. Discrete equations: equilibrium solutions, stability; examples including the logistic map. [4]

Higher-order linear differential equations
Complementary function and particular integral, linear independence, Wronskian (for second-order equations), Abel's theorem. Equations with constant coefficients and examples including radioactive sequences, comparison in simple cases with difference equations, reduction of order, resonance, transients, damping. Homogeneous equations. Response to step and impulse function inputs; introduction to the notions of the Heaviside step-function and the Dirac delta-function. Series solutions including statement only of the need for the logarithmic solution. [8]

Multivariate functions: applications
Directional derivatives and the gradient vector. Statement of Taylor series for functions on $\mathbb{R}^n$. Local extrema of real functions, classification using the Hessian matrix. Coupled first order systems: equivalence to single higher order equations; solution by matrix methods. Non-degenerate phase portraits local to equilibrium points; stability.

Simple examples of first- and second-order partial differential equations, solution of the wave equation in the form $f(x+ct)+g(x-ct)$. [5]

Appropriate books

J. Robinson *An introduction to Differential Equations.* Cambridge University Press, 2004
W.E. Boyce and R.C. DiPrima *Elementary Differential Equations and Boundary-Value Problems (and associated web site: google Boyce DiPrima).* Wiley, 2004
G.F.Simmons *Differential Equations (with applications and historical notes).* McGraw-Hill 1991
D.G. Zill and M.R. Cullen *Differential Equations with Boundary Value Problems.* Brooks/Cole 2001

ANALYSIS I

24 lectures, Lent term

Limits and convergence
Sequences and series in $\mathbb{R}$ and $\mathbb{C}$. Sums, products and quotients. Absolute convergence; absolute convergence implies convergence. The Bolzano-Weierstrass theorem and applications (the General Principle of Convergence). Comparison and ratio tests, alternating series test. [6]

Continuity
Continuity of real- and complex-valued functions defined on subsets of $\mathbb{R}$ and $\mathbb{C}$. The intermediate value theorem. A continuous function on a closed bounded interval is bounded and attains its bounds. [3]

Differentiability
Differentiability of functions from $\mathbb{R}$ to $\mathbb{R}$. Derivative of sums and products. The chain rule. Derivative of the inverse function. Rolle's theorem; the mean value theorem. One-dimensional version of the inverse function theorem. Taylor's theorem from $\mathbb{R}$ to $\mathbb{R}$; Lagrange's form of the remainder. Complex differentiation. [5]

Power series
Complex power series and radius of convergence. Exponential, trigonometric and hyperbolic functions, and relations between them. *Direct proof of the differentiability of a power series within its circle of convergence*. [4]

Integration
Definition and basic properties of the Riemann integral. A non-integrable function. Integrability of monotonic functions. Integrability of piecewise-continuous functions. The fundamental theorem of calculus. Differentiation of indefinite integrals. Integration by parts. The integral form of the remainder in Taylor's theorem. Improper integrals. [6]

Appropriate books

T.M. Apostol *Calculus, vol 1.* Wiley 1967-69

† J.C. Burkill *A First Course in Mathematical Analysis.* Cambridge University Press 1978

D.J.H.Garling *A Course in Mathematical Analysis (Vol 1).* Cambridge University Press 2013

J.B. Reade *Introduction to Mathematical Analysis.* Oxford University Press

M. Spivak *Calculus.* Addison–Wesley/Benjamin–Cummings 2006

David M. Bressoud *A Radical Approach to Real Analysis.* Mathematical Association of America Textbooks

PROBABILITY

24 lectures, Lent term

Basic concepts
Classical probability, equally likely outcomes. Combinatorial analysis, permutations and combinations. Stirling's formula (asymptotics for $\log n!$ proved). [3]

Axiomatic approach
Axioms (countable case). Probability spaces. Inclusion-exclusion formula. Continuity and subadditivity of probability measures. Independence. Binomial, Poisson and geometric distributions. Relation between Poisson and binomial distributions. Conditional probability, Bayes's formula. Examples, including Simpson's paradox. [5]

Discrete random variables
Expectation. Functions of a random variable, indicator function, variance, standard deviation. Covariance, independence of random variables. Generating functions: sums of independent random variables, random sum formula, moments.

Conditional expectation. Random walks: gambler's ruin, recurrence relations. Difference equations and their solution. Mean time to absorption. Branching processes: generating functions and extinction probability. Combinatorial applications of generating functions. [7]

Continuous random variables
Distributions and density functions. Expectations; expectation of a function of a random variable. Uniform, normal and exponential random variables. Memoryless property of exponential distribution.

Joint distributions: transformation of random variables (including Jacobians), examples. Simulation: generating continuous random variables, independent normal random variables. Geometrical probability: Bertrand's paradox, Buffon's needle. Correlation coefficient, bivariate normal random variables. [6]

Inequalities and limits
Markov's inequality, Chebyshev's inequality. Weak law of large numbers. Convexity: Jensen's inequality for general random variables, AM/GM inequality.

Moment generating functions and statement (no proof) of continuity theorem. Statement of central limit theorem and sketch of proof. Examples, including sampling. [3]

Appropriate books

W. Feller *An Introduction to Probability Theory and its Applications, Vol. I.* Wiley 1968

† G. Grimmett and D. Welsh *Probability: An Introduction.* Oxford University Press 2nd Edition 2014

† S. Ross *A First Course in Probability.* Prentice Hall 2009

D.R. Stirzaker *Elementary Probability.* Cambridge University Press 1994/2003

VECTOR CALCULUS **24 lectures, Lent term**

Curves in $\mathbb{R}^3$
Parameterised curves and arc length, tangents and normals to curves in $\mathbb{R}^3$, the radius of curvature. [1]

Integration in $\mathbb{R}^2$ and $\mathbb{R}^3$
Line integrals. Surface and volume integrals: definitions, examples using Cartesian, cylindrical and spherical coordinates; change of variables. [4]

Vector operators
Directional derivatives. The gradient of a real-valued function: definition; interpretation as normal to level surfaces; examples including the use of cylindrical, spherical *and general orthogonal curvilinear* coordinates.

Divergence, curl and ∇^2 in Cartesian coordinates, examples; formulae for these operators (statement only) in cylindrical, spherical *and general orthogonal curvilinear* coordinates. Solenoidal fields, irrotational fields and conservative fields; scalar potentials. Vector derivative identities. [5]

Integration theorems
Divergence theorem, Green's theorem, Stokes's theorem, Green's second theorem: statements; informal proofs; examples; application to fluid dynamics, and to electromagnetism including statement of Maxwell's equations. [5]

Laplace's equation
Laplace's equation in $\mathbb{R}^2$ and $\mathbb{R}^3$: uniqueness theorem and maximum principle. Solution of Poisson's equation by Gauss's method (for spherical and cylindrical symmetry) and as an integral. [4]

Cartesian tensors in $\mathbb{R}^3$
Tensor transformation laws, addition, multiplication, contraction, with emphasis on tensors of second rank. Isotropic second and third rank tensors. Symmetric and antisymmetric tensors. Revision of principal axes and diagonalization. Quotient theorem. Examples including inertia and conductivity. [5]

Appropriate books

H. Anton *Calculus.* Wiley Student Edition 2000
T.M. Apostol *Calculus.* Wiley Student Edition 1975
M.L. Boas *Mathematical Methods in the Physical Sciences.* Wiley 1983
† D.E. Bourne and P.C. Kendall *Vector Analysis and Cartesian Tensors.* 3rd edition, Nelson Thornes 1999
E. Kreyszig *Advanced Engineering Mathematics.* Wiley International Edition 1999
J.E. Marsden and A.J.Tromba *Vector Calculus.* Freeman 1996
P.C. Matthews *Vector Calculus.* SUMS (Springer Undergraduate Mathematics Series) 1998
† K. F. Riley, M.P. Hobson, and S.J. Bence *Mathematical Methods for Physics and Engineering.* Cambridge University Press 2002
H.M. Schey *Div, grad, curl and all that: an informal text on vector calculus.* Norton 1996
M.R. Spiegel *Schaum's outline of Vector Analysis.* McGraw Hill 1974

DYNAMICS AND RELATIVITY **24 lectures, Lent term**

[Note that this course is omitted from Option (b) of Part IA.]
Familiarity with the topics covered in the non-examinable Mechanics course is assumed.

Basic concepts
Space and time, frames of reference, Galilean transformations. Newton's laws. Dimensional analysis. Examples of forces, including gravity, friction and Lorentz. [4]

Newtonian dynamics of a single particle
Equation of motion in Cartesian and plane polar coordinates. Work, conservative forces and potential energy, motion and the shape of the potential energy function; stable equilibria and small oscillations; effect of damping.

Angular velocity, angular momentum, torque.

Orbits: the $u(\theta)$ equation; escape velocity; Kepler's laws; stability of orbits; motion in a repulsive potential (Rutherford scattering).

Rotating frames: centrifugal and Coriolis forces. *Brief discussion of Foucault pendulum.* [8]

Newtonian dynamics of systems of particles
Momentum, angular momentum, energy. Motion relative to the centre of mass; the two body problem. Variable mass problems; the rocket equation. [2]

Rigid bodies
Moments of inertia, angular momentum and energy of a rigid body. Parallel axis theorem. Simple examples of motion involving both rotation and translation (e.g. rolling). [3]

Special relativity
The principle of relativity. Relativity and simultaneity. The invariant interval. Lorentz transformations in $(1+1)$-dimensional spacetime. Time dilation and length contraction. The Minkowski metric for $(1+1)$-dimensional spacetime.

Lorentz transformations in $(3+1)$ dimensions. 4–vectors and Lorentz invariants. Proper time. 4–velocity and 4–momentum. Conservation of 4–momentum in particle decay. Collisions. The Newtonian limit. [7]

Appropriate books

† D. Gregory *Classical Mechanics.* Cambridge University Press 2006
G.F.R. Ellis and R.M. Williams *Flat and Curved Space-times.* Oxford University Press 2000
A.P. French and M.G. Ebison *Introduction to Classical Mechanics.* Kluwer 1986
T.W.B. Kibble and F.H. Berkshire *Introduction to Classical Mechanics.* Kluwer 1986
M.A. Lunn *A First Course in Mechanics.* Oxford University Press 1991
P.J. O'Donnell *Essential Dynamics and Relativity.* CRC Press 2015
† W. Rindler *Introduction to Special Relativity.* Oxford University Press 1991
E.F. Taylor and J.A. Wheeler *Spacetime Physics: introduction to special relativity.* Freeman 1992

COMPUTATIONAL PROJECTS — 8 lectures, Easter term of Part IA

The Computational Projects course is examined in Part IB. However introductory practical sessions are offered at the end of Lent Full Term and the beginning of Easter Full Term of the Part IA year (students are advised by email how to register for a session), and lectures are given in the Easter Full Term of the Part IA year. The lectures cover an introduction to algorithms and aspects of the MATLAB programming language. The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July at the end of the Part IA year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at `http://www.maths.cam.ac.uk/undergrad/catam/`. Full credit may obtained from the submission of the two core projects and a further two additional projects. Once the manual is available, these projects may be undertaken at any time up to the submission deadlines, which are near the start of the Full Lent Term in the IB year for the two core projects, and near the start of the Full Easter Term in the IB year for the two additional projects.

A list of suitable books can be found in the manual.

MECHANICS (non-examinable) — 10 lectures, Michaelmas term

This course is intended for students who have taken fewer than three A-level Mechanics modules (or the equivalent). The material is prerequisite for Dynamics and Relativity in the Lent term.

Lecture 1
Brief introduction

Lecture 2: Kinematics of a single particle
Position, velocity, speed, acceleration. Constant acceleration in one-dimension. Projectile motion in two-dimensions.

Lecture 3: Equilibrium of a single particle
The vector nature of forces, addition of forces, examples including gravity, tension in a string, normal reaction (Newton's third law), friction. Conditions for equilibrium.

Lecture 4: Equilibrium of a rigid body
Resultant of several forces, couple, moment of a force. Conditions for equilibrium.

Lecture 5: Dynamics of particles
Newton's second law. Examples of pulleys, motion on an inclined plane.

Lecture 6: Dynamics of particles
Further examples, including motion of a projectile with air-resistance.

Lecture 7: Energy
Definition of energy and work. Kinetic energy, potential energy of a particle in a uniform gravitational field. Conservation of energy.

Lecture 8: Momentum
Definition of momentum (as a vector), conservation of momentum, collisions, coefficient of restitution, impulse.

Lecture 9: Springs, strings and SHM
Force exerted by elastic springs and strings (Hooke's law). Oscillations of a particle attached to a spring, and of a particle hanging on a string. Simple harmonic motion of a particle for small displacement from equilibrium.

Lecture 10: Motion in a circle
Derivation of the central acceleration of a particle constrained to move on a circle. Simple pendulum; motion of a particle sliding on a cylinder.

Appropriate books

J. Hebborn and J. Littlewood *Mechanics 1, Mechanics 2 and Mechanics 3 (Edexel).* Heinemann, 2000
Peter J O'Donnell *Essential Dynamics and Relativity.* CRC Press, 2014
Anything similar to the above, for the other A-level examination boards

CONCEPTS IN THEORETICAL PHYSICS (non-examinable) 8 lectures, Easter term

This course is intended to give a flavour of the some of the major topics in Theoretical Physics. It will be of interest to all students.

The list of topics below is intended only to give an idea of what might be lectured; the actual content will be announced in the first lecture.

Principle of Least Action
A better way to do Newtonian dynamics. Feynman's approach to quantum mechanics.

Quantum Mechanics
Principles of quantum mechanics. Probabilities and uncertainty. Entanglement.

Statistical Mechanics
More is different: $1 \neq 10^{24}$. Entropy and the Second Law. Information theory. Black hole entropy.

Electrodynamics and Relativity
Maxwell's equations. The speed of light and relativity. Spacetime. A hidden symmetry.

Particle Physics
A new periodic table. From fields to particles. From symmetries to forces. The origin of mass and the Higgs boson.

Symmetry
Symmetry of physical laws. Noether's theorem. From symmetries to forces.

General Relativity
Equivalence principle. Gravitational time dilation. Curved spacetime. Black holes. Gravity waves.

Cosmology
From quantum mechanics to galaxies.

Part IB

GENERAL ARRANGEMENTS

Structure of Part IB

Seventeen courses, including Computational Projects, are examined in Part IB. The schedules for Complex Analysis and Complex Methods cover much of the same material, but from different points of view: students may attend either (or both) sets of lectures. One course, Optimisation, can be taken in the Easter term of either the first year or the second year. Two other courses, Metric and Topological Spaces and Variational Principles, can also be taken in either Easter term, but it should be noted that some of the material in Metric and Topological Spaces will prove useful for Complex Analysis, and the material in Variational Principles forms a good background for many of the theoretical physics courses in Part IB.

The Faculty Board guidance regarding choice of courses in Part IB is as follows:

> *Part IB of the Mathematical Tripos provides a wide range of courses from which students should, in consultation with their Directors of Studies, make a selection based on their individual interests and preferred workload, bearing in mind that it is better to do a smaller number of courses thoroughly than to do many courses scrappily.* The table of dependencies on the next page may also help you with your choice.

Computational Projects

The lectures for Computational Projects will normally be attended in the Easter term of the first year, the Computational Projects themselves being done in the Michaelmas and Lent terms of the second year (or in the summer, Christmas and Easter vacations).

No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of reports. The maximum credit obtainable is 160 marks and there are no alpha or beta quality marks. Credit obtained is added directly to the credit gained on the written papers. The maximum contribution to the final merit mark is thus 160, which is roughly the same (averaging over the alpha weightings) as for a 16-lecture course. The Computational Projects are considered to be a single piece of work within the Mathematical Tripos.

Examination

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

Each of the four papers is divided into two sections. Candidates may attempt at most four questions from Section I and at most six questions from Section II.

The number of questions set on each course varies according to the number of lectures given, as shown:

Number of lectures	Section I	Section II
24	3	4
16	2	3
12	2	2

Examination Papers

Questions on the different courses are distributed among the papers as specified in the following table. The letters S and L appearing in the table denote a question in Section I and a question in Section II, respectively.

	Paper 1	Paper 2	Paper 3	Paper 4
Linear Algebra	L+S	L+S	L	L+S
Groups, Rings and Modules	L	L+S	L+S	L+S
Analysis II	L	L+S	L+S	L+S
Metric and Topological Spaces	L	S	S	L
Complex Analysis	L+S*	L*	L	S
Complex Methods			S	L
Geometry	S	L	L+S	L
Variational Principles	S	L	S	L
Methods	L	L+S	L+S	L+S
Quantum Mechanics	L	L	L+S	S
Electromagnetism	L	L+S	L	S
Fluid Dynamics	L+S	S	L	L
Numerical Analysis	L+S	L	L	S
Statistics	L+S	S	L	L
Optimization	S	S	L	L
Markov Chains	L	L	S	S

*On Paper 1 and Paper 2, Complex Analysis and Complex Methods are examined by means of common questions (each of which may contain two sub-questions, one on each course, of which candidates may attempt only one ('either/or')).

Approximate class boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines in recent years.

For convenience, we define M_1 and M_2 by

$$M_1 = 30\alpha + 5\beta + m - 120, \qquad M_2 = 15\alpha + 5\beta + m.$$

M_1 is related to the primary classification criterion for the first class and M_2 is related to the primary classification criterion for the upper and lower second and third classes.

The second column of each table shows a sufficient criterion for each class (in terms of M_1 for the first class and M_2 for the other classes). The third and fourth columns show M_1 (for the first class) or M_2 (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline.

The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

Part IB 2014			
Class	**Sufficient condition**	**Borderline candidates**	
1	$M_1 > 783$	784/494,12,10	789/474,13, 9
2.1	$M_2 > 480$	481/366, 4,11	484/364, 5, 9
2.2	$M_2 > 336$	337/232, 4, 9	345/225, 6, 6
3	$M_2 > 228$	229/199, 0, 6	255/185, 1,11

Part IB 2015			
Class	**Sufficient condition**	**Borderline candidates**	
1	$M_1 > 707$	708/458,11, 8	715/445,12, 6
2.1	$M_2 > 486$	487/332, 8, 7	487/377, 4,10
2.2	$M_2 > 354$	355/250, 4, 9	357/282, 3, 6
3	$M_2 > 227$	228/183, 1, 6	234/194, 1, 5

Part II dependencies

The relationships between Part IB courses and Part II courses are shown in the following tables. A blank in the table means that the material in the Part IB course is not directly relevant to the Part II course.

The terminology is as follows:

Essential: (E) a good understanding of the methods and results of the Part IB course is essential;

Desirable: (D) knowledge of some of the results of the Part IB course is required;

Background: (B) some knowledge of the Part IB course would provide a useful background.

	Linear Algebra	Groups, Rings and Modules	Analysis II	Metric and Topological Spaces	Complex Analysis	Complex Methods	Geometry	Variational Principles	Methods	Quantum Mechanics	Electromagnetism	Fluid Dynamics	Numerical Analysis	Statistics	Optimisation	Markov Chains
Number Theory																
Topics in Analysis			B	B												
Coding and Cryptography	D	E														
Automata and Form. Lang.																
Statistical Modelling														E		
Mathematical Biology																
Further Complex Methods						E										
Classical Dynamics								E								
Cosmology																
Logic and Set Theory																
Graph Theory																
Galois Theory	D	E														
Representation Theory	E	E														
Number Fields		E		D												
Algebraic Topology			E	D												
Linear Analysis	E		E	E												
Riemann Surfaces			D		E											
Algebraic Geometry		E														
Differential Geometry			E				D									
Prob. and Measure			E													
Applied Prob.																E
Princ. of Stats														E		
Stochastic FM's									D					D		D
Opt. and Control									D						D	D
Asymptotic Methods						E			D							
Dynamical Systems																
Integrable Systems						D			E							
Principles of QM									D	E						
Applications of QM									B	E						
Statistical Physics										E						
Electrodynamics									D		E					
General Relativity								D	D							
Fluid Dynamics II									E			E				
Waves									E			D				
Numerical Analysis	D		D			D							E			

LINEAR ALGEBRA

24 lectures, Michaelmas term

Definition of a vector space (over $\mathbb{R}$ or $\mathbb{C}$), subspaces, the space spanned by a subset. Linear independence, bases, dimension. Direct sums and complementary subspaces. [3]

Linear maps, isomorphisms. Relation between rank and nullity. The space of linear maps from U to V, representation by matrices. Change of basis. Row rank and column rank. [4]

Determinant and trace of a square matrix. Determinant of a product of two matrices and of the inverse matrix. Determinant of an endomorphism. The adjugate matrix. [3]

Eigenvalues and eigenvectors. Diagonal and triangular forms. Characteristic and minimal polynomials. Cayley-Hamilton Theorem over $\mathbb{C}$. Algebraic and geometric multiplicity of eigenvalues. Statement and illustration of Jordan normal form. [4]

Dual of a finite-dimensional vector space, dual bases and maps. Matrix representation, rank and determinant of dual map [2]

Bilinear forms. Matrix representation, change of basis. Symmetric forms and their link with quadratic forms. Diagonalisation of quadratic forms. Law of inertia, classification by rank and signature. Complex Hermitian forms. [4]

Inner product spaces, orthonormal sets, orthogonal projection, $V = W \oplus W^{\perp}$. Gram-Schmidt orthogonalisation. Adjoints. Diagonalisation of Hermitian matrices. Orthogonality of eigenvectors and properties of eigenvalues. [4]

Appropriate books

C.W. Curtis *Linear Algebra: an introductory approach.* Springer 1984
P.R. Halmos *Finite-dimensional vector spaces.* Springer 1974
K. Hoffman and R. Kunze *Linear Algebra.* Prentice-Hall 1971

GROUPS, RINGS AND MODULES

24 lectures, Lent term

Groups
Basic concepts of group theory recalled from Part IA Groups. Normal subgroups, quotient groups and isomorphism theorems. Permutation groups. Groups acting on sets, permutation representations. Conjugacy classes, centralizers and normalizers. The centre of a group. Elementary properties of finite p-groups. Examples of finite linear groups and groups arising from geometry. Simplicity of A_n.

Sylow subgroups and Sylow theorems. Applications, groups of small order. [8]

Rings
Definition and examples of rings (commutative, with 1). Ideals, homomorphisms, quotient rings, isomorphism theorems. Prime and maximal ideals. Fields. The characteristic of a field. Field of fractions of an integral domain.

Factorization in rings; units, primes and irreducibles. Unique factorization in principal ideal domains, and in polynomial rings. Gauss' Lemma and Eisenstein's irreducibility criterion.

Rings $\mathbb{Z}[\alpha]$ of algebraic integers as subsets of $\mathbb{C}$ and quotients of $\mathbb{Z}[x]$. Examples of Euclidean domains and uniqueness and non-uniqueness of factorization. Factorization in the ring of Gaussian integers; representation of integers as sums of two squares.

Ideals in polynomial rings. Hilbert basis theorem. [10]

Modules
Definitions, examples of vector spaces, abelian groups and vector spaces with an endomorphism. Submodules, homomorphisms, quotient modules and direct sums. Equivalence of matrices, canonical form. Structure of finitely generated modules over Euclidean domains, applications to abelian groups and Jordan normal form. [6]

Appropriate books

P.M.Cohn *Classic Algebra.* Wiley, 2000
P.J. Cameron *Introduction to Algebra.* OUP
J.B. Fraleigh *A First Course in Abstract Algebra.* Addison Wesley, 2003
B. Hartley and T.O. Hawkes *Rings, Modules and Linear Algebra: a further course in algebra.* Chapman and Hall, 1970
I. Herstein *Topics in Algebra.* John Wiley and Sons, 1975
P.M. Neumann, G.A. Stoy and E.C. Thomson *Groups and Geometry.* OUP 1994
M. Artin *Algebra.* Prentice Hall, 1991

ANALYSIS II

24 lectures, Michaelmas term

Uniform convergence
The general principle of uniform convergence. A uniform limit of continuous functions is continuous. Uniform convergence and termwise integration and differentiation of series of real-valued functions. Local uniform convergence of power series. [3]

Uniform continuity and integration
Continuous functions on closed bounded intervals are uniformly continuous. Review of basic facts on Riemann integration (from Analysis I). Informal discussion of integration of complex-valued and $\mathbb{R}^n$-valued functions of one variable; proof that $\| \int_a^b f(x)\, dx\| \leq \int_a^b \|f(x)\|\, dx$. [2]

$\mathbb{R}^n$ as a normed space
Definition of a normed space. Examples, including the Euclidean norm on $\mathbb{R}^n$ and the uniform norm on $\mathcal{C}[a,b]$. Lipschitz mappings and Lipschitz equivalence of norms. The Bolzano–Weierstrass theorem in $\mathbb{R}^n$. Completeness. Open and closed sets. Continuity for functions between normed spaces. A continuous function on a closed bounded set in $\mathbb{R}^n$ is uniformly continuous and has closed bounded image. All norms on a finite-dimensional space are Lipschitz equivalent. [5]

Differentiation from $\mathbb{R}^m$ to $\mathbb{R}^n$
Definition of derivative as a linear map; elementary properties, the chain rule. Partial derivatives; continuous partial derivatives imply differentiability. Higher-order derivatives; symmetry of mixed partial derivatives (assumed continuous). Taylor's theorem. The mean value inequality. Path-connectedness for subsets of $\mathbb{R}^n$; a function having zero derivative on a path-connected open subset is constant. [6]

Metric spaces
Definition and examples. *Metrics used in Geometry*. Limits, continuity, balls, neighbourhoods, open and closed sets. [4]

The Contraction Mapping Theorem
The contraction mapping theorem. Applications including the inverse function theorem (proof of continuity of inverse function, statement of differentiability). Picard's solution of differential equations. [4]

Appropriate books

† J.C. Burkill and H. Burkill *A Second Course in Mathematical Analysis.* Cambridge University Press 2002
A.F. Beardon *Limits: A New Approach to Real Analysis.* Springer 1997
D.J.H.Garling *A Course in Mathematical Analysis (Vol 3).* Cambridge University Press 2014
† W. Rudin *Principles of Mathematical Analysis.* McGraw–Hill 1976
W.A. Sutherland *Introduction to Metric and Topological Spaces.* Clarendon 1975
A.J. White *Real Analysis: An Introduction.* Addison–Wesley 1968
T.W. Körner *A companion to analysis.* AMS, 2004

METRIC AND TOPOLOGICAL SPACES

12 lectures, Easter term

Metrics
Definition and examples. Limits and continuity. Open sets and neighbourhoods. Characterizing limits and continuity using neighbourhoods and open sets. [3]

Topology
Definition of a topology. Metric topologies. Further examples. Neighbourhoods, closed sets, convergence and continuity. Hausdorff spaces. Homeomorphisms. Topological and non-topological properties. Completeness. Subspace, quotient and product topologies. [3]

Connectedness
Definition using open sets and integer-valued functions. Examples, including intervals. Components. The continuous image of a connected space is connected. Path-connectedness. Path-connected spaces are connected but not conversely. Connected open sets in Euclidean space are path-connected. [3]

Compactness
Definition using open covers. Examples: finite sets and $[0,1]$. Closed subsets of compact spaces are compact. Compact subsets of a Hausdorff space must be closed. The compact subsets of the real line. Continuous images of compact sets are compact. Quotient spaces. Continuous real-valued functions on a compact space are bounded and attain their bounds. The product of two compact spaces is compact. The compact subsets of Euclidean space. Sequential compactness. [3]

Appropriate books

† W.A. Sutherland *Introduction to metric and topological spaces.* Clarendon 1975
D.J.H.Garling *A Course in Mathematical Analysis (Vol 2).* Cambridge University Press 2013
A.J. White *Real analysis: an introduction.* Addison-Wesley 1968
B. Mendelson *Introduction to Topology.* Dover, 1990

COMPLEX ANALYSIS — 16 lectures, Lent term

Analytic functions
Complex differentiation and the Cauchy-Riemann equations. Examples. Conformal mappings. Informal discussion of branch points, examples of $\log z$ and z^c. [3]

Contour integration and Cauchy's theorem
Contour integration (for piecewise continuously differentiable curves). Statement and proof of Cauchy's theorem for star domains. Cauchy's integral formula, maximum modulus theorem, Liouville's theorem, fundamental theorem of algebra. Morera's theorem. [5]

Expansions and singularities
Uniform convergence of analytic functions; local uniform convergence. Differentiability of a power series. Taylor and Laurent expansions. Principle of isolated zeros. Residue at an isolated singularity. Classification of isolated singularities. [4]

The residue theorem
Winding numbers. Residue theorem. Jordan's lemma. Evaluation of definite integrals by contour integration. Rouché's theorem, principle of the argument. Open mapping theorem. [4]

Appropriate books

L.V. Ahlfors *Complex Analysis.* McGraw–Hill 1978
† A.F. Beardon *Complex Analysis.* Wiley
D.J.H.Garling *A Course in Mathematical Analysis (Vol 3).* Cambridge University Press 2014
† H.A. Priestley *Introduction to Complex Analysis.* Oxford University Press 2003
I. Stewart and D. Tall *Complex Analysis.* Cambridge University Press 1983

COMPLEX METHODS — 16 lectures, Lent term

Analytic functions
Definition of an analytic function. Cauchy-Riemann equations. Analytic functions as conformal mappings; examples. Application to the solutions of Laplace's equation in various domains. Discussion of $\log z$ and z^a. [5]

Contour integration and Cauchy's Theorem
[*Proofs of theorems in this section will not be examined in this course.*]
Contours, contour integrals. Cauchy's theorem and Cauchy's integral formula. Taylor and Laurent series. Zeros, poles and essential singularities. [3]

Residue calculus
Residue theorem, calculus of residues. Jordan's lemma. Evaluation of definite integrals by contour integration. [4]

Fourier and Laplace transforms
Laplace transform: definition and basic properties; inversion theorem (proof not required); convolution theorem. Examples of inversion of Fourier and Laplace transforms by contour integration. Applications to differential equations. [4]

Appropriate books

M.J. Ablowitz and A.S. Fokas *Complex variables: introduction and applications.* CUP 2003
G. Arfken and H. Weber *Mathematical Methods for Physicists.* Harcourt Academic 2001
G. J. O. Jameson *A First Course in Complex Functions.* Chapman and Hall 1970
T. Needham *Visual complex analysis.* Clarendon 1998
† H.A. Priestley *Introduction to Complex Analysis.* Clarendon 1990
† I. Stewart and D. Tall *Complex Analysis (the hitchhiker's guide to the plane).* Cambridge University Press 1983

GEOMETRY — 16 lectures, Lent term

Parts of Analysis II will be found useful for this course.

Groups of rigid motions of Euclidean space. Rotation and reflection groups in two and three dimensions. Lengths of curves. [2]

Spherical geometry: spherical lines, spherical triangles and the Gauss-Bonnet theorem. Stereographic projection and Möbius transformations. [3]

Triangulations of the sphere and the torus, Euler number. [1]

Riemannian metrics on open subsets of the plane. The hyperbolic plane. Poincaré models and their metrics. The isometry group. Hyperbolic triangles and the Gauss-Bonnet theorem. The hyperboloid model. [4]

Embedded surfaces in $\mathbb{R}^3$. The first fundamental form. Length and area. Examples. [1]

Length and energy. Geodesics for general Riemannian metrics as stationary points of the energy. First variation of the energy and geodesics as solutions of the corresponding Euler-Lagrange equations. Geodesic polar coordinates (informal proof of existence). Surfaces of revolution. [2]

The second fundamental form and Gaussian curvature. For metrics of the form $du^2 + G(u,v)dv^2$, expression of the curvature as $\sqrt{G}_{uu}/\sqrt{G}$. Abstract smooth surfaces and isometries. Euler numbers and statement of Gauss-Bonnet theorem, examples and applications. [3]

Appropriate books

† P.M.H. Wilson *Curved Spaces.* CUP, January 2008
M. Do Carmo *Differential Geometry of Curves and Surfaces.* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1976
A. Pressley *Elementary Differential Geometry.* Springer Undergraduate Mathematics Series, Springer-Verlag London Ltd., 2001
E. Rees *Notes on Geometry.* Springer, 1983
M. Reid and B. Szendroi *Geometry and Topology.* CUP, 2005

VARIATIONAL PRINCIPLES — 12 lectures, Easter Term

Stationary points for functions on $\mathbb{R}^n$. Necessary and sufficient conditions for minima and maxima. Importance of convexity. Variational problems with constraints; method of Lagrange multipliers. The Legendre Transform; need for convexity to ensure invertibility; illustrations from thermodynamics. [4]

The idea of a functional and a functional derivative. First variation for functionals, Euler-Lagrange equations, for both ordinary and partial differential equations. Use of Lagrange multipliers and multiplier functions. [3]

Fermat's principle; geodesics; least action principles, Lagrange's and Hamilton's equations for particles and fields. Noether theorems and first integrals, including two forms of Noether's theorem for ordinary differential equations (energy and momentum, for example). Interpretation in terms of conservation laws. [3]

Second variation for functionals; associated eigenvalue problem. [2]

Appropriate books

D.S. Lemons *Perfect Form.* Princeton Unversity Press 1997
C. Lanczos *The Variational Principles of Mechanics.* Dover 1986
R. Weinstock *Calculus of Variations with applications to physics and engineering.* Dover 1974
I.M. Gelfand and S.V. Fomin *Calculus of Variations.* Dover 2000
W. Yourgrau and S. Mandelstam *Variational Principles in Dynamics and Quantum Theory.* Dover 2007
S. Hildebrandt and A. Tromba *Mathematics and Optimal Form.* Scientific American Library 1985

METHODS

24 lectures, Michaelmas term

Self-adjoint ODEs

Periodic functions. Fourier series: definition and simple properties; Parseval's theorem. Equations of second order. Self-adjoint differential operators. The Sturm-Liouville equation; eigenfunctions and eigenvalues; reality of eigenvalues and orthogonality of eigenfunctions; eigenfunction expansions (Fourier series as prototype), approximation in mean square, statement of completeness. [5]

PDEs on bounded domains: separation of variables

Physical basis of Laplace's equation, the wave equation and the diffusion equation. General method of separation of variables in Cartesian, cylindrical and spherical coordinates. Legendre's equation: derivation, solutions including explicit forms of P_0, P_1 and P_2, orthogonality. Bessel's equation of integer order as an example of a self-adjoint eigenvalue problem with non-trivial weight.

Examples including potentials on rectangular and circular domains and on a spherical domain (axisymmetric case only), waves on a finite string and heat flow down a semi-infinite rod. [5]

Inhomogeneous ODEs: Green's functions

Properties of the Dirac delta function. Initial value problems and forced problems with two fixed end points; solution using Green's functions. Eigenfunction expansions of the delta function and Green's functions. [4]

Fourier transforms

Fourier transforms: definition and simple properties; inversion and convolution theorems. The discrete Fourier transform. Examples of application to linear systems. Relationship of transfer function to Green's function for initial value problems. [4]

PDEs on unbounded domains

Classification of PDEs in two independent variables. Well posedness. Solution by the method of characteristics. Green's functions for PDEs in 1, 2 and 3 independent variables; fundamental solutions of the wave equation, Laplace's equation and the diffusion equation. The method of images. Application to the forced wave equation, Poisson's equation and forced diffusion equation. Transient solutions of diffusion problems: the error function. [6]

Appropriate books

G. Arfken and H.J. Weber *Mathematical Methods for Physicists.* Academic 2005
M.L. Boas *Mathematical Methods in the Physical Sciences.* Wiley 2005
J. Mathews and R.L. Walker *Mathematical Methods of Physics.* Benjamin/Cummings 1970
K. F. Riley, M. P. Hobson, and S.J. Bence *Mathematical Methods for Physics and Engineering: a comprehensive guide.* Cambridge University Press 2002
Erwin Kreyszig *Advanced Engineering Mathematics.* Wiley

QUANTUM MECHANICS

16 lectures, Michaelmas term

Physical background

Photoelectric effect. Electrons in atoms and line spectra. Particle diffraction. [1]

Schrödinger equation and solutions

De Broglie waves. Schrödinger equation. Superposition principle. Probability interpretation, density and current. [2]

Stationary states. Free particle, Gaussian wave packet. Motion in 1-dimensional potentials, parity. Potential step, square well and barrier. Harmonic oscillator. [4]

Observables and expectation values

Position and momentum operators and expectation values. Canonical commutation relations. Uncertainty principle. [2]

Observables and Hermitian operators. Eigenvalues and eigenfunctions. Formula for expectation value. [2]

Hydrogen atom

Spherically symmetric wave functions for spherical well and hydrogen atom.

Orbital angular momentum operators. General solution of hydrogen atom. [5]

Appropriate books

Feynman, Leighton and Sands *vol. 3 Ch 1-3 of the Feynman lectures on Physics.* Addison-Wesley 1970
† S. Gasiorowicz *Quantum Physics.* Wiley 2003
P.V. Landshoff, A.J.F. Metherell and W.G Rees *Essential Quantum Physics.* Cambridge University Press 1997
† A.I.M. Rae *Quantum Mechanics.* Institute of Physics Publishing 2002
L.I. Schiff *Quantum Mechanics.* McGraw Hill 1968

ELECTROMAGNETISM 16 lectures, Lent term

Electrostatics
Currents and the conservation of charge. Lorentz force law and Maxwell's equations. Gauss's law. Application to spherically symmetric and cylindrically symmetric charge distributions. Point, line and surface charges. Electrostatic potentials; general charge distributions, dipoles. Electrostatic energy. Conductors. [4]

Magnetostatics
Magnetic fields due to steady currents. Ampère's law. Simple examples. Vector potentials and the Biot–Savart law for general current distributions. Magnetic dipoles. Lorentz force on current distributions and force between current-carrying wires. [3]

Electrodynamics
Faraday's law of induction for fixed and moving circuits. Ohm's law. Plane electromagnetic waves in vacuum, polarization. Electromagnetic energy and Poynting vector. [4]

Electromagnetism and relativity
Review of special relativity; tensors and index notation. Charge conservation. 4-vector potential, gauge transformations. Electromagnetic tensor. Lorentz transformations of electric and magnetic fields. Maxwell's equations in relativistic form. Lorentz force law. [5]

Appropriate books

D.J. Griffiths *Introduction to Electrodynamics.* Pearson 2013
E.M. Purcell and D.J. Morin *Electricity and Magnetism.* Cambridge University Press 2013
A. Zangwill *Modern Electromagnetism.* Cambridge University Press 2012
J.D. Jackson *Classical Electrodynamics.* Wiley 1975
P. Lorrain and D. Corson *Electromagnetism, Principles and Applications.* Freeman 1990
R. Feynman, R. Leighton and M. Sands *The Feynman Lectures on Physics, Vol 2.* Basic Books 2011

FLUID DYNAMICS 16 lectures, Lent term

Parallel viscous flow
Plane Couette flow, dynamic viscosity. Momentum equation and boundary conditions. Steady flows including Poiseuille flow in a channel. Unsteady flows, kinematic viscosity, brief description of viscous boundary layers (skin depth). [3]

Kinematics
Material time derivative. Conservation of mass and the kinematic boundary condition. Incompressibility; streamfunction for two-dimensional flow. Streamlines and path lines. [2]

Dynamics
Statement of Navier-Stokes momentum equation. Reynolds number. Stagnation-point flow; discussion of viscous boundary layer and pressure field. Conservation of momentum; Euler momentum equation. Bernoulli's equation.

Vorticity, vorticity equation, vortex line stretching, irrotational flow remains irrotational. [4]

Potential flows
Velocity potential; Laplace's equation, examples of solutions in spherical and cylindrical geometry by separation of variables. Translating sphere. Lift on a cylinder with circulation.

Expression for pressure in time-dependent potential flows with potential forces. Oscillations in a manometer and of a bubble. [3]

Geophysical flows
Linear water waves: dispersion relation, deep and shallow water, standing waves in a container, Rayleigh-Taylor instability.

Euler equations in a rotating frame. Steady geostrophic flow, pressure as streamfunction. Motion in a shallow layer, hydrostatic assumption, modified continuity equation. Conservation of potential vorticity, Rossby radius of deformation. [4]

Appropriate books

† D.J. Acheson *Elementary Fluid Dynamics.* Oxford University Press 1990
G.K. Batchelor *An Introduction to Fluid Dynamics.* Cambridge University Press 2000
G.M. Homsey et al. *Multi-Media Fluid Mechanics.* Cambridge University Press 2008
M. van Dyke *An Album of Fluid Motion.* Parabolic Press
M.G. Worster *Understanding Fluid Flow.* Cambridge University Press 2009

NUMERICAL ANALYSIS 16 lectures, Lent term

Polynomial approximation
Interpolation by polynomials. Divided differences of functions and relations to derivatives. Orthogonal polynomials and their recurrence relations. Least squares approximation by polynomials. Gaussian quadrature formulae. Peano kernel theorem and applications. [6]

Computation of ordinary differential equations
Euler's method and proof of convergence. Multistep methods, including order, the root condition and the concept of convergence. Runge-Kutta schemes. Stiff equations and A-stability. [5]

Systems of equations and least squares calculations
LU triangular factorization of matrices. Relation to Gaussian elimination. Column pivoting. Factorizations of symmetric and band matrices. The Newton-Raphson method for systems of non-linear algebraic equations. QR factorization of rectangular matrices by Gram–Schmidt, Givens and Householder techniques. Application to linear least squares calculations. [5]

Appropriate books

† S.D. Conte and C. de Boor *Elementary Numerical Analysis: an algorithmic approach.* McGraw–Hill 1980
G.H. Golub and C. Van Loan *Matrix Computations.* Johns Hopkins University Press 1996
A Iserles *A first course in the Numerical Analysis of Differential Equations.* CUP 2009
E. Suli and D.F. Meyers *An introduction to numerical analysis.* CUP 2003
A. Ralston and P. Rabinowitz *A first course in numerical analysis.* Dover 2001
M.J.D. Powell *Approximation Theory and Methods.* CUP 1981
P.J. Davis *Interpolation and Approximation.* Dover 1975

STATISTICS 16 lectures, Lent term

Estimation
Review of distribution and density functions, parametric families. Examples: binomial, Poisson, gamma. Sufficiency, minimal sufficiency, the Rao–Blackwell theorem. Maximum likelihood estimation. Confidence intervals. Use of prior distributions and Bayesian inference. [5]

Hypothesis testing
Simple examples of hypothesis testing, null and alternative hypothesis, critical region, size, power, type I and type II errors, Neyman–Pearson lemma. Significance level of outcome. Uniformly most powerful tests. Likelihood ratio, and use of generalised likelihood ratio to construct test statistics for composite hypotheses. Examples, including t-tests and F-tests. Relationship with confidence intervals. Goodness-of-fit tests and contingency tables. [4]

Linear models
Derivation and joint distribution of maximum likelihood estimators, least squares, Gauss-Markov theorem. Testing hypotheses, geometric interpretation. Examples, including simple linear regression and one-way analysis of variance. ∗Use of software∗. [7]

Appropriate books

D.A. Berry and B.W. Lindgren *Statistics, Theory and Methods.* Wadsworth 1995
G. Casella and J.O. Berger *Statistical Inference.* Duxbury 2001
M.H. DeGroot and M.J. Schervish *Probability and Statistics.* Pearson Education 2001

MARKOV CHAINS **12 lectures, Michaelmas term**

Discrete-time chains

Definition and basic properties, the transition matrix. Calculation of n-step transition probabilities. Communicating classes, closed classes, absorption, irreducibility. Calculation of hitting probabilities and mean hitting times; survival probability for birth and death chains. Stopping times and statement of the strong Markov property. [5]

Recurrence and transience; equivalence of transience and summability of n-step transition probabilities; equivalence of recurrence and certainty of return. Recurrence as a class property, relation with closed classes. Simple random walks in dimensions one, two and three. [3]

Invariant distributions, statement of existence and uniqueness up to constant multiples. Mean return time, positive recurrence; equivalence of positive recurrence and the existence of an invariant distribution. Convergence to equilibrium for irreducible, positive recurrent, aperiodic chains *and proof by coupling*. Long-run proportion of time spent in given state. [3]

Time reversal, detailed balance, reversibility; random walk on a graph. [1]

Appropriate books

G.R. Grimmett and D.R. Stirzaker *Probability and Random Processes.* OUP 2001
G.R. Grimmett and D. Welsh *Probability, An Introduction.* OUP, 2nd edition, 2014
J.R. Norris *Markov Chains.* CUP 1997

OPTIMISATION **12 lectures, Easter term**

Lagrangian methods

General formulation of constrained problems; the Lagrangian sufficiency theorem. Interpretation of Lagrange multipliers as shadow prices. Examples. [2]

Linear programming in the nondegenerate case

Convexity of feasible region; sufficiency of extreme points. Standardization of problems, slack variables, equivalence of extreme points and basic solutions. The primal simplex algorithm, artificial variables, the two-phase method. Practical use of the algorithm; the tableau. Examples. The dual linear problem, duality theorem in a standardized case, complementary slackness, dual variables and their interpretation as shadow prices. Relationship of the primal simplex algorithm to dual problem. Two person zero-sum games. [6]

Network problems

The Ford-Fulkerson algorithm and the max-flow min-cut theorems in the rational case. Network flows with costs, the transportation algorithm, relationship of dual variables with nodes. Examples. Conditions for optimality in more general networks; ∗the simplex-on-a-graph algorithm∗. [3]

Practice and applications

∗Efficiency of algorithms∗. The formulation of simple practical and combinatorial problems as linear programming or network problems. [1]

Appropriate books

† M.S. Bazaraa, J.J. Jarvis and H.D. Sherali *Linear Programming and Network Flows.* Wiley 1988
D. Luenberger *Linear and Nonlinear Programming.* Addison–Wesley 1984
S. Boyd and L. Vandenberghe *Convex Optimization.* Cambridge University Press 2004
D. Bertsimas, J.N. Tsitsiklis *Introduction to Linear Optimization.* Athena Scientific 1997

COMPUTATIONAL PROJECTS — 8 lectures, Easter term of Part IA

Practical sessions are offered and lectures given in the Part IA year. The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July preceding the Part IB year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at http://www.maths.cam.ac.uk/undergrad/catam/. Full credit may obtained from the submission of the two core projects and a further two additional projects. Once the manual is available, these projects may be undertaken at any time up to the submission deadlines, which are near the start of the Full Lent Term in the IB year for the two core projects, and near the start of the Full Easter Term in the IB year for the two additional projects.

A list of suitable books can be found in the CATAM manual.

CONCEPTS IN THEORETICAL PHYSICS (non-examinable) — 8 lectures, Easter term

This course is intended to give a flavour of the some of the major topics in Theoretical Physics. It will be of interest to all students.

The list of topics below is intended only to give an idea of what might be lectured; the actual content will be announced in the first lecture.

Principle of Least Action
A better way to do Newtonian dynamics. Feynman's approach to quantum mechanics.

Quantum Mechanics
Principles of quantum mechanics. Probabilities and uncertainty. Entanglement.

Statistical Mechanics
More is different: $1 \neq 10^{24}$. Entropy and the Second Law. Information theory. Black hole entropy.

Electrodynamics and Relativity
Maxwell's equations. The speed of light and relativity. Spacetime. A hidden symmetry.

Particle Physics
A new periodic table. From fields to particles. From symmetries to forces. The origin of mass and the Higgs boson.

Symmetry
Symmetry of physical laws. Noether's theorem. From symmetries to forces.

General Relativity
Equivalence principle. Gravitational time dilation. Curved spacetime. Black holes. Gravity waves.

Cosmology
From quantum mechanics to galaxies.

Part II

GENERAL ARRANGEMENTS

Structure of Part II

There are two types of lecture courses in Part II: C courses and D courses. C courses are intended to be straightforward and accessible, and of general interest, whereas D courses are intended to be more demanding. The Faculty Board recommend that students who have not obtained at least a good second class in Part IB should include a significant number of C courses amongst those they choose.

There are 9 C courses and 26 D courses. All C courses are 24 lectures; of the D courses, 20 are 24 lectures and 6 are 16 lectures. The complete list of courses is as follows (an asterisk denotes a 16-lecture course):

C courses	**D courses**	
Number Theory	Logic and Set Theory	Stochastic Financial Models
Topics in Analysis	Graph Theory	*Optimisation and Control
Coding and Cryptography	Galois Theory	*Asymptotic Methods
Automata and Formal Lang.	Representation Theory	Dynamical Systems
Statistical Modelling	*Number Fields	*Integrable Systems
Mathematical Biology	Algebraic Topology	Principles of Quantum Mechanics
Further Complex Methods	Linear Analysis	Applications of Quantum Mechanics
Classical Dynamics	*Riemann Surfaces	Statistical Physics
Cosmology	Algebraic Geometry	*Electrodynamics
	Differential Geometry	General Relativity
	Probability and Measure	Fluid Dynamics
	Applied Probability	Waves
	Principles of Statistics	Numerical Analysis

Computational Projects

In addition to the lectured courses, there is a Computational Projects course.

No questions on the Computational Projects are set on the written examination papers, credit for examination purposes being gained by the submission of reports. The maximum credit obtainable is 150 marks and there are no alpha or beta quality marks. Credit obtained is added directly to the credit gained on the written papers. The maximum contribution to the final merit mark is thus 150, which is the same as the maximum for a 16-lecture course. The Computational Projects are considered to be a single piece of work within the Mathematical Tripos.

Examinations

Arrangements common to all examinations of the undergraduate Mathematical Tripos are given on pages 1 and 2 of this booklet.

There are no restrictions on the number or type of courses that may be presented for examination, but examiners may consider if most of the marks are obtained on only one or two courses. The Faculty Board has recommended to the examiners that no distinction be made, for classification purposes, between quality marks obtained on the Section II questions for C course questions and those obtained for D course questions.

Candidates may answer no more than six questions in Section I on each paper; there is no restriction on the number of questions in Section II that may be answered.

The number of questions set on each course is determined by the type and length of the course, as shown in the following table:

	Section I	Section II
C course, 24 lectures	4	2
D course, 24 lectures	—	4
D course, 16 lectures	—	3

In Section I of each paper, there are 9 questions, one on each C course.

In Section II of each paper, there are 4 or 5 questions on C courses, one question on each of the 20 24-lecture D courses and either one question or no questions on each of the 6 16-lecture D courses, giving a total of 29 questions on each paper.

The distribution in Section II of the C course questions and the 16-lecture D course questions is shown in the following table. (**Important note:** Prior to 2015/16 the questions on Topics in Analysis and Asymptotic Methods appeared on different papers from those shown here.)

C courses	P1	P2	P3	P4
Number Theory			*	*
Topics in Analysis		*		*
Coding and Cryptography	*	*		
Automata and Form. Lang.	*		*	
Statistical Modelling	*			*
Mathematical Biology			*	*
Further Complex Methods	*	*		
Classical Dynamics		*		*
Cosmology	*		*	

16-lecture D courses	P1	P2	P3	P4
Number Fields	*	*		*
Riemann Surfaces	*	*	*	
Optimization and Control		*	*	*
Asymptotic Methods		*	*	*
Integrable Systems	*	*	*	
Electrodynamics	*		*	*

Approximate class boundaries

The following tables, based on information supplied by the examiners, show approximate borderlines in recent years.

For convenience, we define M_1 and M_2 by

$$M_1 = 30\alpha + 5\beta + m - 120, \qquad M_2 = 15\alpha + 5\beta + m.$$

M_1 is related to the primary classification criterion for the first class and M_2 is related to the primary classification criterion for the upper and lower second and third classes.

The second column of each table shows a sufficient criterion for each class (in terms of M_1 for the first class and M_2 for the other classes). The third and fourth columns show M_1 (for the first class) or M_2 (for the other classes), raw mark, number of alphas and number of betas of two representative candidates placed just above the borderline.

The sufficient condition for each class is not prescriptive: it is just intended to be helpful for interpreting the data. Each candidate near a borderline is scrutinised individually. The data given below are relevant to one year only; borderlines may go up or down in future years.

Part II 2014			
Class	**Sufficient condition**	**Borderline candidates**	
1	$M_1 > 623$	637/407,11, 4	624/364,12, 4
2.1	$M_2 > 410$	406/296, 5, 7	398/258, 8, 4
2.2	$M_2 > 253$	264/164, 4, 8	254/194, 2, 6
3	$M_2 > 155$	162/117, 1, 6	156/131, 0, 5

Part II 2015			
Class	**Sufficient condition**	**Borderline candidates**	
1	$M_1 > 728$	734/444,12,10	729/374,15, 5
2.1	$M_2 > 467$	468/313, 8, 7	473/303, 9, 7
2.2	$M_2 > 345$	346/236, 4,10	355/220, 7, 6
3	$M_2 > 169$	188/133, 2, 5	206/171, 1, 4

NUMBER THEORY (C) — 24 lectures, Michaelmas term

Review from Part IA Numbers and Sets: Euclid's Algorithm, prime numbers, fundamental theorem of arithmetic. Congruences. The theorems of Fermat and Euler. [2]

Chinese remainder theorem. Lagrange's theorem. Primitive roots to an odd prime power modulus. [3]

The mod-p field, quadratic residues and non-residues, Legendre's symbol. Euler's criterion. Gauss' lemma, quadratic reciprocity. [2]

Proof of the law of quadratic reciprocity. The Jacobi symbol. [1]

Binary quadratic forms. Discriminants. Standard form. Representation of primes. [5]

Distribution of the primes. Divergence of $\sum_p p^{-1}$. The Riemann zeta-function and Dirichlet series. Statement of the prime number theorem and of Dirichlet's theorem on primes in an arithmetic progression. Legendre's formula. Bertrand's postulate. [4]

Continued fractions. Pell's equation. [3]

Primality testing. Fermat, Euler and strong pseudo-primes. [2]

Factorization. Fermat factorization, factor bases, the continued-fraction method. Pollard's method. [2]

Appropriate books

A. Baker *A Concise Introduction to the Theory of Numbers.* Cambridge University Press 1984
Alan Baker *A Comprehensive Course in Number Theory.* Cambridge University Press 2012
G.H. Hardy and E.M. Wright *An Introduction to the Theory of Numbers.* Oxford University Press
N. Koblitz *A Course in Number Theory and Cryptography.* Springer 1994
T. Nagell *Introduction to Number Theory.* AMS
H. Davenport *The Higher Arithmetic.* Cambridge University Press

TOPICS IN ANALYSIS (C) — 24 lectures, Michaelmas term

Analysis courses from IB will be helpful, but it is intended to introduce and develop concepts of analysis as required.

Discussion of metric spaces; compactness and completeness. Brouwer's fixed point theorem. Proof(s) in two dimensions. Equivalent formulations, and applications. The degree of a map. The fundamental theorem of algebra, the Argument Principle for continuous functions, and a topological version of Rouché's Theorem. [6]

The Weierstrass Approximation Theorem. Chebychev polynomials and best uniform approximation. Gaussian quadrature converges for all continuous functions. Review of basic properties of analytic functions. Runge's Theorem on the polynomial approximation of analytic functions. [8]

Liouville's proof of the existence of transcendentals. The irrationality of e and π. The continued fraction expansion of real numbers; the continued fraction expansion of e. [4]

Review of countability, topological spaces, and the properties of compact Hausdorff spaces. The Baire category theorem for a complete metric space. Applications. [6]

Appropriate books

A.F. Beardon *Complex Analysis: the Argument Principle in Analysis and Topology.* John Wiley & Sons, 1979
E.W. Cheney *Introduction to Approximation Theory.* AMS, 1999
G.H. Hardy and E.M. Wright *An Introduction to the Theory of Numbers.* Clarendon Press, Oxford, fifth edition, reprinted 1989
T. Sheil-Small *Complex Polynomials.* Cambridge University. Press, 2002

CODING AND CRYPTOGRAPHY (C) — 24 lectures, Lent term

Part IB Linear Algebra is useful and Part IB Groups, Rings and Modules is very useful.

Introduction to communication channels, coding and channel capacity. [1]

Data compression; decipherability. Kraft's inequality. Huffman and Shannon-Fano coding. Shannon's noiseless coding theorem. [2]

Applications to gambling and the stock market. [1]

Codes, error detection and correction, Hamming distance. Examples: simple parity, repetition, Hamming's original [7,16] code. Maximum likelihood decoding. The Hamming and Gilbert-Shannon-Varshamov bounds. Shannon entropy. [4]

Information rate of a Bernoulli source. Capacity of a memoryless binary symmetric channel; Shannon's noisy coding theorem for such channels. [3]

Linear codes, weight, generator matrices, parity checks. Syndrome decoding. Dual codes. Examples: Hamming, Reed-Muller. [2]

Cyclic codes. Discussion without proofs of the abstract algebra (finite fields, polynomial rings and ideals) required. Generator and check polynomials. BCH codes. Error-locator polynomial and decoding. Recurrence (shift-register) sequences. Berlekamp-Massey algorithm. Secret sharing. [5]

Introduction to cryptography. Unicity distance, one-time pad. Shift-register based pseudo-random sequences. Linear complexity. Simple cipher machines. [2]

Secret sharing via simultaneous linear equations. Public-key cryptography. Secrecy and authentication. The RSA system. The discrete logarithm problem and Elgamal signatures (and the Digital Signature Algorithm). Bit commitment and coin tossing. [4]

Appropriate books

† G.M. Goldie and R.G.E. Pinch *Communication Theory* . Cambridge University Press 1991
D. Welsh *Codes and Cryptography.* OUP 1988
T.M. Cover and J.A. Thomas *Elements of Information Theory.* Wiley 1991
W. Trappe and L.C. Washington *Introduction to Cryptography with Coding Theory.* Prentice Hall, 2002

AUTOMATA AND FORMAL LANGUAGES (C) — 24 lectures, Michaelmas term

Part IA Numbers and Sets is essential.

Register machines
Definition of a register machine; recursive functions, recursively enumerable sets. Undecidability of the halting problem for register machines. Other undecidable problems: reduction and Rice's theorem. Post systems; the λ-calculus; Church's thesis. [8]

Regular languages and finite-state automata
Deterministic finite-state automata (DFA) and regular languages. Nondeterminism. Regular expressions. Limitations of finite-state automata: homomorphisms and closure properties; the pumping lemma; examples of non-regular languages. Minimization and the use of equivalence relations and quotients to construct minimal DFAs. Myhill–Nerode theorem (statement only). Two-way finite automata. [7]

Pushdown automata and context-free languages
Context-free grammars and context-free languages: generating languages by replacement rules. Normal forms for context-free languages, the Chomsky and Greibach normal forms; left- or right-linear grammars are equivalent to regular languages; regular languages are context-free. Limitations of context-free grammars: the pumping lemma for context-free languages; examples of non-context-free languages. Pushdown automata. [8]

Other topics
Statement and *informal discussion* of the P versus NP problem. *Turing machines*. [1]

Appropriate books

J.E. Hopcroft, R. Motwani and J.D. Ullman *Introduction to automata theory, languages and computation, 3rd edn.* Pearson 2006
P.T. Johnstone *Notes on logic and set theory (Chapter 4).* CUP 1987
† D.C. Kozen *Automata and computability.* Springer 1997
M. Sipser *Introduction to the theory of computation.* Wadsworth Publishing Co 2012
R.I. Soare *Recursively enumerable sets and degrees: a study of computable functions and computably generated sets.* Springer (Perspectives in mathematical logic) 1987

STATISTICAL MODELLING (C) — 24 lectures, Lent term

Part IB Statistics is essential. About two thirds of this course will be lectures, with the remaining hours as practical classes, using R in the CATAM system. R may be downloaded at no cost via http://cran.r-project.org

Introduction to the statistical package R
The use of R for graphical summaries of data, e.g. histograms, and for classical tests such as t-tests and F-tests. [2]

Review of basic inferential techniques
Asymptotic distribution of the maximum likelihood estimator. Approximate confidence intervals and hypothesis tests. Wilks' theorem. Posterior distributions and credible intervals. [3]

Linear models
Maximum likelihood, least squares, and projection matrices. Prediction. Multiple linear regression, analysis of variance, transformations of variables including Box-Cox and model selection. Orthogonality of sets of parameters. Factors, interactions and their interpretation. Leverages, residuals, qq-plots, multiple R^2 and Cook's distances. [5]

Exponential dispersion families and generalised linear models (glm)
Exponential families and mean-variance relationship. Dispersion parameter and generalised linear models. Canonical link function. Iterative solution of likelihood equations. Regression for binomial data; use of logit and other link functions. Poisson regression models, and their surrogate use for multinomial data. Application to 2- and 3-way contingency tables. Hypothesis tests and model selection, including deviance and Akaike's Information Criteria. Residuals and model checking. [8]

Examples in R
Linear and generalised linear models. Inference, model selection and criticism. [6]

Appropriate books

A.J.Dobson *An Introduction to Generalized Linear Models.* Chapman and Hall 2002

J.Faraway *Practical Regression and Anova in R.* http://cran.r-project.org/doc/contrib/Faraway-PRA.pdf

A.C. Davison *Statistical Models.* CUP 2008

MATHEMATICAL BIOLOGY (C) — 24 lectures, Lent term

Part II Dynamical Systems is useful.

Introduction to the role of mathematics in biology [1]

Systems without spatial structure: deterministic systems
Examples: population dynamics, epidemiology, chemical reactions, physiological systems.

Continuous and discrete population dynamics governed by deterministic ordinary differential equations or difference equations. Single population models: the logistic model and bifurcation to chaos; systems with time delay; age-structured populations. Two-species models: predator-prey interactions, competition, enzyme kinetics, infectious diseases. Phase-plane analysis, null-clines and stability of equilibrium. Systems exhibiting nonlinear oscillations: limit cycles; excitable systems. [9]

Stochastic systems
Discrete stochastic models of birth and death processes. Master equations and Fokker-Planck equations. The continuum limit and the importance of fluctuations. Comparison of deterministic and stochastic models, including implications for extinction/invasion. Simple random walk and derivation of the diffusion equation. [6]

Systems without spatial structure: diffusion and reaction-diffusion systems
The general transport equation. Fundamental solutions for steady and unsteady diffusion. Models with density-dependent diffusion. Fischer-Kolmogorov equation: propagation of reaction-diffusion waves. Chemotaxis and the growth of chemotactic instability. General conditions for diffusion-driven (Turing) instability: linear stability analysis and evolution of spatial pattern. [8]

Appropriate books

L. Edelstein-Keshet *Mathematical Models in Biology.* SIAM classics in applied mathematics reprint, 2005

J.D. Murray *Mathematical Biology (3rd edition), especially volume 1.* Springer, 2002

S.P. Ellner and J. Guckenheimer *Dynamic Models in Biology.* Princeton University Press, 2006

FURTHER COMPLEX METHODS (C) — 24 lectures, Lent term

Complex Methods (or Complex Analysis) is essential.

Complex variable

Revision of complex variable. Analyticity of a function defined by an integral (statement and discussion only). Analytic and meromorphic continuation.

Cauchy principal value of finite and infinite range improper integrals. The Hilbert transform. Kramers-Kronig relations.

Multivalued functions: definitions, branch points and cuts, integration; examples, including inverse trigonometric functions as integrals and elliptic integrals. [8]

Special functions

Gamma function: Euler integral definition; brief discussion of product formulae; Hankel representation; reflection formula; discussion of uniqueness (e.g. Wielandt's theorem). Beta function: Euler integral definition; relation to the gamma function. Riemann zeta function: definition as a sum; integral representations; functional equation; *discussion of zeros and relation to $\pi(x)$ and the distribution of prime numbers*. [6]

Differential equations by transform methods

Solution of differential equations by integral representation; Airy equation as an example. Solution of partial differential equations by transforms; the wave equation as an example. Causality. Nyquist stability criterion. [4]

Second order ordinary differential equations in the complex plane

Classification of singularities, exponents at a regular singular point. Nature of the solution near an isolated singularity by analytic continuation. Fuchsian differential equations. The Riemann P-function, hypergeometric functions and the hypergeometric equation, including brief discussion of monodromy. [6]

Appropriate books

† M.J. Ablowitz and A.S. Fokas *Complex Variables: Introduction and Applications.* CUP 2003
† E.T. Whittaker and G.N. Watson *A course of modern analysis.* CUP 1996
E.T. Copson *Functions of a Complex Variable.* Oxford University Press 1935
B. Spain and M.G. Smith *Functions of Mathematical Physics.* Van Nostrand 1970

CLASSICAL DYNAMICS (C) — 24 lectures, Michaelmas term

Part IB Variational Principles is essential.

Review of Newtonian mechanics

Newton's second law. Motion of N particles under mutual interactions. Euclidean and Galilean symmetry. Conservation laws of momentum, angular momentum and energy. [2]

Lagrange's equations

Configuration space, generalized coordinates and velocities. Holonomic constraints. Lagrangian, Hamilton's principle and Euler-Lagrange equations of motion. Examples: N particles with potential forces, planar and spherical pendulum, charged particle in a background electromagnetic field, purely kinetic Lagrangians and geodesics. Ignorable coordinates. Symmetries and Noether's theorem. [5]

Quadratic Lagrangians, oscillations, normal modes. [1]

Motion of a rigid body

Kinematics of a rigid body. Angular momentum, kinetic energy, diagonalization of inertia tensor. Euler top, conserved angular momentum, Euler equations and their solution in terms of elliptic integrals. Lagrange top, steady precession, nutation. [6]

Hamilton's equations

Phase space. Hamiltonian and Hamilton's equations. Simple examples. Poisson brackets, conserved quantities. Principle of least action. Liouville theorem. Action and angle variables for closed orbits in 2-D phase space. Adiabatic invariants (proof not required). Mention of completely integrable systems, and their action-angle variables. [7]

Hamiltonian systems in nonlinear phase spaces, e.g. classical spin in a magnetic field. 2-D motion of ideal point vortices. *Connections between Lagrangian/Hamiltonian dynamics and quantum mechanics.* [3]

Appropriate books

† L.D. Landau and E.M. Lifshitz *Mechanics.* Butterworth-Heinemann 2002
F. Scheck *Mechanics: from Newton's laws to deterministic chaos.* Springer 1999
L.N. Hand and J.D. Finch *Analytical Mechanics.* CUP 1999
H. Goldstein, C. Poole and J. Safko *Classical Mechanics.* Pearson 2002
V.I. Arnold *Mathematical methods of classical mechanics.* Springer 1978

COSMOLOGY (C)

24 lectures, Michaelmas term

Introduction

∗Astronomical terminology (parsecs, magnitudes). The matter and radiation distribution in the universe. Hierarchy of structures in the universe from nucleons to galaxy clusters. Dark matter and dark energy. The four forces of nature, brief mention of the standard model, quarks and leptons, and unification.∗ [2]

The expanding universe

The Cosmological Principle: homogeneity and isotropy. Derivation of Hubble's law. Scale factor of the universe. Hubble parameter. Kinematic effects, including redshift. [2]

Simple Newtonian account of an expanding fluid. The cosmological constant. Derivation of Friedmann and Raychaudhuri equations for perfect fluid models. Observational parameters: H, q, Ω. Possible worlds: open, closed, and flat models. Einstein, Friedmann, Lemaître and De Sitter universes. Age of the universe. Luminosity distance. The acceleration of the universe. Horizons. [4]

Statistical physics

Summary (only) of thermodynamics and statistical physics needed for the early universe: temperature and entropy, blackbody radiation, density of states, fermion and boson distributions for mass, number and entropy densities in classical and relativistic limits. [3]

Thermal history of the universe

Cosmic microwave background. Hydrogen recombination and photon decoupling. Helium, deuterium and primordial nucleosynthesis. Survival of massive particles (light and heavy neutrinos, hadrons). Conservation of entropy and particle number. Matter/antimatter asymmetry and the early universe. Inflation and the problems of the standard cosmology. Significance of the Planck time. The initial singularity. [8]

Origin of structure in the universe

Jeans instability. Definition of density perturbation. Derivation of cosmological perturbation equation. Simple growing mode solution. Non-zero pressure, damping of perturbations and the Jeans length. Non-linear dust collapse. ∗Anisotropies in the cosmic microwave background. Simple description of inflationary fluctuations and the constant curvature spectrum.∗ [5]

Appropriate books

J.D. Barrow *The Book of Universes.* Vintage 2012
E. Linder *First Principles of Cosmology.* Addison Wesley 1997
R. Belusevic *Relativity, Astrophysics and Cosmology 1st vol.* Wiley 2008
B. Ryden *Introduction to Cosmology.* Addison-Wesley 2003
E.Kolb and M.S.Turner *The Early Universe.* Addison-Wesley 1990 (Advanced text to consult for particular topics)
E.R. Harrison *Cosmology: The Science of the Universe.* CUP 2000

LOGIC AND SET THEORY (D)

24 lectures, Lent term

No specific prerequisites.

Ordinals and cardinals

Well-orderings and order-types. Examples of countable ordinals. Uncountable ordinals and Hartogs' lemma. Induction and recursion for ordinals. Ordinal arithmetic. Cardinals; the hierarchy of alephs. Cardinal arithmetic. [5]

Posets and Zorn's lemma

Partially ordered sets; Hasse diagrams, chains, maximal elements. Lattices and Boolean algebras. Complete and chain-complete posets; fixed-point theorems. The axiom of choice and Zorn's lemma. Applications of Zorn's lemma in mathematics. The well-ordering principle. [5]

Propositional logic

The propositional calculus. Semantic and syntactic entailment. The deduction and completeness theorems. Applications: compactness and decidability. [3]

Predicate logic

The predicate calculus with equality. Examples of first-order languages and theories. Statement of the completeness theorem; *sketch of proof*. The compactness theorem and the Löwenheim-Skolem theorems. Limitations of first-order logic. Model theory. [5]

Set theory

Set theory as a first-order theory; the axioms of ZF set theory. Transitive closures, epsilon-induction and epsilon-recursion. Well-founded relations. Mostowski's collapsing theorem. The rank function and the von Neumann hierarchy. [5]

Consistency

∗Problems of consistency and independence∗. [1]

Appropriate books

B.A. Davey and H.A. Priestley *Lattices and Order.* Cambridge University Press 2002
T. Forster *Logic, Induction and Sets.* Cambridge University Press
A. Hajnal and P. Hamburger *Set Theory.* LMS Student Texts number 48, CUP 1999
A.G. Hamilton *Logic for Mathematicians.* Cambridge University Press 1988
† P.T. Johnstone *Notes on Logic and Set Theory.* Cambridge University Press 1987
D. van Dalen *Logic and Structure.* Springer-Verlag 1994

GRAPH THEORY (D) — 24 lectures, Michaelmas term

No specific prerequisites.

Introduction
Basic definitions. Trees and spanning trees. Bipartite graphs. Euler circuits. Elementary properties of planar graphs. Statement of Kuratowski's theorem. [3]

Connectivity and matchings
Matchings in bipartite graphs; Hall's theorem and its variants. Connectivity and Menger's theorem. [3]

Extremal graph theory
Long paths, long cycles and Hamilton cycles. Complete subgraphs and Turán's theorem. Bipartite subgraphs and the problem of Zarankiewicz. The Erdős-Stone theorem; *sketch of proof*. [5]

Eigenvalue methods
The adjacency matrix and the Laplacian. Strongly regular graphs. [2]

Graph colouring
Vertex and edge colourings; simple bounds. The chromatic polynomial. The theorems of Brooks and Vizing. Equivalent forms of the four colour theorem; the five colour theorem. Heawood's theorem for surfaces; the torus and the Klein bottle. [5]

Ramsey theory
Ramsey's theorem (finite and infinite forms). Upper bounds for Ramsey numbers. [3]

Probabilistic methods
Basic notions; lower bounds for Ramsey numbers. The model $\mathcal{G}(n, p)$; graphs of large girth and large chromatic number. The clique number. [3]

Appropriate books

† B.Bollobás *Modern Graph Theory.* Springer 1998
R.Diestel *Graph Theory.* Springer 2000
D.West *Introduction to Graph Theory.* Prentice Hall 1999

GALOIS THEORY (D) — 24 lectures, Michaelmas term

Groups, Rings and Modules is essential.

Field extensions, tower law, algebraic extensions; irreducible polynomials and relation with simple algebraic extensions. Finite multiplicative subgroups of a field are cyclic. Existence and uniqueness of splitting fields. [6]

Existence and uniquness of algebraic closure. [1]

Separability. Theorem of primitive element. Trace and norm. [3]

Normal and Galois extensions, automorphic groups. Fundamental theorem of Galois theory. [3]

Galois theory of finite fields. Reduction mod p. [2]

Cyclotomic polynomials, Kummer theory, cyclic extensions. Symmetric functions. Galois theory of cubics and quartics. [4]

Solubility by radicals. Insolubility of general quintic equations and other classical problems. [3]

Artin's theorem on the subfield fixed by a finite group of automorphisms. Polynomial invariants of a finite group; examples. [2]

Appropriate books

E. Artin *Galois Theory.* Dover Publications
I. Stewart *Galois Theory.* Taylor & Francis Ltd Chapman & Hall/CRC 3rd edition
B. L. van der Waerden *Modern Algebra.* Ungar Pub 1949
S. Lang *Algebra (Graduate Texts in Mathematics).* Springer-Verlag New York Inc
I. Kaplansky *Fields and Rings.* The University of Chicago Press

REPRESENTATION THEORY (D) **24 lectures, Lent term**

Linear Algebra, and Groups, Rings and Modules are esssential.

Representations of finite groups
Representations of groups on vector spaces, matrix representations. Equivalence of representations. Invariant subspaces and submodules. Irreducibility and Schur's Lemma. Complete reducibility for finite groups. Irreducible representations of Abelian groups.

Character theory
Determination of a representation by its character. The group algebra, conjugacy classes, and orthogonality relations. Regular representation. Permutation representations and their characters. Induced representations and the Frobenius reciprocity theorem. Mackey's theorem. Frobenius's Theorem. [12]

Arithmetic properties of characters
Divisibility of the order of the group by the degrees of its irreducible characters. Burnside's p^aq^b theorem. [2]

Tensor products
Tensor products of representations and products of characters. The character ring. Tensor, symmetric and exterior algebras. [3]

Representations of S^1 and SU_2
The groups S^1, SU_2 and $SO(3)$, their irreducible representations, complete reducibility. The Clebsch-Gordan formula. *Compact groups.* [4]

Further worked examples
The characters of one of $GL_2(F_q)$, S_n or the Heisenberg group. [3]

Appropriate books

J.L. Alperin and R.B. Bell *Groups and representations.* Springer 1995
I.M. Isaacs *Character theory of finite groups.* Dover Publications 1994
G.D. James and M.W. Liebeck *Representations and characters of groups.* Second Edition, CUP 2001
J-P. Serre *Linear representations of finite groups.* Springer-Verlag 1977
M. Artin *Algebra.* Prentice Hall 1991

NUMBER FIELDS (D) **16 lectures, Lent term**

Part IB Groups, Rings and Modules is essential and Part II Galois Theory is desirable.

Definition of algebraic number fields, their integers and units. Norms, bases and discriminants. [3]

Ideals, principal and prime ideals, unique factorisation. Norms of ideals. [3]

Minkowski's theorem on convex bodies. Statement of Dirichlet's unit theorem. Determination of units in quadratic fields. [2]

Ideal classes, finiteness of the class group. Calculation of class numbers using statement of the Minkowski bound. [3]

Dedekind's theorem on the factorisation of primes. Application to quadratic fields. [2]

Discussion of the cyclotomic field and the Fermat equation or some other topic chosen by the lecturer. [3]

Appropriate books

Alan Baker *A Comprehensive Course in Number Theory.* Cambridge University Press 2012
† Z.I. Borevich and I.R. Shafarevich *Number Theory.* Elsevier 1986
† J. Esmonde and M.R. Murty *Problems in Algebraic Number Theory.* Springer 1999
E. Hecke *Lectures on the Theory of Algebraic Numbers.* Springer 1981
† D.A. Marcus *Number Fields.* Springer 1977
I.N. Stewart and D.O. Tall *Algebraic Number Theory and Fermat's Last Theorem.* A K Peters 2002

ALGEBRAIC TOPOLOGY (D) — 24 lectures, Michaelmas term

Part IB Analysis II is essential, and Metric and Topological Spaces is highly desirable.

The fundamental group

Homotopy of continuous functions and homotopy equivalence between topological spaces. The fundamental group of a space, homomorphisms induced by maps of spaces, change of base point, invariance under homotopy equivalence. [3]

Covering spaces

Covering spaces and covering maps. Path-lifting and homotopy-lifting properties, and ther application to the calculation of fundamental groups. The fundamental group of the circle; topological proof of the fundamental theorem of algebra. *Construction of the universal covering of a path-connected, locally simply connected space*. The correspondence between connected coverings of X and conjugacy classes of subgroups of the fundamental group of X. [5]

The Seifert–Van Kampen theorem

Free groups, generators and relations for groups, free products with amalgamation. Statement *and proof* of the Seifert–Van Kampen theorem. Applications to the calculation of fundamental groups. [4]

Simplicial complexes

Finite simplicial complexes and subdivisions; the simplicial approximation theorem. [3]

Homology

Simplicial homology, the homology groups of a simplex and its boundary. Functorial properties for simplicial maps. *Proof of functoriality for continuous maps, and of homotopy invariance*. [4]

Homology calculations

The homology groups of S^n, applications including Brouwer's fixed-point theorem. The Mayer-Vietoris theorem. *Sketch of the classification of closed combinatorical surfaces*; determination of their homology groups. Rational homology groups; the Euler–Poincaré characteristic and the Lefschetz fixed-point theorem [5]

Appropriate books

M. A. Armstrong *Basic topology.* Springer 1983
W. Massey *A basic course in algebraic topology.* Springer 1991
C. R. F. Maunder *Algebraic Topology.* Dover Publications 1980
A. Hatcher *Algebraic Topology.* Cambridge University Press, 2001

LINEAR ANALYSIS (D) — 24 lectures, Michaelmas term

Part IB Linear Algebra, Analysis II and Metric and Topological Spaces are essential.

Normed and Banach spaces. Linear mappings, continuity, boundedness, and norms. Finite-dimensional normed spaces. [4]

The Baire category theorem. The principle of uniform boundedness, the closed graph theorem and the inversion theorem; other applications. [5]

The normality of compact Hausdorff spaces. Urysohn's lemma and Tiezte's extension theorem. Spaces of continuous functions. The Stone–Weierstrass theorem and applications. Equicontinuity: the Ascoli–Arzelà theorem. [5]

Inner product spaces and Hilbert spaces; examples and elementary properties. Orthonormal systems, and the orthogonalization process. Bessel's inequality, the Parseval equation, and the Riesz–Fischer theorem. Duality; the self duality of Hilbert space. [5]

Bounded linear operations, invariant subspaces, eigenvectors; the spectrum and resolvent set. Compact operators on Hilbert space; discreteness of spectrum. Spectral theorem for compact Hermitian operators. [5]

Appropriate books

† B. Bollobás *Linear Analysis.* 2nd Edition, Cambridge University Press 1999
C. Goffman and G. Pedrick *A First Course in Functional Analysis.* 2nd Edition, Oxford University Press 1999
W. Rudin *Real and Complex Analysis.* McGraw–Hill International Editions: Mathematics Series

RIEMANN SURFACES (D) **16 lectures, Lent term**

Complex Analysis is essential and Analysis II desirable.

The complex logarithm. Analytic continuation in the plane; natural boundaries of power series. Informal examples of Riemann surfaces of simple functions (via analytic continuation). Examples of Riemann surfaces, including the Riemann sphere, and the torus as a quotient surface. [4]

Analytic, meromorphic and harmonic functions on a Riemann surface; analytic maps between two Riemann surfaces. The open mapping theorem, the local representation of an analytic function as $z \mapsto z^k$. Complex-valued analytic and harmonic functions on a compact surface are constant. [2]

Germs of an analytic map between two Riemann surfaces; the space of germs as a covering surface (in the sense of complex analysis). The monodromy theorem (statement only). The analytic continuation of a germ over a simply connected domain is single-valued. [3]

The degree of a map between compact Riemann surfaces; Branched covering maps and the Riemann-Hurwitz relation (assuming the existence of a triangulation). The fundamental theorem of algebra. Rational functions as meromorphic functions from the sphere to the sphere. [3]

Meromorphic periodic functions; elliptic functions as functions from a torus to the sphere. The Weierstrass P-function. [3]

Statement of the Uniformization Theorem; applications to conformal structure on the sphere, $*$to tori, and the hyperbolic geometry of Riemann surfaces$*$. [1]

Appropriate books

L.V.Ahlfors *Complex Analysis.* McGraw-Hill, 1979

A.F.Beardon *A Primer on Riemann Surfaces.* Cambridge University Press, 2004

G.A.Jones and D.Singerman *Complex functions: an algebraic and geometric viewpoint.* Cambridge University Press, 1987

E.T.Whittaker and G.N.Watson *A Course of Modern Analysis Chapters XX and XXI, 4th Edition.* Cambridge University Press, 1996

ALGEBRAIC GEOMETRY (D) **24 lectures, Lent term**

Groups Rings and Modules is essential.

Affine varieties and coordinate rings. Projective space, projective varieties and homogenous coordinates. Rational and regular maps. [4]

Discussion of basic commutative algebra. Dimension, singularities and smoothness. [4]

Conics and plane cubics. Quadric surfaces and their lines. Segre and Veronese embeddings. [4]

Curves, differentials, genus. Divisors, linear systems and maps to projective space. The canonical class. [8]

Statement of the Riemann-Roch theorem, with applications. [4]

Appropriate books

† K. Hulek *Elementary Algebraic Geometry.* American Mathematical Scoiety, 2003

F. Kirwan *Complex Algebraic Curves.* Cambridge University Press, 1992

M. Reid *Undergraduate Algebraic Geometry.* Cambridge University Press 1989

B. Hassett *Introduction to Algebraic Geometry.* Cambridge University Press, 2007

K. Ueno *An Introduction to Algebraic Geometry.* American Mathematical Society 1977

R. Hartshorne *Algebraic Geometry, chapters 1 and 4.* Springer 1997

DIFFERENTIAL GEOMETRY (D) — 24 lectures, Lent term

Analysis II and Geometry are very useful.

Smooth manifolds in $\mathbb{R}^n$, tangent spaces, smooth maps and the inverse function theorem. Examples, regular values, Sard's theorem (statement only). Transverse intersection of submanifolds. [4]

Manifolds with boundary, degree mod 2 of smooth maps, applications. [3]

Curves in 2-space and 3-space, arc-length, curvature, torsion. The isoperimetric inequality. [2]

Smooth surfaces in 3-space, first fundamental form, area. [1]

The Gauss map, second fundamental form, principal curvatures and Gaussian curvature. Theorema Egregium. [3]

Minimal surfaces. Normal variations and characterization of minimal surfaces as critical points of the area functional. Isothermal coordinates and relation with harmonic functions. The Weierstrass representation. Examples. [3]

Parallel transport and geodesics for surfaces in 3-space. Geodesic curvature. [2]

The exponential map and geodesic polar coordinates. The Gauss-Bonnet theorem (including the statement about classification of compact surfaces). [4]

Global theorems on curves: Fenchel's theorem (the total curvature of a simple closed curve is greater than or equal to 2π); the Fary-Milnor theorem (the total curvature of a simple knotted closed curve is greater than 4π). [2]

Appropriate books

P.M.H. Wilson *Curved Spaces.* CUP, January 2008
M. Do Carmo *Differential Geometry of Curves and Surfaces.* Pearson Higher Education, 1976
V. Guillemin and A. Pollack *Differential Topology,.* Pearson Higher Education, 1974
J. Milnor *Topology from the differentiable viewpoint.* Revised reprint of the 1965 original. Princeton Landmarks in Mathematics. Princeton University Press, Princeton, NJ, 1997
B. O'Neill *Elementary Differential Geometry.* Harcourt 2nd ed 1997
A. Pressley *Elementary Differential Geometry,.* Springer Undergraduate Mathematics Series, Springer-Verlag London Ltd, 2000
I.M. Singer and J.A. Thorpe *Lecture notes on elementary topology and geometry.* Undergraduate Texts in Mathematics. Springer-Verlag, New York-Heidelberg, 1996
M. Spivak *A Comprehensive Introduction to Differential Geometry.* Vols. I-V, Publish or Perish, Inc. 1999
J.A. Thorpe *Elementary Topics in Differential Geometry.* Springer-Verlag 1994

PROBABILITY AND MEASURE (D) — 24 lectures, Michaelmas term

Analysis II is essential.

Measure spaces, σ-algebras, π-systems and uniqueness of extension, statement $*$and proof$*$ of Carathéodory's extension theorem. Construction of Lebesgue measure on $\mathbb{R}$. The Borel σ-algebra of $\mathbb{R}$. Existence of non-measurable subsets of $\mathbb{R}$. Lebesgue-Stieltjes measures and probability distribution functions. Independence of events, independence of σ-algebras. The Borel–Cantelli lemmas. Kolmogorov's zero-one law. [6]

Measurable functions, random variables, independence of random variables. Construction of the integral, expectation. Convergence in measure and convergence almost everywhere. Fatou's lemma, monotone and dominated convergence, differentiation under the integral sign. Discussion of product measure and statement of Fubini's theorem. [6]

Chebyshev's inequality, tail estimates. Jensen's inequality. Completeness of L^p for $1 \leqslant p \leqslant \infty$. The Hölder and Minkowski inequalities, uniform integrability. [4]

L^2 as a Hilbert space. Orthogonal projection, relation with elementary conditional probability. Variance and covariance. Gaussian random variables, the multivariate normal distribution. [2]

The strong law of large numbers, proof for independent random variables with bounded fourth moments. Measure preserving transformations, Bernoulli shifts. Statements $*$and proofs$*$ of maximal ergodic theorem and Birkhoff's almost everywhere ergodic theorem, proof of the strong law. [4]

The Fourier transform of a finite measure, characteristic functions, uniqueness and inversion. Weak convergence, statement of Lévy's convergence theorem for characteristic functions. The central limit theorem. [2]

Appropriate books

P. Billingsley *Probability and Measure.* Wiley 1995
R.M. Dudley *Real Analysis and Probability.* Cambridge University Press 2002
R.T. Durrett *Probability: Theory and Examples.* Wadsworth and Brooks/Cole 1991
D. Williams *Probability with Martingales.* Cambridge University Press

APPLIED PROBABILITY (D) — 24 lectures, Lent term

Markov Chains is essential

Finite-state continuous-time Markov chains: basic properties. Q-matrix, backward and forward equations. The homogeneous Poisson process and its properties (thinning, superposition). Birth and death processes. [6]

General continuous-time Markov chains. Jump chains. Explosion. Minimal Chains. Communicating classes. Hitting times and probabilities. Recurrence and transience. Positive and null recurrence. Convergence to equilibrium. Reversibility. [6]

Applications: the $M/M/1$ and $M/M/\infty$ queues. Burke's theorem. Jackson's theorem for queueing networks. The $M/G/1$ queue. [4]

Renewal theory: renewal theorems, equilibrium theory (proof of convergence only in discrete time). Renewal-reward processes. Little's formula. [4]

Moran and Wright-Fisher models. Kingmans's coalescent. Infinite sites and infinite alleles models. Ewens' sampling formula. [4]

Appropriate books

G.R. Grimmett and D.R. Stirzaker *Probability and Random Processes.* OUP 2001
J.R. Norris *Markov Chains.* CUP 1997
R. Durrett *Probability Models for DNA Sequences.* Springer 2008

PRINCIPLES OF STATISTICS (D) — 24 lectures, Michaelmas term

Part IB Statistics is essential

The Likelihood Principle
Basic inferential principles. Likelihood and score functions, Fisher information, Cramer-Rao lower bound, review of multivariate normal distribution. Maximum likelihood estimators and their asymptotic properties: stochastic convergence concepts, consistency, efficiency, asymptotic normality. Wald, score and likelihood ratio tests, confidence sets, Wilks' theorem, profile likelihood. Examples. [8]

Bayesian Inference
Prior and posterior distributions. Conjugate families, improper priors, predictive distributions. Asymptotic theory for posterior distributions. Point estimation, credible regions, hypothesis testing and Bayes factors. [3]

Decision Theory
Basic elements of a decision problem, including loss and risk functions. Decision rules, admissibility, minimax and Bayes rules. Finite decision problems, risk set. Stein estimator. [4]

Multivariate Analysis
Correlation coefficient and distribution of its sample version in a bivariate normal population. Partial correlation coefficients. Classification problems, linear discriminant analysis. Principal component analysis. [5]

Nonparametric Inference and Monte Carlo Techniques
Glivenko-Cantelli theorem, KolmogorovSmirnov tests and confidence bands. Bootstrap methods: jackknife, roots (pivots), parametric and nonparametric bootstrap. Monte Carlo simulation and the Gibbs sampler. [4]

Appropriate books

G. Casella and R.L. Berger *Statistical Inference.* Duxbury (2001)
A.W. van der Vaart *Asymptotic Statistics.* CUP (1998)
T.W. Anderson *An introduction to multivariate statistical analysis.* Wiley (2003)
E.L. Lehmann and G Casell *Theory of Point estimation.* Springer (1998)

STOCHASTIC FINANCIAL MODELS (D) — 24 lectures, Lent term

Methods, Statistics, Probability and Measure, and Markov Chains are desirable.

Utility and mean-variance analysis
Utility functions; risk aversion and risk neutrality. Portfolio selection with the mean-variance criterion; the efficient frontier when all assets are risky and when there is one riskless asset. The capital-asset pricing model. Reservation bid and ask prices, marginal utility pricing. Simplest ideas of equilibrium and market clearning. State-price density. [5]

Martingales
Conditional expectation, definition and basic properties. Conditional expectation, definition and basic properties. Stopping times. Martingales, supermartingales, submartingales. Use of the optional sampling theorem. [3]

Dynamic models
Introduction to dynamic programming; optimal stopping and exercising American puts; optimal portfolio selection. [3]

Pricing contingent claims
Lack of arbitrage in one-period models; hedging portfolios; martingale probabilities and pricing claims in the binomial model. Extension to the multi-period binomial model. Axiomatic derivation. [4]

Brownian motion
Introduction to Brownian motion; Brownian motion as a limit of random walks. Hitting-time distributions; changes of probability. [3]

Black-Scholes model
The Black-Scholes formula for the price of a European call; sensitivity of price with respect to the parameters; implied volatility; pricing other claims. Binomial approximation to Black-Scholes. Use of finite-difference schemes to compute prices [6]

Appropriate books

J. Hull *Options, Futures and Other Derivative Securities.* Prentice-Hall 2003
J. Ingersoll *Theory of Financial Decision Making.* Rowman and Littlefield 1987
A. Rennie and M. Baxter *Financial Calculus: an introduction to derivative pricing.* Cambridge University Press 1996
P. Wilmott, S. Howison and J. Dewynne *The Mathematics of Financial Derivatives: a student introduction.* Cambridge University Press 1995

OPTIMISATION AND CONTROL (D) — 16 lectures, Lent term

Optimisation and Markov Chains are very helpful.

Dynamic programming
The principle of optimality. The dynamic programming equation for finite-horizon problems. Interchange arguments. Markov decision processes in discrete time. Infinite-horizon problems: positive, negative and discounted cases. Value interation. Policy improvment algorithm. Stopping problems. Average-cost programming. [6]

LQG systems
Linear dynamics, quadratic costs, Gaussian noise. The Riccati recursion. Controllability. Stabilizability. Infinite-horizon LQ regulation. Observability. Imperfect state observation and the Kalman filter. Certainty equivalence control. [5]

Continuous-time models
The optimality equation in continuous time. Pontryagin's maximum principle. Heuristic proof and connection with Lagrangian methods. Transversality conditions. Optimality equations for Markov jump processes and diffusion processes. [5]

Appropriate books

D.P. Bertsekas *Dynamic Programming and Optimal Control, Volumes I and II.* Athena Scientific, 2001
L. M. Hocking *Optimal Control: An introduction to the theory with applications.* Oxford 1991
S. Ross *Introduction to Stochastic Dynamic Programming.* Academic Press 1995
P. Whittle *Optimisation over Time Vols. I and II.* Wiley 1983
P. Whittle *Optimal Control: Basics and Beyond.* Wiley 1995

ASYMPTOTIC METHODS (D) — 16 lectures, Lent term

Either Complex Methods or Complex Analysis is essential, Part II Further Complex Methods is useful.

Asymptotic expansions

Definition (Poincare) of $\phi(z) \sim \sum a_n z^{-n}$; examples; elementary properties; uniqueness; Stokes' phenomenon. [4]

Asymptotics behaviour of functions defined by integrals

Integration by parts. Watson's lemma and Laplace's method. Riemann–Lebesgue lemma and method of stationary phase. The method of steepest descent (including derivation of higher order terms). Airy function, *and application to wave theory of a rainbow*. [7]

Asymptotic behaviour of solutions of differential equations

Asymptotic solution of second-order linear differential equations, including Liouville–Green functions (proof that they are asymptotic not required) and WKBJ with the quantum harmonic oscillator as an example. [4]

Recent developments

Further discussion of Stokes' phenomenon. *Asymptotics 'beyond all orders'*. [1]

Appropriate books

† M.J. Ablowitz and A.S. Fokas *Complex Variables: Introduction and Applications.* CUP 2003
J.D. Murray *Asymptotic Analysis.* Springer 1984
A. Erdelyi *Asymptotic Expansions.* Dover 1956
P.D. Miller *Applied Asymptotic Analysis.* American Math. Soc. 2006
F.W.J. Olver *Asymptotics and Special Functions.* A K Peters 1997
† C.M. Bender and S.A. Orszag *Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory.* Springer 1999

DYNAMICAL SYSTEMS (D) — 24 lectures, Michaelmas term

General introduction

The notion of a dynamical system and examples of simple phase portraits. Relationship between continuous and discrete systems. Reduction to autonomous systems. Initial value problems, uniqueness, finite-time blowup, examples. Flows, orbits, invariant sets, limit sets and topological equivalence. [3]

Fixed points of flows

Linearization. Classification of fixed points in $\mathbb{R}^2$, Hamiltonian case. Effects of nonlinearity; hyperbolic and non-hyperbolic cases; Stable-manifold theorem (statement only), stable and unstable manifolds in $\mathbb{R}^2$. Phase-plane sketching. [3]

Stability

Lyapunov, quasi-asymptotic and asymptotic stability of invariant sets. Lyapunov and bounding functions. Lyapunov's 1st theorem; La Salle's invariance principle. Local and global stability. [2]

Periodic orbits in $\mathbb{R}^2$

The Poincaré index; Dulac's criterion; the Poincaré–Bendixson theorem (*and proof*). Nearly Hamiltonian flows.

Stability of periodic orbits; Floquet multipliers. Examples; van der Pol oscillator. [5]

Bifurcations in flows and maps

Non-hyperbolicity and structural stability. Local bifurcations of fixed points: saddle-node, transcritical, pitchfork and Andronov-Hopf bifurcations. Construction of centre manifold and normal forms. Examples. Effects of symmetry and symmetry breaking. *Bifurcations of periodic orbits.* Fixed points and periodic points for maps. Bifurcations in 1-dimensional maps: saddle-node, period-doubling, transcritical and pitchfork bifurcations. The logistic map. [5]

Chaos

Sensitive dependence on initial conditions, topological transitivity. Maps of the interval, the sawtooth map, horseshoes, symbolic dynamics. Period three implies chaos, the occurrence of N-cycles, Sharkovsky's theorem (statement only). The tent map. Unimodal maps and Feigenbaum's constant. [6]

Appropriate books

D.K. Arrowsmith and C.M. Place *Introduction to Dynamical Systems.* CUP 1990
P.G. Drazin *Nonlinear Systems.* CUP1992
† P.A. Glendinning *Stability, Instability and Chaos.* CUP1994
D.W. Jordan and P. Smith *Nonlinear Ordinary Differential Equations.* OUP 1999
J. Guckenheimer and P. Holmes *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields.* Springer, second edition 1986

INTEGRABLE SYSTEMS (D) — 16 lectures, Michaelmas term

Part IB Methods, and Complex Methods or Complex Analysis are essential; Part II Classical Dynamics is desirable.

Integrability of ordinary differential equations: Hamiltonian systems and the Arnol'd–Liouville Theorem (sketch of proof). Examples. [3]

Integrability of partial differential equations: The rich mathematical structure and the universality of the integrable nonlinear partial differential equations (Korteweg-de Vries, sine-Gordon). Backlund transformations and soliton solutions. [2]

The inverse scattering method: Lax pairs. The inverse scattering method for the KdV equation, and other integrable PDEs. Multi soliton solutions. Zero curvature representation. [6]

Hamiltonian formulation of soliton equations. [2]

Painleve equations and Lie symmetries: Symmetries of differential equations, the ODE reductions of certain integrable nonlinear PDEs, Painleve equations. [3]

Appropriate books

† Dunajski, M *Solitons, Instantons and Twistors.* (Ch 1–4) Oxford Graduate Texts in Mathematics, ISBN 9780198570639, OUP, Oxford 2009

S. Novikov, S.V. Manakov, L.P. Pitaevskii, V. Zaharov *Theory of Solitons.* for KdF and Inverse Scattering

P.G. Drazin and R.S. Johnson *Solitons: an introduction.* (Ch 3, 4 and 5) Cambridge University Press 1989

V.I. Arnol'd *Mathematical Methods of Classical Mechanics.* (Ch 10) Springer, 1997

P.R. Hydon *Symmetry Methods for Differential Equations:A Beginner's Guide.* Cambridge University Press 2000

P.J. Olver *Applications of Lie groups to differential equations.* Springeri 2000

MJ Ablowitz and P Clarkson *Solitons, Nonlinear Evolution Equations and Inverse Scattering.* CUP 1991

MJ Ablowitz and AS Fokas *Complex Variables.* CUP, Second Edition 2003

PRINCIPLES OF QUANTUM MECHANICS (D) — 24 lectures, Michaelmas term

IB Quantum Mechanics is essential.

Dirac formalism

Bra and ket notation, operators and observables, probability amplitudes, expectation values, complete commuting sets of operators, unitary operators. Schrödinger equation, wave functions in position and momentum space. [3]

Time evolution operator, Schrödinger and Heisenberg pictures, Heisenberg equations of motion. [2]

Harmonic oscillator

Analysis using annihilation, creation and number operators. Significance for normal modes in physical examples. [2]

Multiparticle systems

Composite systems and tensor products, wave functions for multiparticle systems. Symmetry or anti-symmetry of states for identical particles, Bose and Fermi statistics, Pauli exclusion principle. [3]

Perturbation theory

Time-independent theory; second order without degeneracy, first order with degeneracy. [2]

Angular momentum

Analysis of states $|jm\rangle$ from commutation relations. Addition of angular momenta, calculation of Clebsch–Gordan coefficients. Spin, Pauli matrices, singlet and triplet combinations for two spin half states. [4]

Translations and rotations

Unitary operators corresponding to spatial translations, momenta as generators, conservation of momentum and translational invariance. Corresponding discussion for rotations. Reflections, parity, intrinsic parity. [3]

Time-dependent perturbation theory

Interaction picture. First-order transition probability, the golden rule for transition rates. Application to atomic transitions, selection rules based on angular momentum and parity, *absorption, stimulated and spontaneous emission of photons*. [3]

Quantum basics

Quantum data, qubits, no cloning theorem. Entanglement, pure and mixed states, density matrix. Classical determinism versus quantum probability, Bell inequality for singlet two-electron state, GHZ state. [2]

Appropriate books

† E. Merzbacher *Quantum Mechanics, 3rd edition.* Wiley 1998

† B.H. Bransden and C.J. Joachain *Quantum Mechanics, 2nd edition.* Pearson

J. Binney and D. Skinner *The Physics of Quantum Mechanics.* Cappella Archive, 3rd edition

P.A.M. Dirac *The Principles of Quantum Mechanics.* Oxford University Press 1967, reprinted 2003

C.J. Isham *Lectures on Quantum Theory: Mathematical and Structural Foundations.* Imperial College Press 1995

APPLICATIONS OF QUANTUM MECHANICS (D) — 24 lectures, Lent term

Principles of Quantum Mechanics is essential.

Variational Principle
Variational principle, examples. [2]

Bound states and scattering states in one dimension
Bound states, reflection and transmission amplitudes. Examples. Relation between bound states and transmission amplitude by analytic continuation. [3]

Scattering theory in three dimensions
Classical scattering, definition of differential cross section. Asymptotic wavefunction for quantum scattering, scattering amplitude, cross section. Green's function, Born approximation to scattering on a potential. Spherically symmetric potential, partial waves and phase shifts, optical theorem. Low energy scattering, scattering length. Bound states and resonances as zeros and poles of S-matrix. [5]

Electrons in a magnetic field
Vector potential and Hamiltonian. Quantum Hamiltonian, inclusion of electron spin, gauge invariance, Zeeman splitting. Landau levels, effect of spin, degeneracy and filling effects, use of complex variable for lowest Landau level. Aharonov-Bohm effect. [4]

Particle in a one-dimensional periodic potential
Discrete translation group, lattice and reciprocal lattice, periodic functions. Bloch's theorem, Brillouin zone, energy bands and gaps. Floquet matrix, eigenvalues. Band gap in nearly-free electron model, tight-binding approximation. [3]

Crystalline solids
Introduction to crystal symmetry groups in three dimensions, Voronoi/Wigner-Seitz cell. Primitive, body-centred and face-centred cubic lattices. Reciprocal lattice, periodic functions, lattice planes, Brillouin zone. Bloch states, electron bands, Fermi surface. Basics of electrical conductivity: insulators, semiconductors, conductors. Extended zone scheme. [4]

Bragg scattering. Vibrations of crystal lattice, quantization, phonons. [3]

Appropriate books

D.J. Griffiths *Introduction to Quantum Mechanics.* 2nd edition, Pearson Education 2005
N.W. Ashcroft and N.D. Mermin *Solid State Physics.* Holt–Saunders 1976
L.D. Landau and E.M. Lifshitz *Quantum Mechanics (Course in Theoretical Physics Vol. 3).* Butterworth–Heinemann, 1981

STATISTICAL PHYSICS (D) — 24 lectures, Lent term

Part IB Quantum Mechanics and "Multiparticle Systems" from Part II Principles of Quantum Mechanics are essential.

Fundamentals of statistical mechanics
Microcanonical ensemble. Entropy, temperature and pressure. Laws of thermodynamics. Example of paramagnetism. Boltzmann distribution and canonical ensemble. Partition function. Free energy. Specific heats. Chemical Potential. Grand Canonical Ensemble. [5]

Classical gases
Density of states and the classical limit. Ideal gas. Maxwell distribution. Equipartition of energy. Diatomic gas. Interacting gases. Virial expansion. Van der Waal's equation of state. Basic kinetic theory. [3]

Quantum gases
Density of states. Planck distribution and black body radiation. Debye model of phonons in solids. Bose-Einstein distribution. Ideal Bose gas and Bose-Einstein condensation. Fermi-Dirac distribution. Ideal Fermi gas. Pauli paramagnetism. [8]

Thermodynamics
Thermodynamic temperature scale. Heat and work. Carnot cycle. Applications of laws of thermodynamics. Thermodynamic potentials. Maxwell relations. [4]

Phase transitions
Liquid-gas transitions. Critical point and critical exponents. Ising model. Mean field theory. First and second order phase transitions. Symmetries and order parameters. [4]

Appropriate books

F. Mandl *Statistical Physics.* Wiley 1988
R.K. Pathria *Statistical Mechanics, 2nd ed.*. Butterworth-Heinemann 1996
L.D. Landau and E.M. Lifshitz *Statistical Physics, Part 1 (Course of Theoretical Physics volume 5).* Butterworth-Heinemann 1996
F. Reif *Fundamentals of Thermal and Statistical Physics.* McGraw-Hill 1965
A.B. Pippard *Elements of Classical Thermodynamics.* Cambridge University Press, 1957
K. Huang *Introduction to Statistical Physics.* Taylor and Francis 2001

ELECTRODYNAMICS (D) — 16 lectures, Michaelmas term

IB Electromagnetism and IA Dynamics and Relativity are essential. IB Methods is desirable.

Classical Field Theory

Revision of Maxwell's equations in relativistic form. Action principle for Maxwell's equations with prescribed current. Action principle for charged particles. Motion of relativistic charged particles in constant electric and magnetic fields. Electromagnetic field energy, momentum and stress tensor. Poynting's theorem. Energy and momentum density of a plane electromagnetic wave. Radiation pressure. [6]

Electromagnetic Radiation

Revision of multipole expansions in electrostatics and magnetostatics. Retarded potential of a time-dependent charge distribution. The radiation field. Dipole radiation. Energy radiated. Liénard–Wiechert potentials for an arbitrarily moving point charge. Larmor formula. Scattering. [5]

Electromagnetism in Media

Electric fields in matter and polarisation. Magnetic fields in matter and bound currents. Macroscopic Maxwell equations. Reflection and refraction. Dispersion. Causality and Kramers–Kronig relation. Electromagnetic waves in conductors; the Drude model, plasma oscillations. [5]

Appropriate books

A. Zangwill *Modern Electrodynamics.* CUP 2012
J.D. Jackson *Electrodynamics.* Wiley 1999
L.D. Landau and E.M. Lifshitz *The Classical Theory of Fields (Course of Theoretical Physics volume 2).* Butterworth-Heinemann 1996
R. Feynman, R. Leighton and M. Sands *The Feynman Lectures in Physics, Vol 2.* . Basic Books 2011

GENERAL RELATIVITY (D) — 24 lectures, Lent term

Part IB Methods and Variational Principles are very useful.

Brief review of Special Relativity

Notion of proper time. Equation of motion for free point particle derivable from a variational principle. Noether's Theorem. [1]

Introduction and motivation for General Relativity

Curved and Riemannian spaces. The Pound–Rebka experiment. Introduction to general relativity: interpretation of the metric, clock hypothesis, geodesics, equivalence principles. Static spacetimes. Newtonian limit. [4]

Tensor calculus

Covariant and contravariant tensors, tensor manipulation, partial derivatives of tensors. Metric tensor, magnitudes, angles, duration of curve, geodesics. Connection, Christoffel symbols, covariant derivatives, parallel transport, autoparallels as geodesics. Curvature. Riemann and Ricci tensors, geodesic deviation. [5]

Vacuum field equations

Spherically symmetric spacetimes, the Schwarzschild solution. Birkhoff's Theorem. Rays and orbits, gravitational red-shift, light deflection, perihelion advance. Shapiro time delay. [4]

Einstein Equations coupled to matter

Concept of an energy momentum tensor. Maxwell stress tensor and perfect fluid as examples. Importance of Bianchi identities. The emergence of the cosmological term. Simple exact solutions: Friedmann-Lemaitre metrics, the Einstein Static Universe. Hubble expansion and redshift. De-Sitter spacetime, mention of Dark Energy and the problem of Dark matter. Notion of geodesic completeness and definition of a spacetime singularity. Schwarzschild and Friedmann-Lemaitre spacetimes as examples of spacetimes with singularities. [4]

Linearized theory

Linearized form of the vacuum equations. De-Donder gauge and reduction to wave equation. Comparison of linearized point mass solution with exact Schwarzschild solution and identification of the mass parameter. Gravitational waves in linearized theory. *The quadrupole formula for energy radiated.* Comparison of linearized gravitational waves with the exact pp-wave metric. [4]

Gravitational collapse and black holes

Non-singular nature of the surface $r = 2M$ in the Schwarzschild solution using Finkelstein and Kruskal coordinates. The idea of an event horizon and the one-way passage of timelike geodesics through it. Qualitative account of idealized spherically symmetric collapse. The final state: statement of Israel's Theorem. *Qualitative description of Hawking radiation.* [2]

Appropriate books

S.M. Carroll *Spacetime and Geometry.* Addison-Wesley 2004
J.B. Hartle *Gravity: An introduction to Einstein's General Relativity.* Addison–Wesley 2002
L.P. Hughston and K.P. Tod *An Introduction to General Relativity.* Cambridge University Press 1990
R. d'Inverno *Introducing Einstein's Relativity.* Clarendon 1992
† W. Rindler *Relativity: Special, General and Cosmological.* Oxford University Press 2001
H. Stephani *Relativity: An introduction to Special and General Relativity.* Cambridge University Press, 2004

FLUID DYNAMICS II (D) — 24 lectures, Michaelmas term

Methods and Fluid Dynamics are essential.

It is recommended that students attend the associated Laboratory Demonstrations in Fluid Dynamics, which take place in the Michaelmas term.

Governing equations for an incompressible Newtonian fluid
Stress and rate-of-strain tensors and hypothesis of linear relation between them for an isotropic fluid; equation of motion; conditions at a material boundary; dissipation; flux of mass, momentum and energy; the Navier-Stokes equations. Dynamical similarity; steady and unsteady Reynolds numbers. [4]

Unidirectional flows
Couette and Poiseuille flows; the Stokes layer; the Rayleigh problem. [2]

Stokes flows
Flow at low Reynolds number; linearity and reversibility; uniqueness and minimum dissipation theorems. Flow in a corner; force and torque relations for a rigid particle in arbitrary motion; case of a rigid sphere and a spherical bubble. [4]

Flow in a thin layer
Lubrication theory; simple examples; the Hele-Shaw cell; gravitational spreading on a horizontal surface. [3]

Generation and confinement of vorticity
Vorticity equation; vortex stretching; flow along a plane wall with suction; flow toward a stagnation point on a wall; flow in a stretched line vortex. [3]

Boundary layers at high Reynolds number
The Euler limit and the Prandtl limit; the boundary layer equation for two-dimensional flow. Similarity solutions for flow past a flat plate and a wedge. ∗Discussion of the effect of acceleration of the external stream, separation.∗ Boundary layer at a free surface; rise velocity of a spherical bubble. [6]

Stability of unidirectional inviscid flow
Instability of a vortex sheet and of simple jets (e.g. vortex sheet jets). [2]

Appropriate books

D.J. Acheson *Elementary Fluid Dynamics.* Oxford University Press 1990
G.K. Batchelor *An Introduction to Fluid Dynamics.* Cambridge University Press 2000
E. Guyon, J-P Hulin, L. Petit and C.D. Mitescu *Physical Hydrodynamics.* Oxford University Press 2000

WAVES (D) — 24 lectures, Lent term

Part IB Methods is essential and Part IB Fluid Dynamics is very helpful.

Sound waves
Equations of motion of an inviscid compressible fluid (without discussion of thermodynamics). Mach number. Linear acoustic waves; wave equation; wave-energy equation; plane waves; spherically symmetric waves. [3]

Elastic waves
Momentum balance; stress and infinitesimal strain tensors and hypothesis of a linear relation between them for an isotropic solid. Wave equations for dilatation and rotation; dilatation and shear potentials. Compressional and shear plane waves; simple problems of reflection and transmission; Rayleigh waves. [6]

Dispersive waves
Rectangular acoustic wave guide; Love waves; cut-off frequency. Representation of a localised initial disturbance by a Fourier integral (one-dimensional case only); modulated wave trains; stationary phase. Group velocity as energy propagation velocity; dispersing wave trains. Water waves; internal gravity waves. [5]

Ray theory
Group velocity from wave-crest kinematics; ray tracing equations. Doppler effect; ship wave pattern. Cases where Fermat's Principle and Snell's law apply. [4]

Non-linear waves
One-dimensional unsteady flow of a perfect gas. Water waves. Riemann invariants; development of shocks; rarefaction waves; 'piston' problems. Rankine-Hugoniot relations for a steady shock. Hydraulic jumps. [6]

Appropriate books

J.D. Achenbach *Wave Propagation in Elastic Solids.* North Holland 1973
† J. Billingham and A.C. King *Wave Motion: Theory and application.* Cambridge University Press 2000
D.R. Bland *Wave Theory and Applications.* Clarendon Press 1988
M.J. Lighthill *Waves in Fluids.* Cambridge University Press 1978
G.B. Whitham *Linear and Nonlinear Waves.* Wiley 1999

NUMERICAL ANALYSIS (D) **24 lectures, Michaelmas term**

Part IB Numerical Analysis is essential and Analysis II, Linear Algebra and Complex Methods or Complex Analysis are all desirable.

Finite difference methods for the Poisson's equation

Approximation of ∇^2 by finite differences. The accuracy of the five-point method in a square. Higher order methods. Solution of the difference equations by iterative methods, including multigrid. Fast Fourier transform (FFT) techniques. [5]

Finite difference methods for initial value partial differential equations

Difference schemes for the diffusion equation and the advection equation. Proof of convergence in simple cases. The concepts of well posedness and stability. Stability analysis by eigenvalue and Fourier techniques. Splitting methods. [6]

Spectral methods

Brief review of Fourier expansions. Calculation of Fourier coefficients with FFT. Spectral methods for the Poisson equation in a square with periodic boundary conditions. Chebyshev polynomials and Chebyshev methods. Spectral methods for initial-value PDEs. [5]

Iterative methods for linear algebraic systems

Iterative methods, regular splittings and their convergence. Jacobi and Gauss-Seidel methods. Krylov spaces. Conjugate gradients and preconditioning. [5]

Computation of eigenvalues and eigenvectors

The power method and inverse iteration. Transformations to tridiagonal and upper Hessenberg forms. The QR algorithm for symmetric and general matrices, including shifts. [3]

Appropriate books

G.H. Golub and C.F. van Loan *Matrix Computations.* Johns Hopkins Press 1996

A. Iserles *A First Course in the Numerical Analysis of Differential Equations.* Cambridge University Press 1996

K.W. Morton and D.F. Mayers *Numerical Solution of Partial Differential Equations: an Introduction.* Cambridge University Press 2005

COMPUTATIONAL PROJECTS

The projects that need to be completed for credit are published by the Faculty in a manual usually by the end of July preceding the Part II year. The manual contains details of the projects and information about course administration. The manual is available on the Faculty website at `http://www.maths.cam.ac.uk/undergrad/catam/`. Each project is allocated a number of units of credit. Full credit may obtained from the submission of projects with credit totalling 30 units. Credit for submissions totalling less than 30 units is awarded proportionately. There is no restriction on the choice of projects. Once the manual is available, the projects may be done at any time up to the submission deadline, which is near the beginning of the Easter Full Term.

A list of suitable books can be found in the manual