COURSES IN PART IB

OF THE

MATHEMATICAL TRIPOS

This document contains a list of all the courses which are examinable in Part IB of the Mathematical Tripos together with an informal description of each course and suggestions for preliminary reading.

A formal syllabus is given in the booklet *Schedules for the Mathematical Tripos*.

All the documentation (including Schedules for the Mathematical Tripos) is available on the Faculty web site (http://www.maths.cam.ac.uk/).

The provisional lecture timetable is likely to appear towards the end of June
(see http://www.maths.cam.ac.uk/undergrad/lecturelists/)

June 17, 2015

Introduction

Contents

You will find here a list of all Part IB courses, together with non–technical summaries, a summary of the learning outcomes of the course and suggestions for vacation reading. The full learning outcome is that you should understand the material described in the formal syllabuses given in the *Schedules of Lecture Courses for the Mathematical Tripos* and be able to apply it to the sort of problems that can be found on previous Tripos papers.

Changes from last year

- There will now be three “accelerated” 16 lecture courses in Lent term, with three lectures on Mondays, Wednesdays and Fridays for the first five (and a third!) weeks of term: Complex Methods, Complex Analysis and Numerical Analysis. This acceleration is intended to avoid a pile-up of example sheets and supervisions at the end of term, and the accelerations of Complex Methods and Complex Analysis proved popular with students when they were tested for the first time in 2014/15.

Preliminary reading

Any mathematics that you manage to do over the summer vacation will be immensely helpful for next year. Although revision of the Part IA courses would be useful, it would probably be more helpful either to work on any of the Part IB courses which you attended this Easter term or to do some preliminary reading for some of next year’s courses. The suggestions in this document are only intended to give an idea of the appropriate level and approach. They should all be in your college library. By browsing in the library, you will no doubt find other books which you find at least as useful as those listed here.

One of the most useful things you can do over the summer is to get going on the computational projects, which can be extremely time-consuming and, if left to the last moment, can be very distracting. If you don’t know how to program in MATLAB, then you should make sure that you have cracked it before October.

Choice of courses

You are not expected to take all the IB courses. You should choose the number of courses by comparison with Part IA, for which you were expected to attend two lectures per day for two terms (total 192 lectures). If you were comfortable with that, then this might be a realistic target for Part IB. However, many students prefer to take fewer courses and learn them more thoroughly; and some may wish to take more. The structure of the Part IB examination is such that you do not need to take as many lectures as this in order to obtain full marks. But you might like to check the distribution of questions on the four examination papers before making your final choice: this is given in the Schedules booklet.

You should consult your Director of Studies about your choice of courses, because it will have consequences for your choices in Part II.

Courses in Part IB

Linear Algebra — Michaelmas, 24 lectures

The first year course Vectors and Matrices includes a concrete introduction to vector spaces. Here, vector spaces are investigated from an abstract axiomatic point of view. This has two purposes: firstly to provide an introduction to abstract algebra in an already familiar context and secondly to provide a foundation for the study of infinite-dimensional vector spaces which are required for advanced courses in analysis and physics. One important application is to function spaces and differential and difference operators. A striking result is the Cayley-Hamilton theorem which says (roughly) that any square matrix satisfies the same equation as its eigenvalues (the characteristic equation).

The spaces studied for the first parts of the course have nothing corresponding to length or angle. These are introduced by defining an inner product (i.e. a 'dot' product) on the vector space. This is generalised to the notion of a bilinear form ('lengths' do not have to be positive) and even further. There are direct applications to quantum mechanics and statistics.

The last part of the course covers the theory of bilinear and hermitian forms, and inner products on vector spaces. An important example is the quadratic form. The discussion of orthogonality of eigenvectors and properties of eigenvalues of Hermitian matrices has consequences in many areas of mathematics and physics, including quantum mechanics.

There are many suitable books on linear algebra: for example *Finite-dimensional Vector Spaces* by Halmos (Springer, 1974), Birkhoff and MacLane's *Algebra* (Macmillan, 1979) and Strang's *Linear Algebra* (Academic Press, 1980).

Learning outcomes

By the end of this course, you should:

- understand the concepts of, and be able to prove results in the theory of, real and complex vector spaces;
- understand the concepts of, and be able to prove results in the theory of, linear maps between and endomorphisms of real and complex vector spaces, including the role of eigenvectors and eigenvalues and Jordan canonical form;
- understand, and be able to prove and apply, the Cayley-Hamilton theorem;
- understand, and be able to prove results in the theory of, dual vector spaces;
- understand bilinear forms and their connection with the dual space, and be able to derive their basic properties;
- know the theory of canonical forms for symmetric, alternating and hermitian forms, and be able to find them in simple cases;
- understand the theory of hermitian endomorphisms of a complex inner product space, and know and be able to apply the Gram-Schmidt orthogonalisation process;

Groups, Rings and Modules — Lent, 24 lectures

This course unites a number of useful and important algebraic and geometric ideas by developing three concepts which are fundamental in abstract algebra. Firstly there is the notion of a *group* which you met in Part IA Groups and which is found in so much of mathematics, both pure and applied. The basic concepts of group theory are recalled from the first year and then built upon, resulting in beautiful theorems that reveal much about the structure of finite groups.

Whereas a group has only one operation, a *ring* is a set that is equipped with two operations: that of addition and multiplication, such as the integers. The next third of the course develops this idea in a way that mirrors the approach to groups, as well as considering examples such as fields and the important case of a ring of polynomials in one, and in many, variables.

The last part of the course defines and deals with the notion of a *module*, which can be described as the immediate generalisation of a vector space where the scalars form a ring rather than a field. The advantage of this approach is that it allows proof of general results which can then be used to unify theorems in specific cases, as shown at the end of the course where applications to Jordan Normal Form are given, along with a proof of the classification of finitely generated abelian groups.

For an introduction to groups, J. F. Humphreys, *A course in group theory* (Oxford Science Publications) amongst others is very readable whereas B. Hartley and T. O. Hawkes, *Rings, Modules and Linear Algebra* (Chapman and Hall), although somewhat dry, contains nearly all of the rings part of the course and more than all of the material on modules.

The course also lays the foundations for most of the algebra options in Part II. In particular it is essential for Galois Theory, and highly desirable for areas such as Number Fields and Representation Theory.

Learning outcomes

By the end of this course, you should:

- have a firm understanding of the fundamental concepts of group theory and be comfortable applying these to groups of small order.
- know the definition of a ring, a field and an ideal, and be able to determine whether an ideal is principal, maximal or prime.
- be able to factorise elements in specific rings, including cases where factorisation is non-unique.
- understand the concept of a module and its application to finitely generated abelian groups.

Analysis II — Michaelmas, 24 lectures

In the Analysis I course in Part IA, you encountered for the first time the rigorous mathematical study of the concepts of limit, continuity and derivative, applied to functions of a single real variable. This course extends that study in two different ways. First, it introduces the important notions of uniform convergence and uniform continuity, which help to explain various problematic aspects of limiting processes for functions of one variable. Then the fundamental ideas of analysis are extended from the real line $\mathbb{R}$, first to finite-dimensional Euclidean spaces $\mathbb{R}^n$ (thus providing the logical underpinnings for the results — such as symmetry of the mixed partial derivatives — which you met in Part IA Vector Calculus), and then to still more general 'metric spaces' whose 'points' may be objects such as functions or sets. The advantages of this more general point of view are demonstrated using Banach's Contraction Mapping Theorem, whose applications include a general existence and uniqueness theorem for solutions of differential equations, and the inverse function theorem, a result of fundamental importance.

If you wish to do some vacation reading, W.A. Sutherland's *Introduction to Metric and Topological Spaces* (OUP, 1975) provides a good introduction to analysis on more general spaces.

Learning outcomes

By the end of this course, you should:

- understand and be able to prove the basic results about convergence and the properties of continuous functions in $\mathbb{R}^n$;
- understand and be able to prove the basic results about differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$ and be able to calculate derivatives in simple cases;
- understand the notion of uniform convergence of functions and appreciate its significance in the theory of integration;
- understand the basic theory of metric spaces, be able to prove the contraction mapping theorem and apply it to the solution of differential equations and the inverse function theorem.

Metric and Topological Spaces **Easter, 12 lectures**

This course may be taken in the Easter term of either the first year or the second year; however, if you are planning to take Complex Analysis (i.e. the course on complex variable theory which has a pure approach; Complex Methods covers roughly the same material with an applied approach), you will find the material in Metric and Topological Spaces very useful.

Continuity is one of the basic ideas developed in Analysis I, and this course shows the value of a very abstract formulation of that idea. It starts with the general notion of distance in the theory of metric spaces and uses that to motivate the definition of topological space. The key topological ideas of connectedness and compactness are introduced and their applications explained. In particular a fresh view emerges of the important result (from Analysis I) that a continuous function on a closed and bounded interval is bounded and attains its bounds.

The recommended book for this course is W.A. Sutherland's short and readable *Introduction to Metric and Topological Spaces* (Oxford, 1975).

Learning outcomes

By the end of this course, you should:

- appreciate the definitions of metric and topological space and be able to distinguish between standard topological and non-topological properties;
- understand the topological notion of connectedness and its relation to path-connectedness;
- understand the topological notion of compactness, know its significance in basic analysis and be able to apply it to identify standard quotients of topological spaces.

Geometry **Lent, 16 lectures**

Geometry here means the study of curved surfaces. In particular, the course studies spherical geometry (where the curvature is constant and positive) and geometry of the hyperbolic plane (where the curvature is constant and negative), before moving on to the study of the geometry of embedded surfaces in 3-space and other generalizations. This material is not only appealing in itself, but serves as an introduction to the fascinating area of differential geometry, which is of vital importance both in pure mathematics and various branches of theoretical physics, such as General Relativity and its modern development, string theory. Indeed, the last fifty years is marked by the increasing importance of geometry both in mathematics and theoretical physics.

This course provides an introduction to some of the basic ideas of geometry. The course begins by developing some of the ideas from the Vectors and Matrices course in Part IA, moving from Euclidean geometry to the geometry of the sphere and torus. The hyperbolic plane is then introduced, and its geometrical properties studied. These classical geometries motivate the more general differential geometry of surfaces studied in the last part of the course, where in particular the curvature of surfaces embedded in 3-space will be explicitly described, and a deep connection between the curvature of a surface and its global geometry, given in terms of the Euler number, is sketched.

For the first half of the course, *Notes on Geometry* by Rees (Springer, 1983) will be found useful, and a good reference for the second half is *Elementary Differential Geometry* by Pressley (Springer, 2001). The best book is *Curved Spaces* by P M H Wilson (CUP).

Learning outcomes

By the end of this course, you should:

- know and be able to derive the basic properties of spherical geometry, and of the hyperbolic plane, and the description of their isometry groups;
- understand and be able to calculate the Euler number in simple cases;
- understand what is meant by a Riemannian metric and a geodesic;
- understand what is meant by, and how to calculate, the first and second fundamental forms for surfaces embedded in 3-space, and the Gaussian curvature.

Complex Analysis — Lent, 16 lectures: accelerated

This course covers about 2/3 of the material in Complex Methods, from a more rigorous point of view. The main omissions are applications applications of conformal mappings to solutions of Laplace's equations and the theory of Fourier and Laplace transforms.

The theory of complex variable is exceptionally elegant. It is used in many branches of pure mathematics, including number theory. It also forms one of the guiding models for the modern development of geometry.

A rigorous course not only provides a firm foundation for, and makes clear the underlying structure of, this material but also allows a deeper appreciation of the links with material in other analysis courses — in particular, IB Metric and Topological Spaces.

An excellent book both for the course and for preliminary reading is Hilary Priestley's *Introduction to Complex Analysis* (OUP, paperback). The books by Stewart and Tall (*Complex Analysis*) and by Jameson (*A First Course in Complex Functions*) are also good.

Learning outcomes

By the end of this course, you should:

- understand the concept of analyticity;
- prove rigorously the main theorems in the course;
- be able to use the theory of contour integration, including the residue theorem, to evaluate integrals;

Complex Methods — Lent, 16 lectures: accelerated

Complex variable theory was introduced briefly in Analysis I (for example, complex power series). Here, the subject is developed without the full machinery of a pure analysis course. Rigorous justification of the results used is given in the parallel course, Complex Analysis.

The course starts with a the definition of analyticity and the Cauchy Riemann equations (which must be satisfied by the real and imaginary parts of a complex function in order for it to be analytic; i.e. in order for it to be expressible as a power series. There follows a brief discussion of conformal mapping with applications to Laplace's equation. Then a heuristic version of Cauchy's theorem leads, via Cauchy's integral formula, to the residue calculus. This is a remarkable technique for evaluating integrals in the complex plane, which can also be used to calculate definite integrals on the real line. It allows the calculation of integrals which one would not have a hope of calculating by other means, as well as remarkably simple and elegant derivations of standard results such as $\int_{-\infty}^{\infty} \exp(-x^2/2 + ikx)dx = \sqrt{(2\pi)} \exp(-k^2/2)$ and $\int_0^{\infty} (\sin x)/x dx = \pi/2$.

An important application is to Fourier (which was introduced in the Methods course) and Laplace transform theory. The transforms are used to represent, for example, time dependent signal as a sum (in fact, an integral unless the function is periodic) over its frequency components. This is important because one often knows how a system responds to pure frequency signals rather than to an arbitrary input. In many situations, the use of a transform simplifies a physical problem by reducing a partial differential equation to an ordinary differential equation. This is a particularly important technique for numerous branches of physics, including acoustics, optics and quantum mechanics.

For a fairly applied approach, look at chapters 6 and 7 of *Mathematical Methods for Physicists* by Arfken (Academic Press, 1985). This material is also sympathetically dealt with in: *Mathematical Methods in the Physical Sciences* by Boas (Wiley, 1983).

Learning outcomes

By the end of this course, you should:

- understand the concept of analyticity;
- be able to use conformal mappings to find solutions of Laplace's equations;
- be able to use the theory of contour integration, including the residue theorem, to evaluate integrals;
- understand the theory of Fourier and Laplace transforms and apply it to the solution of ordinary and partial differential equations.

Methods **Michaelmas, 24 lectures**

This course continues the development of mathematical methods which can be applied to physical systems. The material is fundamental to nearly all areas of applied mathematics and theoretical physics.

The course introduces the important class of ordinary differential equations that are self-adjoint. The equivalent in the complex domain, used in Quantum Mechanics, are Hermitian operators. Self-adjoint equations have nice properties such as having real eigenvalues and orthogonal eigenfunctions, which allow eigenfunction expansions, the prototype being Fourier series. Fourier series generalise, for non-periodic functions,to Fourier transforms which provide a useful way of solving linear differential ordinary and partial differential equations.

Much of the remainder of the course concentrates on second order partial differential equations: classification into wave, diffusion and Laplace type equations; the fundamental solutions of the three different types, solution by separation of variable which ties in with the earlier work on self-adjoint equations.

The course also introduces the famous Dirac δ, or spike, function and the Green's function which can be regarded as the inverse operator to a differential equation: it is used to express the solution in terms of an integral. Many courses later, it will reappear as a basic tool in quantum field theory.

It would be particularly worthwhile to get to grips early with the major new ideas introduced here: Fourier series/transforms; the Sturm-Liouville equations. Reasonably friendly accounts can be found in *Mathematical Methods in the Physical Sciences* by Boas (Wiley, 1983); *Mathematical Methods for Physicists* by Arfken (Academic Press, 1985) and *Mathematical Methods for Physicists and Engineers* by Riley, Hobson and Bence (CUP, 98). It would also be very worthwhile to revise thoroughly the Variational Principles course from the Easter term.

Learning outcomes

By the end of this course, you should:

- be able to apply the theory of Green's functions to ordinary differential equations;
- understand the basic properties of Sturm-Liouville equations;
- be able to apply the method of separation of variables to partial differential equations;
- to be able to use standard methods to solve partial differential equations.
- be able to solve wave problems using Fourier analysis and advanced/retarded coordinates.

Variational Principles **Easter, 12 Lectures**

The techniques addressed in this course are of fundamental importance throughout physics and applied mathematics, and also in many areas of pure and applicable mathematics (and are related to some of the material in the Part IB Optimisation course). They generalise the idea of a stationary point of a function of one variable to functions of many variables and to functions (called functionals) defined by integrals.

In the first part of the course, stationary points in $\mathbb{R}^n$ are discussed, extending the treatment in Part IA Differential Equations by allowing constraints; for example, a point constrained to lie on a surface in $\mathbb{R}^3$. Applications include thermodynamics and the theory of supply and demand in economics.

The second part of the course deals with functionals, and functional derivatives. One example is the problem of working out the path bewteen two points in a gravitational field that gives the shortest time of travel.

Learning outcomes

By the end of this course, you should:

- understand the concepts of a functional, and of a functional derivative;
- be able to apply constraints to variational problems;
- appreciate the relationship between variational statements, conservation laws and symmetries in physics.

Quantum Mechanics — Michaelmas, 16 lectures

Quantum Mechanics is the theory which describes the behaviour of elementary particles. In fact, it supersedes Newton's laws of motion for all bodies, but the difference (in practice — not of course in concept) is generally only significant on the atomic scale.

For a single particle, the basic equation is the Schrödinger equation, which expresses the conservation of total energy as a second order differential equation. The solution of this equation is the wave function of the particle. It carries all the available information about the motion of the particle, but this information comes in the form of a probability distribution; one cannot predict where exactly the particle will be, but one can give a probability that it will be found in any given volume.

Using the wave function, one can work out the expected position and momentum of the particle at any time but there is always an uncertainty in the result of any measurement. This uncertainty is enshrined as a basic principle of quantum mechanics.

This course sets up the mathematical framework required to discuss the theory of quantum mechanics. The Schrödinger equation is then solved in certain simple but important cases including the square well potential and the hydrogen atom.

For a very readable non-mathematical account with lots of pictures, which goes well beyond the IB course, see *The Quantum Universe* by Hey and Walters (CUP, 1987). A cheap and well-written text which covers the course is Davies's *Quantum Mechanics* (Routledge, 1984). The first three chapters of Feynman's *Lectures on Physics, Volume III* (Addison-Wesley, 1964) give a good physical discussion of the subject.

Learning outcomes

By the end of this course, you should:

- understand the basic theory of quantum mechanics, including the role of the Schrödinger equation, observables, operators and their eigenvectors and eigenvalues, and expectation values;
- be able to solve, and interpret the solution, of the Schrödinger equation in simple cases, including: 1-dimensional potential wells and steps; the harmonic oscillator; and the hydrogen atom.

Electromagnetism — Lent, 16 lectures

Maxwells equations of electromagnetism are among the great triumphs of nineteenth century physics. These equations unify the electric and magnetic forces and provide an explanation for many natural phenomena, including the existence of light itself. The equations also hold the seed of the theory of special relativity. This course gives the first opportunity in the Tripos to study a modern physical field theory.

After a brief discussion of electric and magnetic forces, Maxwell's equations are introduced. A key idea is the use of potentials to represent the electric and magnetic fields and it is shown how Maxwells equations imply the existence of such potential functions. The equations are solved in special cases of physical interest. First, time independent situations are covered: for example, point charges, bar magnets, currents in wires. Next, time varying situations are investigated: for example, induction. It is also shown how Maxwells equations have wave-like solutions which we identify as light. The course ends with a discussion of special relativity in the context of electromagnetism. When viewed through the lens of relativity, the Maxwell equations become remarkably simple.

The course relies heavily on vector calculus. The latter part of the course also uses the theory of tensors from Part IA Vector Calculus and special relativity from Part IA Dynamics and Relativity. Electromagnetism is important for all of the theoretical physics courses in Part II, and is particularly relevant to General Relativity through its use of 4-vectors and tensors.

Learning outcomes

By the end of this course, you should:

- understand the physical significance of and be able to manipulate Maxwell's equations (including deriving the integral forms);
- solve simple problems in electrostatics including calculation of electrostatic energy, capacity and force;

- derive, and apply to simple situations, the Biot-Savart law;
- use Gauss's law and Ampère's law to calculate electric and magnetic fields in symmetrical situations;
- calculate forces using the Lorentz force;
- derive and apply Faraday's law of induction to simple circuits;
- solve Maxwell's equations to obtain plane waves.

Fluid Dynamics — Lent, 16 lectures

Fluid dynamics investigates the motion of liquids and gases, such as the motion that enables aircraft to fly. Newton's laws of motion apply – acceleration equals force per unit mass – but a subtlety arises because acceleration means the rate of change of velocity of a fluid particle. It does not mean the rate of change of the fluid velocity at a fixed point in space. A special mathematical operator, the material derivative, expresses the required rate of change using vector calculus. The forces entering Newton's laws can be external, such as gravity, or internal, arising from pressure or from viscosity (internal friction). When the viscosity is small enough to be negligible, the motion is often irrotational as well as incompressible: both the curl and divergence of velocity field vanish. In this situation, the fluid velocity can be described by a potential, and standard potential theory applies, including in some cases solutions of Laplace's equation.

The topics studied include jets, bubbles, waves, vortices, flow around aircraft wings, and flow in weather systems. Suitable introductory reading material can be found in Worster's *Understanding Fluid Flow* (CUP) or in Acheson's *Elementary Fluid Dynamics* (Oxford). For background motivation, see also the visionary discussion in the Feynman Lectures on Physics, last two chapters of Volume II (Addison-Wesley).

Learning outcomes

By the end of this course, you should:

- understand the basic principles governing the dynamics of parallel viscous flows and flows in which viscosity in negligible;
- be able to derive and deduce the consequences of the equation of conservation of mass;
- be able solve kinematic problems such as finding particle paths and streamlines;
- be able to apply Bernoulli's theorem and the momentum integral to simple problems including river flows;
- understand the concept of vorticity and the conditions in which it may be assumed to be zero;
- calculate velocity fields and forces on bodies for simple steady and unsteady flows derived from potentials;
- understand the theory of interfacial waves and be able to use it to investigate, for example, standing waves in a container;
- understand fundamental ideas relating to flows in rotating frames of reference, particularly geostrophy.

Numerical Analysis — Lent, 16 lectures: accelerated

An important aspect of the application of mathematics to problems in the real world is the ability to compute answers as accurately as possible subject to the errors inherent in the data presented and the limits on the accuracy of calculation. Numerical analysis is the branch of mathematics studying such computations.

The course commences from approximation theory, focussing on the approximation of functions and data by polynomials, continues with the numerical solution of ordinary differential equations and concludes with the solution of linear algebraic systems. Although computational algorithms form a central part of the course, so do mathematical theories underlying them and investigating their behaviour: computation and approximation at their best should be done with proper mathematical justification.

An Introduction to Numerical Analysis by Suli & Mayers (CUP, 2003) and *Interpolation and Approximation* by Davis (Dover, 1975) are two excellent introductory texts.

Learning outcomes

By the end of this course, you should:

- understand the role of algorithms in numerical analysis;
- understand the role and basic theory (including orthogonal polynomials and the Peano kernel theorem) of polynomial approximation;
- understand multistep and Runge–Kutta methods for ordinary differential equations and the concepts of convergence, order and stability;
- understand the theory of algorithms such as LU and QR factorisation, and be able to apply them, for example to least squares calculations.

Statistics — Lent, 16 lectures

Statistics is the study of what can be learnt from data. We regard our data as realisations of random variables, and consider models for the (joint) distribution of these random variables. In this course, we focus entirely on *parametric* models, where the class of distributions considered can be indexed by a finite-dimensional parameter. As a simple example, the family of normal distributions can be indexed by a two-dimensional parameter, representing the mean and variance. Nonparametric models are treated in more advanced courses.

Our aim is to make inference about the unknown parameter by, for example, providing a point estimate, a confidence interval or conducting a hypothesis test. Building on Part IA Probability, this course will present basic techniques of inference, together with their theoretical justification. The final chapter will cover the ubiquitous *linear model*, with its elegant theory of orthogonal projection and application of results from linear algebra.

The most appropriate book for the course is *Statistical inference* by Casella and Berger (Duxbury, 2001).

Learning outcomes

By the end of this course, you should:

- understand the basic concepts involved in point estimation, the construction of confidence intervals and Bayesian inference;
- understand and be able to apply the ideas of hypothesis testing, including the Neyman-Pearson lemma, and generalised likelihood ratio tests, including applications to goodness of fit tests and contingency tables.
- understand and be able to apply the theory of the linear model, including examples of linear regression and one-way analysis of variance.

Markov Chains — Michaelmas, 12 lectures

A Markov process is a random process for which the future (the next step) depends only on the present state; it has no memory of how the present state was reached. A typical example is a random walk (in two dimensions, the drunkard's walk).

The course is concerned with Markov chains in discrete time, including periodicity and recurrence. For example, a random walk on a lattice of integers returns to the initial position with probability one in one or two dimensions, but in three or more dimensions the probability of recurrence in zero. Some Markov chains settle down to an equilibrium state and these are the next topic in the course.

The material in this course will be essential if you plan to take any of the applicable courses in Part II. Further introductory material and notes on the course are available via the webpage "Study in DPMMS".

Learning outcomes

By the end of this course, you should:

- understand the notion of a discrete-time Markov chain and be familiar with both the finite state-space case and some simple infinite state-space cases, such as random walks and birth-and-death chains;

- know how to compute for simple examples the n-step transition probabilities, hitting probabilities, expected hitting times and invariant distribution;
- understand the notions of recurrence and transience, and the stronger notion of positive recurrence;
- understand the notion of time-reversibility and the role of the detailed balance equations;
- know under what conditions a Markov chain will converge to equilibrium in long time;
- be able to calculate the long-run proportion of time spent in a given state.

Optimisation — Easter, 12 lectures

A typical problem in optimisation is to find the cheapest way of supplying a set of supermarkets from a set of warehouses: in more general terms, the problem is to find the minimum (or maximum) value of a quantity when the variables are subject to certain constraints. Many real-world problems are of this type and the theory discussed in the course are practically extremely important as well as being interesting applications of ideas introduced earlier in Numbers and Sets and in Vectors and Matrices.

The theory of Lagrange multipliers, linear programming and network analysis is developed. Topics covered include the simplex algorithm, the theory of two-person games and some algorithms particularly well suited to solving the problem of minimising the cost of flow through a network.

Whittle's *Optimisation under Constraints* (Wiley, 1971) gives a good idea of the scope and range of the subject but is a little advanced mathematically; Luenberger's *Introduction to Linear and and Non-linear Programming* (Addison-Wesley, 1973) is at the right level but provides less motivation.

Learning outcomes

By the end of this course, you should:

- understand the nature and importance of convex optimisation;
- be able to apply Lagrangian methods to solve problems involving constraints;
- be able to solve problems in linear programming by methods including the simplex algorithm and duality;
- be able to solve network problems by methods using, for example, the Ford-Fulkerson algorithm and min-cut max-flow theorems.

Computational Projects

This course consists mainly of practical computational projects carried out and written up for submission a week after the beginning of the Lent and Easter terms. For full credit, you do four projects. The first two are written up on pro-formas and submitted in the Lent term. The remaining two are chosen from a list of projects and are submitted in the Easter term.

The emphasis is on understanding the mathematical problems being modelled rather than on the details of computer programming. You will have been given the booklet of projects and if you have access to a suitable computer over the summer, it will be extremely helpful to get the first two projects out of the way: some students find the projects very time consuming, especially those who are not used to programming. If you are wondering whether to take the Projects, you may like to know that over 95% of Part IB students submit projects (not necessarily complete); and the corresponding figure for the Part II Projects course is over 90%. The amount of credit available for the Computational Projects course in Part IB examinations will be 160 marks and no quality marks.

Learning outcomes

By the end of this course, you should:

- be able to programme using a traditional programming language;
- understand the limitations of computers in relation to solving mathematical problems;
- be able to use a computers to solve problems in both pure and applied mathematics involving, for example, solution of ordinary differential equations and manipulation of matrices.