

Complex Manifolds (L24)

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A preliminary outline of the course is as follows, but this will almost certainly be subject to change.

- Basic concepts of complex manifolds, holomorphic vector bundles, holomorphic tangent and cotangent bundles (for which corresponding concepts from the real smooth manifolds will be assumed). Canonical line bundles, normal bundle for a submanifold and the adjunction formula.
- Brief description of sheaf cohomology, with deduction of de Rham and Dolbeault cohomology for complex manifolds.
- Hermitian metrics, connections, curvature and Chern classes for complex vector bundles. Case of holomorphic vector bundles.
- Harmonic forms: the Hodge theorem and Serre duality (general results on elliptic operators will be assumed).
- Compact Kahler manifolds. Hodge and Lefschetz decompositions on cohomology, Kodaira - Nakano vanishing, Kodaira embedding theorem.

Pre-requisites

A knowledge of basic Differential Geometry from the Michaelmas term will be highly desirable.

Literature

1. J. P. Demailly, *Complex analytic and differential geometry*. Available as a pdf at <https://www-fourier.ujf-grenoble.fr/~demailly/documents.html>.
2. D. Huybrechts, *Complex Geometry – an introduction*. Springer, 2004.
3. C. Voisin, *Hodge Theory and Complex Algebraic Geometry, I*. Cambridge Studies in Adv. Mathematics, 2007.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.