Combinatorics (M16)

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This course can be viewed as a continuation of the Part II Graph Theory course, although we shall not rely on many of the results from that course.

In the extremal part of the course, we shall study collections of subsets of a finite set, with special emphasis on size, intersection and containment. There are many very natural and fundamental questions to ask about families of subsets; although many of these remain unsolved, several have been answered using a great variety of elegant techniques.

We shall cover a number of ‘classical’ extremal theorems, such as those of Erdős-Ko-Rado and Kruskal-Katona, together with more recent results concerning such topics as ‘concentration of measure’ and hereditary properties of hypergraphs. There will be several indications of open problems.

Much of the course will be on the following material.

**Extremal Combinatorics**

- Antichains; Sperner’s lemma and related results. Shadows; compression operators and the Kruskal-Katona theorem. Intersecting families; the Erdős-Ko-Rado theorem.

**Combinatorial Probability**

- Harris’s Lemma, the van den Berg–Kesten Inequality and the Four Functions Theorem. The KKL Inequality and the Friedgut–Kalai Sharp Threshold Theorem.

**Random Graphs**

- The basic models. Small subgraphs. The component structure. Connectedness. The phase transition.

**Prerequisites**

The basic concepts of graph theory and probability theory, and mathematical maturity.

**Introductory Reading**


**Further Reading**

The material in the course is covered by (small parts of) the following books.

