UNIVERSITY OF CAMBRIDGE

MAMA307, NST3AS/307, MAAS/307

MAT3

MATHEMATICAL TRIPOS Part III

Wednesday 10 June 2026 2:00pm to 4:00pm

PAPER 307

SUPERSYMMETRY

Before you begin please read these instructions carefully

Candidates have TWO HOURS to complete the written examination.

Attempt no more than **TWO** questions.
There are **THREE** questions in total.
The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Write the component field expansion of a $N = 1$ SUSY chiral superfield Φ written in terms of the special coordinate $y^\mu = x^\mu + i\bar{\theta}\sigma^\mu\theta$. Here, the superspace coordinates are $(x^\mu,\ \theta^\alpha,\ \bar{\theta}^{\dot{\alpha}})$ and $\sigma^\mu = (I, \underline{\sigma})$, I being the 2 by 2 identity matrix and $\underline{\sigma}$ being a vector of the three Pauli matrices. Label and explain each component field and calculate the number of off-shell real degrees of freedom in the boson fields and in the fermion fields.

Consider a $N = 1$ SUSY theory containing three chiral superfields Φ_i, $i \in \{1, 2, 3\}$.

(i) Write down the most general renormalisable superpotential that is invariant under the transformation

$$\Phi_i \to \exp(i\lambda^a T_a)^j{}_i \Phi^i.$$

for real constants λ^a and where T_a are the 3×3 Gell-Mann matrices and $a = 1, 2, \ldots, 8$. Identify the symmetry of the superpotential (we shall henceforth call this symmetry G).

(ii) Write down the Kähler potential, assuming that it is of canonical form and respects G.

(iii) Compute the theory's scalar potential in the supersymmetric limit.

(iv) What is meant by "soft" supersymmetry breaking?

(v) Assume that some other sector of the theory yields general soft supersymmetry breaking terms in the Lagrangian which respect G. Write these down.

(vi) Write the Feynman rule and draw a Feynman diagram for:

(a) All interactions involving only scalar field components,

(b) All interactions involving both the fermionic field components and scalar field components.

(vii) Briefly explain whether or not the theory as a whole respects G once quantum corrections are included. What implications does this have for the viability of the theory?

2 Consider a single chiral superfield Φ with $N = 1$ supergravity and a superpotential

$$W = m^2(\Phi + \beta),$$

where $\beta > 0$ is a real constant. The Kähler potential is $K = |\Phi|^2$. Throughout this question you should work in units where the Planck mass is equal to 1. The $N = 1$ supergravity scalar potential is then given by

$$V = e^K \left(K^{-1}_{ij} D_i W (D_j W)^* - 3|W|^2 \right).$$

where $D_i W = \partial_i W + W \partial_i K$.

(i) If ϕ, the scalar component of Φ, develops a vacuum expectation value $\langle\phi\rangle \neq 0$, does the model break or preserve supersymmetry? Give any necessary conditions involving the parameters.

(ii) Calculate the scalar potential $V(\phi)$ and argue that the extremisation conditions are compatible with $\langle\phi\rangle \in \mathbb{R}$ (you may then assume this for the rest of the question).

(iii) Observations of the cosmological constant are equivalent to the constraint

$$V(\langle\phi\rangle) \approx 0.$$

Imposing this as an equality, show that a sub-Planckian vacuum solution (i.e. $|\langle\phi\rangle| < 1$), exists for

$$\beta = -\sqrt{X} + Y, \qquad \langle\phi\rangle = \sqrt{U} + Z,$$

where X, Y, U and Z are integers which you should find.

(iv) Check that any extremum you find is of the correct type to be a vacuum solution along the $\phi \in \mathbb{R}$ direction. Sketch the potential along this direction.

3 Consider a single chiral superfield Φ in a $N = 1$ SUSY theory which is defined at high energies by the superpotential:

$$W_{\text{tree}} \,=\, \frac{1}{2}\, m\, \Phi^2 \,+\, \frac{1}{3}\, g\, \Phi^3 \;, \tag{1}$$

where $m \in \mathbb{R}$ and $g \in \mathbb{C}$. Write down the action in terms of superspace integrals, assuming a canonical form for the Kähler potential.

Explain what a *spurion* is. In the following, think of m and g as being components of spurion superfields.

(i) There are two distinct $U(1)$ symmetries of the theory. Defining each $U(1)$ charge of Φ to be unity, find the charges of the superpotential. Then find the two $U(1)$ charges of each spurion.

(ii) Calculate the charge of

$$\frac{g\,\Phi}{m}$$

under the action of each $U(1)$. Argue that the most general low energy effective action must take the form:

$$W_{\text{eff}} \,=\, \frac{1}{2}\, A\, f\left(\frac{g\,\Phi}{m}\right) \;, \tag{2}$$

for some function $f(z) = \sum_{-\infty}^{\infty} f_n z^n$ — you should determine the combination of parameters A in such a way that W_{eff} has the correct charge under each symmetry. By taking appropriate limits, constrain the form of $f(z)$.

(iii) Find the form of $f(z)$ required to match with the microscopic superpotential W_{tree} at weak coupling, and thus conclude that

$$W_{\text{eff}} \,=\, W_{\text{tree}} \;. \tag{3}$$

What does this imply in terms of renormalisation, including non-perturbative corrections?

(iv) Give the explicit tree-level potential of the theory.

(v) Briefly explain in terms of the component fields how SUSY cancels quadratic divergences in one-loop corrections to the scalar propagator in this model. You should include Feynman diagrams in your explanation, but you need not calculate them explicitly.

END OF PAPER