UNIVERSITY OF CAMBRIDGE

MAMA/343, NST3AS/343, MAAS/343

MAT3

MATHEMATICAL TRIPOS Part III

Monday 10 June 2024 9:00 am to 11:00 am

PAPER 343

QUANTUM ENTANGLEMENT IN MANY-BODY PHYSICS

Before you begin please read these instructions carefully

Candidates have TWO HOURS to complete the written examination.

Attempt no more than **THREE** questions.
There are **FOUR** questions in total.
The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

a. Discuss and define completely positive (CP) maps, and show that they can be characterized by specifying a finite set of Kraus operators $\{A^i\}$.

b. Define a matrix product state (MPS), and demonstrate that a MPS can be interpreted as an unravelling or, equivalently, a purifiction of a CP-map.

c. Take Kraus operators $A^1 = \sigma_x, A^2 = \sigma_y, A^3 = \sigma_z$; create a MPS with these Kraus operators, and construct a Hamiltonian for which this is the ground state.

2

a. Consider the Heisenberg spin-1/2 antiferromagnetic spin model defined on an infinite dimensional lattice (which means that every spin has infinitely many nearest-neigbours). What is its ground state energy density, and how do you calculate it?

b. State the mean-field ansatz as a corollary of the quantum de Finetti theorem. Explain how this ansatz justifies your solution to part (a). Does your answer in part (a) saturate the bound in the de Finetti theorem?

3

a. Consider the nearest-neigbour spin-1/2 Heisenberg antiferromagnet, defined on an infinite 1-dimensional quantum spin chain. Show that the Lieb-Robinson bound for the Heisenberg ferromagnet defined on the same chain gives rise to exactly the same bound as for the antiferromagnet.

b. Lieb-Robinson bounds can be used to prove lower bounds on the depth of quantum circuits to create certain states. One such example is the quantum circuit to map a product state $|00\cdots 0\rangle$ to the GHZ-state $\left(|00\cdots 0\rangle + |11\cdots 1\rangle\right)/\sqrt{2}$. Make use of Lieb-Robinson bounds to argue that there is no constant-depth quantum circuit that can do this. What is the significance of this result for the characterization of phases of matter?

4

a. State and prove the fundamental theorem of matrix product states.

b. The most famous matrix product states are the AKLT state and the cluster state. Define these states. How are these states related to each other? What kind of topological order do these states exhibit?

END OF PAPER