UNIVERSITY OF CAMBRIDGE

MAMA/325, NST3AS/325, MAAS/325

MAT3

MATHEMATICAL TRIPOS Part III

Monday 10 June 2024 1:30 pm to 3:30 pm

PAPER 325

QUANTUM INFORMATION, FOUNDATIONS AND GRAVITY

Before you begin please read these instructions carefully

Candidates have TWO HOURS to complete the written examination.

Attempt no more than **THREE** questions.
There are **FOUR** questions in total.
The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 State the measurement postulate for projective measurements in quantum theory, and explain how this applies to a projective measurement on one subsystem of a system S comprising two subsystems, S_1 and S_2. Explain what is meant by saying subsystems S_1 and S_2 are isolated.

Explain carefully what is meant by the *reduced density matrix* ρ_1 of subsystem S_1. Show carefully that if S_1 and S_2 are isolated, no projective measurement on S_1 can give any information about whether or not a projective measurement was previously carried out on S_2. Explain the significance of this for relativistic quantum theory.

A hypothetical device D has a classical input setting that allows any hermitian observable A to be specified. It has the property that, when applied to a localised quantum subsystem S_j for which A is defined, it produces as output one of the eigenvalues λ_i of A, chosen randomly with probability $p_i = \mathrm{Tr}(P_i \rho_j)$, where P_i is the projection onto the eigenspace of λ_i and ρ_j is the reduced density matrix of S_j. It does not alter the quantum state of the overall system S that S_j is part of. The device may be used as many times as required, with different observables if required. Show that no such device can be constructed according to the laws of quantum theory.

In a hypothetical world in which quantum theory was extended by such devices, would superluminal signalling necessarily be possible? Justify your answer carefully.

2 Separated parties A and B carry out spin measurements about axes $\mathbf{a}, \mathbf{b}$ on a singlet state of two spin-1/2 particles. Explain carefully the implications of the Einstein-Podolsky-Rosen (EPR) argument for this experiment. Give a careful account of what is meant, in the context of the EPR argument, by the hypotheses that these measurements satisfy (i) outcome determinism, (ii) parameter independence, (iii) measurement independence.

Show that the predictions of quantum theory are inconsistent with the combination of these three hypotheses, justifying your argument and calculations carefully.

3 (a) Describe briefly the Page-Geilker experiment, explaining the actual outcome and the alternative outcome that was discussed by Page and Geilker but not observed. Explain carefully what Page and Geilker intended the experiment to test, beyond what was previously implied by experiment and observation.

(b) Write down the Schrödinger equation for a neutron of mass m_n in the Earth's gravitational field Φ_E. Describe the set-up of the Colella-Overhauser-Werner interference experiment. Derive an expression for the phase difference between the two paths in the experiment in terms of the horizontal interference path segment length s, the height difference r between the two horizontal path segments, the gravitational acceleration g due to Earth, the wavelength λ of the source neutron beam and Planck's constant h. (You may neglect effects due to the thickness of the reflecting slabs in the experiment.)

Obtain an expression for the number of oscillations (to the nearest integer) expected in the interference pattern as the apparatus is rotated from horizontal to vertical, assuming that the apparatus does not deform during rotation, when $\lambda = 1.42 \times 10^{-10}$m, $s = \sqrt{10}$cm, and the maximum height difference $r = \sqrt{10}$cm. (You may take $g = 9.8\text{ms}^{-1}$, $h = 6.6 \times 10^{-34}\ \text{m}^2\text{kg s}^{-1}$, $m_n = 1.67 \times 10^{-27}$kg.)

(c) Does the Page-Geilker experiment test anything (however implausible) not tested by the Colella-Overhauser-Werner experiment, and vice versa? Justify your answers.

4 Write down the Schrödinger equation for two particles of masses m_1, m_2 interacting gravitationally, assuming that all other forces (including external gravitational fields) are negligible. Describe how the state evolves from an initial state in which particle i is in a superposition state of the form $\frac{1}{\sqrt{2}}(\psi_{0i} + \psi_{1i})$, where ψ_{ai} is localised around horizontal position x_{ai} for $i = 1, 2$ and $a = 0, 1$, and the particles are allowed to fall freely.

Suppose now that the separation $|x_{11} - x_{02}|$ is much smaller than the other three separations. Estimate how long it will take to create a near-maximally entangled state if $m_1 = m_2 = 10^{-14}$kg and $|x_{11} - x_{02}| = 2 \times 10^{-4}$m, justifying your calculations carefully.

Could a near-maximally entangled state be created in this way if the Schrödinger equation instead involved a single classical gravitational potential throughout the space-time region in which the experiment takes place? Justify your answer.

Explain what is meant by an entanglement witness. Give an example of an entanglement witness that would distinguish the near-maximally entangled state generated by the procedure above from any separable state that is a linear combination of product states $\psi_{a1}\psi_{b2}$, where $a, b \in \{0, 1\}$. Explain briefly the relevance for obtaining evidence about the quantum nature of gravity.

(You may take Newton's gravitational constant $G = 6.7 \times 10^{-11}\ \text{m}^3\ \text{kg}^{-1}\ \text{s}^{-2}$ and Planck's constant $h = 6.6 \times 10^{-34}\ \text{m}^2\text{kg s}^{-1}$, with $\hbar = h/2\pi$.)

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