MAT3

MATHEMATICAL TRIPOS Part III

Tuesday, 6 June, 2023 9:00 am to 12:00 pm

PAPER 349

THE LIFE AND DEATH OF GALAXIES

Before you begin please read these instructions carefully

Candidates have THREE HOURS to complete the written examination.

Attempt no more than **THREE** questions.
There are **FOUR** questions in total.
The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 The Jeans equations are used to predict the behaviour of the radial and tangential velocity dispersion components σ_r and σ_t of a chosen set of luminous tracers given the galaxy's overal potential $\Phi(r)$ and and their density $\nu(r)$.

In a spherically symmetric galaxy in equilibrium, velocity dispersion components must satisfy the Jeans equation:

$$\frac{1}{\nu}\frac{\mathrm{d}}{\mathrm{d}r}\left(\nu\sigma_r^2\right)+\frac{2(\sigma_r^2-\sigma_t^2)}{r}=-\frac{\mathrm{d}\Phi(r)}{\mathrm{d}r}$$

Assuming that both the tracer density and the circular velocity V_c follow power laws, namely $\nu \propto r^{-\alpha}$ and $V_c \propto r^{\gamma}$ and the orbital anisotropy $\beta = 1-\sigma_t^2\sigma_r^{-2}$ is constant, derive the dependence of $\sigma^2(r)$ on V_c^2 and α, β, γ. Discuss physically possible solutions.

For a canonical first-order linear differential equation $y'+yA(x)=B(x)$, the solution for y can be found with the help of integrating factor $f(x)=e^{\int A(x)dx}$

$$y=\frac{1}{f(x)}\int B(x)f(x)dx$$

Discuss the observational aspects of using the Jeans equation in the Milky Way's halo and focus on the mass-anisotropy-density degeneracy arising in Jeans kinematic modeling. You might want to use the $\sigma_r = f(V_c, \alpha, \beta, \gamma)$ relation derived above to illustrate the degeneracy. Start with the simple case when the power laws describing tracer density, anisotropy and circular velocity remain constant throughout the galaxy. Then consider a more complicated case when these can change behaviour at some radius.

What other, more flexible and less degenerate solutions of the Collisionless Boltzmann Equation are available for modeling of the tracer kinematics instead of the Jeans equations?

2

Stellar feedback by massive star winds and core-collapse supernovae is the process of expulsion of metal-enriched gas from sites of star formation. List and describe known pieces of direct and indirect observational evidence in favor of stellar feedback in galaxies. Discuss the effects non-adiabatic removal of large amounts of gas can have on orbits of stars and dark matter particles in a galaxy undergoing bursty star formation. Why is it important for our understanding of Dark Matter properties in dwarf galaxies?

Consider a dwarf galaxy with a mass M and size R losing gas non-adiabatically as a result of a strong burst of star formation. Some time after the mass loss, the dwarf virializes and reaches its new state with a size R'. Assume that mass changes abruptly from M to $M' = \epsilon M$ and use the Virial theorem to deduce the dependence of new size R' on ϵ. What values of ϵ lead to a complete disruption of the dwarf galaxy?

Alternatively, the dwarf can reach its new, lower-mass state slowly. Assuming small changes in mass, integrate the Taylor-expanded version of the expression for the size evolution you obtained above to find the size dependence on ϵ when the total mass change is achieved adiabatically.

Focusing on the rapid mass loss in a Keplerian potential, calculate the evolution of the orbital eccentricity e as a function of ϵ assuming that initially the orbit is circular.

3

Star Formation law (also known as Schmidt-Kennicutt law) describes an empirical connection between the rate at which the gas is converted into stars and the state of the gas in a star-forming galaxy. Describe the SF law and the details of obtaining the observational ingredients used to build the correlation. In terms of the timescales involved in gas consumption, how efficient is star formation? You may wish to compare typical star-formation timescales to free-fall times of giant molecular clouds.

First, assume that the galaxy's star formation is described by a constant timescale, i.e.:

$$\tau_* = \frac{M_g}{\dot{M}_*} = \text{const},$$

where M_g is the gas mass and $\dot{M}_*$ is the star formation rate. Using the Closed Box model of galactic chemical evolution, derive $M_g(t)$, $\dot{M}_*$ and $Z(t)$, in other words deduce how the gas mass, the star formation rate and the metallicity evolve with time. Using the heavy element yield $y_Z = 0.006$, what is the star formation timescale in the Solar neighborhood of the Milky Way where the Sun is the most metal rich star at $Z_\odot = 0.014$?

For the second model, use an effective SF timescale $\tilde{\tau}_*$ corresponding to increasing τ_* with time:

$$\tilde{\tau}_* = \frac{M_g}{\dot{M}_*}\frac{M_g}{M_g(0)} = \text{const}$$

How does the gas mass, the SFR and the metallicity evolve with time in this model? What is the SF timescale $\tilde{\tau}_*$ in the Solar neighborhood?

Derive expressions for the metallicity distribution $\frac{dN}{dZ}$ in each case. What do you conclude?

4

Spectra of star-forming galaxies look very different from those that have ceased all star formation activity. In terms of the sources of radiation, what is the main difference? Describe the principal mechanisms contributing to the radiation from star-forming regions.

To model emission from a bubble with uniform hydrogen density n_{H} and recombination rate density α around a radiation source with ionisation rate $\dot{N}_{\mathrm{ion}}$, the so-called Strömgren sphere is invoked. Describe the Strömgren sphere model. Focusing on the expansion phase of the sphere, derive the differential equation governing the change in its size with time $R(t)$ and solve it to obtain the dependence of R on t. You might find useful to switch from physical size and time to the size relative to the Strömgren sphere radius R_{S}, and the time relative to the recombination time $t_{\mathrm{rec}} = (\alpha n_{\mathrm{H}})^{-1}$

END OF PAPER