UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 14 June, 2022 9:00 am to 11:00 am

PAPER 322

BINARY STARS

Before you begin please read these instructions carefully

Candidates have TWO HOURS to complete the written examination.

Attempt no more than **TWO** questions.
There are **THREE** questions in total.
The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

A cataclysmic variable comprises a white dwarf of mass M_1 and a low-mass main-sequence star of mass M_2, that is filling its Roche lobe, in a circular orbit of separation a and period $P = 2\pi/\Omega$. Assume that the spin angular momentum of the stars is negligible and show that the orbital angular momentum of the system is

$$J = \frac{M_1 M_2}{M} a^2 \Omega,$$

where $M = M_1 + M_2$.

The system is losing angular momentum by magnetic braking at a rate

$$-\frac{\dot{J}}{J} = \alpha$$

and the radius of star 2's Roche lobe can be written as

$$R_{\mathrm{L}} = 0.462 \left(\frac{M_2}{M}\right)^{\frac{1}{3}} a.$$

Show that conservative mass transfer is dynamically stable as long as

$$q = \frac{M_2}{M_1} < \frac{3\zeta_{\mathrm{ad}} + 5}{6}, \qquad (*)$$

where

$$\zeta_{\mathrm{ad}} = \frac{\partial \log R_2}{\partial \log M_2}$$

is the adiabatic response of star 2 to mass loss.

Now suppose $(*)$ is satisfied and that on the time scale of the mass transfer star 2 responds so that

$$\frac{\dot{R}_2}{R_2} = \beta > 0.$$

Show that, when the total system mass is conserved, mass transfer proceeds at a rate

$$-\frac{\dot{M}_2}{M_2} = \frac{3(2\alpha + \beta)}{5 - 6q}.$$

Explain briefly why cataclysmic variables experience nova eruptions and why they can eject all the accreted material.

Now, in addition, suppose that all the material transferred to the white dwarf is periodically ejected and carries off the specific angular momentum of the white dwarf in its orbit. On a timescale that is long compared with that between eruptions show that the mass transfer rate becomes

$$-\frac{\dot{M}_2}{M_2} = \frac{3(1+q)(2\alpha + \beta)}{5 + 3q - 6q^2}.$$

2

A binary system of total mass M consists of two stars of masses M_1 and M_2 in an elliptical orbit and sufficiently separated that they behave as point mass objects. Instantaneously, the vector from star 2 to star 1 is $\boldsymbol{r}$ and the velocity of star 1 relative to star 2 is $\boldsymbol{v} = \dot{\boldsymbol{r}}$. Show that the energy of the orbit is

$$E = \mu \left(\frac{1}{2} v^2 - \frac{GM}{r} \right),$$

where $\mu = M_1 M_2 / M$ is the reduced mass, $r = |\boldsymbol{r}|$ and $v = |\boldsymbol{v}|$ are the instantaneous separation and relative speed of the stars and G is Newton's constant. Show further that the orbit can be parametrised by

$$r = \frac{l}{1 + e\cos\theta},$$

where l and e are constants and $0 \leqslant \theta < 2\pi$, and that

$$E = -\frac{GM\mu}{2a} = \text{const},$$

where $l = a(1 - e^2)$ and a is the semi-major axis. What do θ and e represent?

In a system with a circular orbit, star 1 of mass M_1 undergoes a supernova explosion to leave a neutron star of mass M_1' in a time much shorter than the orbital period. The mass lost to the gravitational binding energy of the neutron star may be neglected. Matter is ejected in an asymmetric manner such that the neutron star experiences a kick of speed $u = \alpha v$, with $\alpha < 1$, at an angle ψ to $\boldsymbol{v}$. Show that the new total mass M' and semi-major axis a' are related by

$$\frac{M'}{a'} = \frac{2M'}{a} - (1 + 2\alpha\cos\psi + \alpha^2)\frac{M}{a}$$

and from there that the system always remains bound if

$$M' > \frac{1}{2}(1+\alpha)^2 M$$

but always unbinds if

$$M' < \frac{1}{2}(1-\alpha)^2 M.$$

When the system becomes unbound, show that the stars eventually recede from one another at a speed

$$V = \left\{ (1 + 2\alpha\cos\psi + \alpha^2) - \frac{2M'}{M} \right\}^{\frac{1}{2}} v.$$

3

Explain why a process of common envelope evolution in binary stars is postulated and discuss what initiates it, how it proceeds and what are its possible outcomes.

Describe how the energy formalism is used to model the outcome of a phase of common envelope evolution.

Discuss how an accreting carbon–oxygen white dwarf can explode as a type Ia supernova paying attention to the energy required to create and power it.

In the double-degenerate scenario for type Ia supernovae the immediate progenitor of the supernova is a close binary system comprising two white dwarfs. Use your knowledge of binary star evolution to describe, without detailed calculation, how a zero-age system comprising 5 and $8\,M_{\odot}$ main-sequence stars could evolve to such a type Ia supernova. In your discussion make clear what physical process is driving the evolution at each stage and how the separation of the two stars evolves with time.

What major difficulty is encountered by this scenario if mass transfer between the white dwarfs is dynamically stable? How might this be overcome if the mass transfer is dynamically unstable?

END OF PAPER