UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 9 June, 2022 9:00 am to 12:00 pm

PAPER 305

THE STANDARD MODEL

Before you begin please read these instructions carefully

Candidates have THREE HOURS to complete the written examination.

Attempt no more than **THREE** questions.
There are **FOUR** questions in total.
The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Provide a short answer (less than roughly 8-10 lines) to each of the following questions:

(i) Explain why the strong and weak interactions are short range while electromagnetic interactions are long range.

(ii) Explain what is meant by asymptotic freedom and Λ_{QCD} and outline their importance.

(iii) Explain why the approximate isospin and eightfold-way symmetries gave rise to QCD and why QCD explains both isospin and eightfold way. Why are there not higher approximate symmetries than the $SU(3)$ of the eightfold-way?

(iv) Explain the origin of baryon and lepton number conservation from the Standard Model.

(v) Explain why the Higgs field provides a mass to quarks, leptons and gauge fields. Which Standard Model fields do not get a mass from the Higgs?

(vi) Identify an operator of dimension 5 that gives a mass to the neutrinos and an operator of dimension 6 that may induce proton decay. What can we learn from this?

(vii) State what is meant by neutrino oscillations and why they are relevant to understand neutrino physics.

(viii) Give two reasons for the existence of an even number of quarks.

2

Consider the unitary irreducible representations of the Poincaré group and write the labels on the one-particle states for massless and massive particles.

(i) Determine the number of degrees of freedom of massless and massive particles of helicity/spin equal to $0, 1/2, 1, 2$ respectively. Identify the constraints, if any, needed to be imposed for fields $\phi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x)$ respectively, in order to capture the appropriate number of degrees of freedom. For the massless field $A_\mu(x)$ state what is meant by a redundancy and relate this to gauge invariance.

(ii) Describe what is meant by the Ward identity for matrix elements involving massless vector fields.

(iii) Use the Ward identity in amplitudes with soft photons ($|\lambda| = 1$) to prove the conservation of charge. What can be said for helicities $|\lambda| \geqslant 2$?

(iv) Outline a general problem for interacting massive spin 1 particles and how the Standard Model solves it.

3 Consider a theory of N scalar fields $\varphi_n, n = 1, \cdots, N$ invariant under the infinitesimal global transformations $\delta\varphi_m = \mathrm{i}\alpha^{\mathrm{a}}\mathrm{T}^{\mathrm{a}}_{\mathrm{mn}}\varphi_{\mathrm{n}}$, where $T^a_{mn}, a = 1, \cdots, \dim G$ are the generators of the algebra of a compact continuous group G acting on the representation of the fields φ_m and α^a the parameters of the transformation. The symmetry under G leads to a conserved Noether's current J^a_μ.

(i) Show that the conserved charges $Q^a = \int d^3x J^a_0$ are also the generators of the infinitesimal transformations $\delta\varphi_m = \mathrm{i}\alpha^{\mathrm{a}}[\varphi_{\mathrm{m}}, \mathrm{Q}^{\mathrm{a}}] = \mathrm{i}\alpha^{\mathrm{a}}\mathrm{T}^{\mathrm{a}}_{\mathrm{mn}}\varphi_{\mathrm{n}}$.

(ii) Use these charges to define the order parameter for spontaneous symmetry breaking (SSB).

(iii) Show that if the symmetry is spontaneously broken, particle states related by the symmetry are not necessarily degenerate in energy and that the broken states $Q^a|0\rangle$ are degenerate with the vacuum $|0\rangle$.

(iv) State and prove Goldstone's theorem.

(v) Consider now the local symmetry $G = U(1)$ with gauge field $A_\mu(x)$. Write down the most general renormalisable Lagrangian in 4-dimensions coupling the charged scalar field φ and A_μ. Identify the number of independent parameters in the Lagrangian density and establish the range of these parameters so that the theory is stable and manifests SSB. Consider fluctuations around the minimum and determine the masses and couplings of the physical particles. Based on this example explain what is meant by the Higgs mechanism.

4 Consider the Standard Model spectrum for which under $SU(3)_c \times SU(2)_L \times U(1)_Y$ the components of each family of matter fields transform as $\left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right) + \left(\bar{\mathbf{3}}, \mathbf{1}, \frac{2}{3}\right) + \left(\bar{\mathbf{3}}, \mathbf{1}, -\frac{1}{3}\right) + \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right) + (\mathbf{1}, \mathbf{1}, -1)$ including also a right handed neutrino $(\mathbf{1}, \mathbf{1}, 0)$.

(i) Show that all the perturbative gauge and gravitational anomalies are cancelled with this spectrum.

(ii) What would have gone wrong if the anomalies would not have cancelled?

(iii) Is baryon number anomalous? How significant is this?

(iv) By considering the couplings of the $W^\pm$ and Higgs bosons to quarks define the Cabibbo-Kobayashi-Maskawa (CKM) matrix and determine the number of independent parameters.

(v) Show two different sources of CP violation in the Standard Model. Can they be related?

END OF PAPER