UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 11 June, 2021 12:00 pm to 3:00 pm

PAPER 315

EXTRASOLAR PLANETS - ATMOSPHERES AND INTERIORS

Before you begin please read these instructions carefully

Candidates have THREE HOURS to complete the written examination.

Attempt no more than ***THREE*** *questions.*
There are ***FOUR*** *questions in total.*
The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

a. A transiting hot jupiter of the same mass as jupiter and equilibrium temperature 1250 K orbits a sun-like star. A thermal emission spectrum of the planet was observed in the 1-25 μm wavelength range with the James Webb Space Telescope (JWST). Atmospheric retrieval of the spectrum provided constraints on the pressure-temperature (P-T) profile in the atmosphere. The temperature structure was found to be isothermal at 800 K for pressures of 1 mbar and lower. For higher pressures, the temperature gradient in the observable atmosphere varied as $dT/d\log P = A\log P$, where P is pressure in mbar and A is a constant. The optical and infrared photosphere is at 1 bar. Estimate the temperature at a pressure of 10 bar. Sketch a plausible P-T profile of the atmosphere between 10^{-5}–100 bar. Estimate a plausible value for A. Make and state any assumptions required.

b. Derive an expression for temperature gradient with distance in the atmosphere, dT/dz, as a function of P.

c. Consider a region of wavelength in a 1 μm wide bin centred at 17 μm where there is no significant molecular line absorption. Estimate the planet-star flux ratio in this spectral bin. Make and state any assumptions required.

d. The planet-star flux ratio in the spectral bin above was measured with JWST at a precision of 100 ppm. Assuming the same precision, determine the signal-to-noise ratio with which thermal emission can be detected in the same bin from an Earth-size bare rock planet at a surface temperature of 600 K transiting an M dwarf star. Assume the radius of the M dwarf star to be 0.1 solar radius and its effective temperature to be 3500 K. Sketch the planet-star flux ratio of this system as a function of wavelength between 1–20 μm as observed at secondary eclipse assuming the star to be emitting as a blackbody .

2

Consider a transiting exoplanet of mass M_p composed of two concentric spherical layers, an inner core of mass M_1 and an outer mantle containing the remaining mass. Both the core and the mantle are of uniform densities ρ_1 and ρ_2, respectively. The temperature and pressure at the surface of the planet are T_0 and P_0, respectively.

a. Derive the mass profile $M(r)$ of the planet where $M(r)$ is the mass interior to a spherical shell of radius r from the centre of the planet. Determine the radius of the core and the surface radius of the planet. Express your answers in terms of any of the above quantities.

b. Derive the pressure at the boundary between the two layers. Make and state any assumptions required.

c. Derive the pressure profile $P(r)$ in the interior and the pressure at the centre of the planet.

d. Consider a thin isothermal atmosphere on top of the planetary surface with a mean molecular weight of μ. If the planetary photosphere in transit is observed to be at a pressure of P_phot, determine the corresponding observed radius of the planet.

e. Derive an expression for the mass of the atmosphere described in part d.

3

a. Consider a one-dimensional atmosphere of an isolated planet in radiative equilibrium. Starting with the radiative transfer equation show that the total flux in the atmosphere is constant with depth. If the specific intensity from the planet may be assumed to be a Planck function, show that the internal flux of the planet may be represented by an effective temperature T_eff. You may state and use results of standard integrals.

b. Consider an exoplanet of mass M_p and radius R_p transiting a star of radius R_s. The atmosphere of the planet is isothermal with a temperature T_p, a constant density ρ with solar abundance elemental composition, and a uniform extinction coefficient k_ν. Assuming plane parallel geometry derive the expression for the transmission spectrum of the planet. You may assume a constant path length l for a ray of starlight passing through the planetary atmosphere. Express your answer in terms of the quantities given here, relevant fundamental constants and any additional quantities assumed.

4

a. An isolated planet of mass $M_{\rm p}$ and radius $R_{\rm p}$ radiates with an effective temperature T_0. The planet migrates into a close-in orbit around a sun-like star at an orbital separation a and becomes tidally locked. Derive the equilibrium temperature of the planet. Make and state any assumptions required.

b. Considering a single-layer homogeneous atmosphere of a planet show how the surface temperature relates to the equilibrium temperature. Make and state any assumptions required.

c. Show that the specific intensity is invariant with distance between the source and receiver. Consider a planet with radius $R_{\rm p}$ and an isothermal atmosphere at a temperature T. Derive the luminosity of the planet.

d. Demonstrate why a thermal inversion causes emission features in an emergent spectrum rather than absorption features. List three factors that can affect thermal inversions in planetary atmospheres.

e. Determine the condition on the Gibbs free energy for a system to be in thermochemical equilibrium at a given pressure and temperature. List examples of three processes that can drive an atmosphere out of thermochemical equilibrium.

END OF PAPER