UNIVERSITY OF CAMBRIDGE

MAT3, MAMA, NST3AS, MAAS

MATHEMATICAL TRIPOS Part III

Monday, 10 June, 2019 9:00 am to 11:00 am

PAPER 349

EVOLUTION OF GALAXIES

Attempt no more than ***TWO*** *questions.*

There are ***THREE*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

An emission nebula of pure hydrogen with a uniform number density n forms around a young massive star. Assuming that the UV photons are generated with a rate $\dot{N}_{\rm ion}$ and the recombination coefficient is α, derive the radius of the Strömgren sphere, i.e. the size of the gas bubble where the ionization balances recombination. Consider the expansion phase of the nebula formation and derive how the radius of the ionization front scaled by the size of the Strömgren sphere changes with time.

For a more realistic example of a nebula, consider a power-law number density of hydrogen with the power-law index β and the density normalization n_0. Derive how the recombination rate depends on the steepness of the nebula density profile. Finally, make the nebula density profile cored with a core radius r_c to avoid singularity in the centre. Show that for a certain choice of n_0, β and r_c, a critical ionization rate exists above which the recombination can never balance ionization.

2

Consider the radial first moment Jeans equation in spherical polar coordinates:

$$
\begin{aligned}
-\nu \frac{\partial \Phi}{\partial r} =& \nu \frac{\partial \overline{v}_r}{\partial t} + \nu \left(\overline{v}_r \frac{\partial \overline{v}_r}{\partial r} + \frac{\overline{v}_\theta}{r} \frac{\partial \overline{v}_r}{\partial \theta} + \frac{\overline{v}_\phi}{r \sin\theta} \frac{\partial \overline{v}_r}{\partial \phi} \right) \\
&+ \frac{\partial}{\partial r}(\nu \sigma_{rr}^2) + \frac{1}{r}\frac{\partial}{\partial \theta}(\nu \sigma_{r\theta}^2) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \phi}(\nu \sigma_{r\phi}^2) \\
&+ \frac{\nu}{r}[2\sigma_{rr}^2 - (\sigma_{\theta\theta}^2 + \sigma_{\phi\phi}^2 + \overline{v}_\theta^2 + \overline{v}_\phi^2) + \sigma_{r\theta}^2 \cot\theta],
\end{aligned}
$$

where Φ is the underlying gravitational potential, ν is the density of tracer objects and v_r, v_θ, v_ϕ are the velocity components.

Simplify the above equation assuming that the spherically symmetric system is in a steady state and introducing the orbital anisotropy parameter $\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$, where $\sigma_t = \sigma_{\theta\theta} = \sigma_{\phi\phi}$

Show that the resulting Jeans equation can be re-written to express the mass enclosed within radius r as follows:

$$M(\leqslant r) = -\frac{r\sigma_{rr}^2}{G}\left[\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_{rr}^2}{d\ln r} + 2\beta(r)\right]$$

Assuming that the velocity anisotropy is constant and that both the tracer density and the galaxy's circular velocity follow power law as a function of r, i.e. $\nu \propto r^{-\gamma}$ and $V_c \propto r^{\alpha}$, show that:

$$\sigma_r^2(r) = \frac{1}{\gamma - 2\beta - 2\alpha} V_c^2(r)$$

The above equations can be used to determine the mass distribution in the Milky Way. The necessary data comprise heliocentric line-of-sight velocities (corrected for solar motion) for a large number of directions on the sky, thus yielding the velocity dispersion $\sigma_\odot$. Before the Jeans equations above can be used to deduce the Galaxy's mass, the heliocentric velocity dispersion must be corrected to take into account the off-centre position of the Sun. By assuming that the Sun is located $R_\odot$ kpc away from the Galactic centre and averaging over many directions on the sphere show that:

$$\sigma_\odot(r) = \sigma_r(r)\sqrt{1 - \beta(r)A(r, R_\odot)},$$

where $A(r, R_\odot)$ is a geometric correction factor.

3

Consider the following simple modification of the Closed Box chemical enrichment model. To mimic the stellar feedback associated with the star-formation, assume that an amount of mass proportional to the star-formation rate, i.e. $\alpha \dot{M}_s$ can be completely removed from the system. Derive the evolution of the gas metallicity $Z = \frac{M_h}{M_g}$ as a function of time in this Leaky Box model with yield p. Here, M_s is the total mass of stars in the galaxy, M_h is the total mass of heavy elements, M_g is the total mass of gas. Compare the Leaky Box solution to the Closed Box solution. State your assumptions (e.g. as to the initial mass in stars and the metallicity of the expelled gas) carefully.

END OF PAPER