UNIVERSITY OF CAMBRIDGE

MAT3, MAMA, NST3AS

MATHEMATICAL TRIPOS Part III

Thursday, 30 May, 2019 1:30 pm to 3:30 pm

PAPER 328

BOUNDARY VALUE PROBLEMS FOR LINEAR PDES

Attempt no more than ***TWO*** *questions.*

There are ***THREE*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Let the real valued function $u(x,y)$ satisfy the Laplace equation

$$u_{xx} + u_{yy} = 0,$$

in the first quadrant of the (x,y) plane, namely in the domain $0 < x < \infty,\ 0 < y < \infty$, together with the boundary conditions

$$u_y(0,y) = g_1(y),\ 0 < y < \infty, \qquad u_x(x,0) = g_2(x),\ 0 < x < \infty,$$

where $g_1(y)$ and $g_2(x)$ are given functions which decay for large y and x, respectively.

Express the derivative $u_z,\ z = x + iy$ as an integral representation of $g_1(y)$ and $g_2(x)$.

2 Let $u(x,t)$ satisfy the equation

$$u_t = u_{xx} + \beta u_x, \qquad 0 < x < L,\ \ 0 < t < T,$$

and the initial-boundary conditions

$$\begin{aligned} u(x,0) &= u_0(x), & 0 < x < L, \\ u_x(0,t) &= g(t), & 0 < t < T, \\ u_x(L,t) &= h(t), & 0 < t < T, \end{aligned}$$

where $\beta,\ L$ and T are given positive constants, the given functions $u_0,\ g$ and h have sufficient smoothness, $\dot{u}_0(0) = g(0)$, and $\dot{u}_0(L) = h(0)$.

Obtain an integral representation for the solution $u(x,t)$ in terms of $u_0(x),\ g(t)$ and $h(t)$.

3 Let $u(x,t)$ satisfy the equation

$$u_t + u_{xxx} = 0, \qquad 0 < x < \infty,\ \ 0 < t < T,$$

and the initial boundary conditions

$$\begin{aligned} u(x,0) &= u_0(x), & 0 < x < \infty, \\ u_x(0,t) &= g(t), & 0 < t < T, \end{aligned}$$

where T is a given positive constant, the given functions u_0 and g_0 have sufficient smoothness and $\dot{u}_0(0) = g(0)$.

Obtain an integral representation for the solution $u(x,t)$ in terms of $u_0(x)$ and $g(t)$.

END OF PAPER