UNIVERSITY OF CAMBRIDGE

MAT3, MAMA, NST3AS, MAAS

MATHEMATICAL TRIPOS Part III

Thursday, 30 May, 2019 1:30 pm to 4:30 pm

PAPER 317

STRUCTURE AND EVOLUTION OF STARS

Attempt no more than ***THREE*** *questions.*

There are ***FOUR*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Consider a group of stars, all of which have the same homogeneous chemical compoition and are built from an ideal gas. Radiation pressure can be neglected. The energy generation occurs through the *p-p* chain with $\epsilon = \epsilon_{\rm pp}\,\varrho T^4$, where $\epsilon_{\rm pp}$ is a constant, ϱ the density and T the temperature. All the energy is transported by radiation. Opacity is given by Kramer's formula $\kappa = \kappa_0\,\varrho T^{-3.5}$, where κ_0 is a constant. M is the total mass of a star, R its radius, L its luminosity and $T_{\rm eff}$ is the stars effecive temperature.

Use the appropriate stellar structure equations to derive the following expressions, clearly stating any additional assumptions you need to achieve your goal:

(i) $M \propto R^{13}$

(ii) $L \propto M^{71/13}$

(iii) $T_{\rm eff} \propto L^{\rm q}$ where you should give the value of q and obtain the slope of a line on the Hertzsprung-Russell diagram occupied by the discussed stars.

(iv) How would your results in (i), (ii) and (iii) change if the energy generation occured through the CNO cycle with $\epsilon = \epsilon_{\rm CNO}\,\varrho T^{16}$ and opacity were given by electron scattering with $\kappa =$ constant?

2

(i) Derive the Schwarzschild and Ledoux criteria for convective instability.

(ii) Consider a stellar atmosphere composed of a perfect monatomic gas of uniform composition. The atmosphere is in radiative equilibrium with temperature profile $T = T(T_{\rm eff}, \tau)$ where $T_{\rm eff}$ is the effective temperature of the star and τ is the optical depth. This temperature profile arises from the Eddington closure approximation for the outer boundary condition at the stellar surface. You should specify the temperature profile because you will need it to solve the problem.

Assume that the pressure profile in the atmosphere is given by $P^2 = P_0^2\,\ln(1 + \frac{3}{2}\tau)$.

Show that the convection sets when $\tau = \frac{2}{3}\left(\exp(\frac{4}{5}) - 1\right)$.

3

(i) Consider a cold white dwarf supported by pressure arising from a fully degenerate electron gas. Derive the appropriate general equation of state and find limits for both the non-relativistic and relativistic limits stating all necessary assumptions.

(ii) Suppose that the white dwarf is supported by non-relativistic degeneracy pressure.

(a) Use the appropriate stellar structure equations to derive an equation describing the structure of this white dwarf.

(b) Continue to obtain an expression that relates the radius R to the total mass M of this white dwarf in a form $M \propto R^{\mathrm{s}}$ and specify s.

4

(i) A spherical star is composed of a gas with equation of state $P = K\varrho^2$, where K is a constant, P the pressure and ϱ the density. Find $\varrho = \varrho(r)$. Derive an expression for the radius of the star

$$R = \left(\frac{K\pi}{2G}\right)^{\frac{1}{2}}.$$

Show that the ratio of its mean density $\overline{\varrho}$ to its central density ϱ_{c} is:

$$\frac{\overline{\varrho}}{\varrho_{\mathrm{c}}} = \frac{3}{\pi^2}$$

(ii) A cubic star of volume L^3 is composed of a gas obeying the same equation of state as above. Show that it is possible to construct a solution to the structure equations for $0 \leqslant x \leqslant L$, $0 \leqslant y \leqslant L$ and $0 \leqslant z \leqslant L$ such that ϱ vanishes on the faces of the cube. Find $\varrho = \varrho(x, y, z)$.

For this cubic star derive an expression

$$\frac{\overline{\varrho}}{\varrho_{\mathrm{c}}} = \frac{8}{\pi^3}.$$

Do you expect cubic stars likely to exist in nature on either theoretical or observational grounds? Explain why?

END OF PAPER