UNIVERSITY OF CAMBRIDGE

MAT3, MAMA, NST3AS, MAAS

MATHEMATICAL TRIPOS Part III

Friday, 31 May, 2019 9:00 am to 12:00 pm

PAPER 315

EXTRASOLAR PLANETS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper
Rough paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

a. Show that the amplitude of a spectral feature in the transmission spectrum of a transiting planet can be approximated by $\delta_\lambda = A(1 - e^{-\tau_\lambda})$. Derive and define all the terms in this equation. Make and state any assumptions required.

b. A giant exoplanet with the size and mass of Jupiter was discovered to be orbiting a sun-like star in a circular orbit at a distance of 10 parsec from the Earth. The largest angular separation between the star and the planet was 0.01 arcseconds and the orbital inclination was inferred to be 90°, i.e., edge on, which meant a transit of the planet can be observed. Assuming the planetary atmosphere to be isothermal, estimate the minimum flux sensitivity required by an infrared telescope at 10 μm to detect the atmosphere in (i) transmission, and (ii) emission. Express your answer in parts per million (ppm) of flux contrast for a 5-σ detection. Make and state any assumptions required.

c. Using the expression derived in (a) show how a transmission spectrum in the optical range can be used to identify the presence of clouds or hazes in the atmosphere.

2

a. Starting from the radiative transfer equation show how the temperature gradient in an atmosphere determines whether emission, absorption, or no features are seen in the thermal emission spectrum of a planet. Make and state any assumptions required.

b. A circumbinary planet orbits a close binary system consisting of two stars with temperatures T_A and T_B and radii R_A and R_B. The mass and radius of the planet are similar to Jupiter, the orbital separation of the planet's orbit is a, and the separation between the individual stars is a_{AB}, where $a \gg a_{AB}$. Derive an expression for the equilibrium temperature of the planet. Make and state any assumptions required.

Assuming the two stars are main sequence stars with spectral types G and M (e.g., T_A = 6000 K and T_B = 3000 K), estimate the equilibrium temperature of the planet for orbital parameters of your choice. Make and state any assumptions required. Assuming the planet to be isothermal at this temperature, sketch the emergent spectrum of the planet.

c. Three Jupiter-like planets were formed around three sun-like stars at separations similar to that of Jupiter in the solar system. After forming in similar conditions each of them had a different path due to close encounters of their host stars with other stars in the neighbourhood. One of the planets was ejected out of its solar system, another migrated to an orbit one hundred times closer, and the third planet remained unperturbed. Discuss what the temperature structures, compositions, and equilibrium temperatures of the three planets would be in steady state several million years after the event.

3

a. Consider a spherical planet with a mass M and radius R. Use one or more of the structure equations to estimate the minimum pressure (P_c) at the centre of the planet. Why is the actual central pressure in a planet expected to be significantly larger than this estimate? Compare the minimum central pressure between the Earth and Jupiter.

b. A certain theoretical model predicts that gas giant planets form in protoplanetary disks within 3 million years. According to the model such planets are born large and hot and contract over time with the radius evolving as $R \propto t^{-\alpha}$ for a given mass and irradiation. Observations of a Jupiter-mass planet orbiting a young star, of age 5 Myr and four times solar luminosity, at an orbital separation of 10 au found the planetary radius to be 2 Jupiter radii. Estimate α if the model is universally valid. Make and state any assumptions required.

c. Discuss two methods by which inflated giant exoplanets are discovered and how their sizes are measured. State two broad categories of explanations for inflated hot Jupiters and discuss three explanations within each category.

4

a. Discuss what governs the day-night temperature contrast in a hot Jupiter orbiting a sun-like star in a close-in orbit. Discuss the dependance of the temperature contrast on (i) the equilibrium temperature of the planet, and (ii) the height in the atmosphere accessible to infrared observations.

b. If the Sun turned off its radiation today Jupiter would reach a steady state low temperature $T_{\rm cold}$ in τ years, where $\tau \gtrsim$ Gyr. A giant exoplanet ten times more massive than Jupiter formed around a nearby star at the same time as Jupiter in a similar orbit but was then ejected into outer space and left free floating without a host star. What is a characteristic timescale on which it will reach a temperature $T_{\rm cold}$ if it radiates at Jupiter's present luminosity. You may assume that half of Jupiter's present luminosity is powered by solar irradiation and that the luminosity is nearly constant in time. Make and state any further assumptions required.

c. Derive the surface temperature of a rocky planet as a function of its equilibrium temperature assuming a simplified isothermal layer of atmosphere with absorption α in the infrared and transparent in the visible.

d. Consider the following equilibrium reaction in the atmosphere of a gas giant planet with the size and mass of Jupiter: $A + B \rightleftharpoons C$. The chemical timescale ($\tau_{\rm chem}$) for the forward reaction at 1 bar pressure is 10^5 s and the atmospheric temperature profile is an isotherm at 1000 K. Estimate the vertical eddy mixing coefficient (K_{zz}) that would be required to quench A in the atmosphere at 1 bar, and loft it to higher regions of the atmosphere where it may be observable.

If such a quenching is achieved, how would the number density of A vary with altitude in the atmosphere for pressures below the quench pressure. Make and state any reasonable assumptions.

State two examples of this process that can occur in exoplanetary atmospheres.

e. A new transiting hot Jupiter was discovered around a nearby sun-like star. Discuss a brief plan for follow-up observations of the planet to characterise its atmosphere in detail. You should identify the observational method, wavelength range, and potential telescope, that would allow you to characterise at least five different aspects of the atmosphere.

END OF PAPER