UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS   Part III

Tuesday, 12 June, 2018   9:00 am to 11:00 am

PAPER 338

OPTICAL AND INFRARED ASTRONOMICAL TELESCOPES AND INSTRUMENTS

Attempt no more than ***TWO*** *questions.*

There are ***THREE*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS

Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS

None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Telescopes

(a) A telescope with a focal length f is used to take an image of a region of the sky. The telescope has no optical field distortion and the central detector pixel column is perfectly aligned with a great circle of constant right ascension. The optical axis of the telescope passes through the centre of the detector and is pointed at a position B on the sky with right ascension A and (declination) D. A star at position S with celestial coordinates α (right ascension) and δ (declination) is imaged at a position (ξ, η) in a coordinate system aligned with the detector pixel grid and with (0,0) at the centre of the detector. The central pixel column defines the η axis. η increases to the north and ξ increases to the east.

(i) Show that

$$\xi = f\frac{\cos q \tan(\alpha - A)}{\cos(q - D)}$$

$$\eta = f \tan(q - D)$$

where

$$\cot q = \cot\delta\cos(\alpha - A).$$

(ii) Let L be a point on the great circle through the celestial pole P and B such that the angle $P\hat{L}S$ is 90^o. Show that q is the declination of the point L.

[You may find the following useful. For a spherical triangle made of three great circle arcs with apex angles, A, B and C and opposite side angular lengths a, b and c, we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$$

$$\cos a \cos C = \sin a \cot b - \sin C \cot B.$$

Trigonometric identities.

$$\sin(\beta - \omega) = \sin\beta\cos\omega - \cos\beta\sin\omega$$

$$\cos(\beta - \omega) = \cos\beta\cos\omega + \sin\beta\sin\omega.]$$

(b) Sketch the optical layout of the following types of telescope.

(i) A Newtonian reflector.

(ii) A classical Cassegrain reflector.

(iii) A Ritchey-Chrétien reflector.

(iv) A classical Schmidt telescope.

(v) A Gregorian telescope.

Your sketches should show (1) the optical axis, (2) a ray parallel to the optical axis that passes through a point near the edge of the telescope pupil, (3) the optical elements and (4) the focal plane. The optical elements should be labelled to indicate their shape.

(ii) Comment on the merits and drawbacks of these designs.

(c) (i) A simple telescope primary mirror is made from a solid, thick piece of glass. Why does the mirror have to be thick?

(ii) Why is it not possible to make a useful version of such a mirror larger than about 4m in diameter?

(iii) List three primary mirror technologies that allow useable larger primary mirror diameter to be made and comment on the currently operational large ground-based telescopes and the future planned extremely large telescopes.

2 Spectrographs and high contrast imagers

(a) A telescope has an aperture (i.e. diameter) D, and it has a spectrograph which uses a reflection grating. The resolving power is $R = \lambda/\Delta\lambda$ where λ is the wavelength and $\Delta\lambda$ is the smallest resolvable wavelength separation of two spectral features. The spectrum has a linear dispersion, q, which has units of distance on the detector per unit wavelength. The angular width of the slit on the sky is θ radians.

(i) Draw a sketch of the optical layout of such a system including the telescope, the slit, the collimator, the grating and the camera. Also draw a sketch of the intensity of light versus position on the detector in the dispersion direction for monochromatic light (assuming the collimator and camera have no aberations, the light uniformly illuminates the slit and that diffraction at the slit is negligible). Use this sketch to explain how R, q, the physical width, p, of the monochromatic slit image on the detector and the slit's apparent wavelength extent are related.

(ii) Define a quantity L by

$$L = R\theta D.$$

Show that

$$L = \frac{RpA_{cam}}{f_{cam}}$$

where A_{cam} is the aperture of the beam entering the camera (in the dispersion direction) and f_{cam} is the focal length of the camera.

(iii) Show that

$$L = \frac{\lambda q A_{cam}}{f_{cam}}.$$

(iv) Show that

$$L = \frac{W m \lambda}{d}$$

where m is the spectral order and d is the groove spacing of the grating.

(v) Finally, show that

$$L = W(\sin\alpha + \sin\beta)$$

where W is the illuminated length on the surface of the grating, α is the angle of incidence on the grating and β is the angle of diffraction from the grating with α and β both measured from the grating normal. What is the physical significance of the quantity L?

(b) Explain the term *integral field spectrograph* and describe the *data cube* produced by such an instrument. Give up to 6 examples of instrumental configurations which can deliver such data cubes.

(c) In high contrast imaging there are two techniques: ADI and SSDI.

(i) Give details for both including an explanation of how they work.

(ii) Discuss how they are affected by systematic errors.

3 Detectors

(a) A photon counting CCD has a multiplication register with r elements and a probability p of producing an extra electron per initial electron at each stage in the register. Write down an expression in terms of r and p for the mean gain $g =< x_n > /n$ for the whole multiplication register where n is the number of electrons entering the register and $< x_n >$ is the mean number of electrons coming out for n input electrons. For large r and small p the probability of x electrons appearing on the output for a single input electron is given by

$$P_1(x) = g^{-1} \exp(-xg^{-1}).$$

(i) Show that the probability distribution for n input electrons is

$$P_n(x) = \frac{x^{n-1} \exp(-xg^{-1})}{g^n(n-1)!}.$$

(ii) At high photon arrival rates and for large values of r and g, the variance of the probability distribution in part (i) is ng^2. Show that in this regime the multiplication register effectively halves the quantum efficiency of the CCD.

(b) Light from a star falls on a patch of n pixels during an observation which has a total exposure time T. Some unwanted background light also falls on the same pixel patch as the star light. Let Q be the total number of photons collected from the star and B be the total number of background photons collected in the patch. During the exposure, another patch of n pixels collects photons from the background only and the number of photons collected in the background-only patch is fB where $f \sim 1$ but is not exactly equal to unity (i.e. there is a systematic error in the background measurement). You may assume that readout noise is negligible.

(i) Assuming that Q is estimated by subtracting one patch measurement from the other, derive an expression for the signal-to-noise ratio Z for this estimate.

(ii) Show that in the limit as the exposure time $T \to \infty$, Z becomes

$$Z = \frac{R_Q}{(1-f)R_B},$$

where R_Q is the average photon arrival rate from the object in the patch and R_B is the average photon arrival rate from the background in the same patch.

(iii) Comment on the significance of this result.

(c) Describe the architecture of a H2RG IR focal plane array and explain how it works.

END OF PAPER