UNIVERSITY OF
CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 7 June, 2018 1:30 pm to 4:30 pm

PAPER 315

EXTRASOLAR PLANETS: ATMOSPHERES AND INTERIORS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Spectroscopy of Transiting Exoplanets

(a) Consider a transiting exoplanet with a radius R_p orbiting a star of radius R_s in a circular orbit. R_p corresponds to the nominal surface radius of the planet. It is given that the observable total height of the atmosphere is H above the surface in the spectral range of interest. The planet hosts a cloud deck covering the entire atmosphere horizontally and the top of the cloud deck is at a height H_c from the surface. The optical depth in the cloud-free region of the atmosphere over the total path length in transmission geometry is τ_λ and that in the cloudy region is $\tau_{\lambda,c}$. You may assume cylindrical geometry and that the optical depth does not depend on the pressure-temperature profile. Derive an expression for the transmission spectrum of the planet.

(b) The transmission spectrum of a hot Neptune observed with the Hubble Space Telescope was found to be flat. One hypothesis put forward to explain the spectrum was that the atmosphere hosts a high-altitude cloud deck. Estimate the pressure at the top of the cloud deck if the pressure at the surface of the planet is given as P_s.

(c) The transit depth of a transiting exoplanet with a cloud-free atmosphere can be approximated to be of the form $\delta_\lambda = A + B(1 - e^{-\tau_\lambda})$, where A and B are constants. Show that the continuum spectral slope of the transmission spectrum for such an atmosphere in the optical is given by $d \log \delta_\lambda / d \log \lambda \sim m$, where m is a number. For a reasonable assumption, which should be stated, what is a typical value of m?

2 Energy transport

(a) The atmospheric temperature profile of a highly irradiated hot jupiter can be approximated to be of the following form.

$$T^4 = \frac{3T_{\rm int}^4}{4}\left[\tau + \frac{2}{3}\right] + \frac{3f_r T_{\rm irr}^4}{4}\left[\frac{2}{3} + \frac{1}{\gamma\sqrt{3}} + \left(\frac{\gamma}{\sqrt{3}} - \frac{1}{\gamma\sqrt{3}}\right)e^{-\gamma\tau\sqrt{3}}\right]$$

(i) Define all the quantities in this expression and state the assumptions that need to be made in deriving this expression.

(ii) Derive the condition for which this atmosphere does or does not have a thermal inversion and how can such a condition be met in a hot Jupiter.

(iii) What is a condition in which the atmosphere is stable against convection and why?

(b) The temperature gradient in the deep atmosphere of an irradiated giant planet where it is still dominated by radiative transport can be expressed in the form $dT/dz = A\rho/T^3$ where A may be assumed to be constant and ρ is the density. Derive an expression for the pressure P_b at the radiative-convective boundary in the deep atmosphere. If P_b is assumed to be at 0.1 bar for Jupiter in the solar system, estimate the value of P_b for a typical hot jupiter orbiting a sun-like star.

3 Atmospheric Chemistry

(a) Describe, in a few sentences for each, four mechanisms of non-equilibrium chemistry in planetary atmospheres. For each mechanism, provide one example in the solar system and one in exoplanets.

(b) The pressure-temperature profile in the atmosphere of a hot Jupiter is given by

$$\begin{aligned} T &= 500\ K, \quad \text{for} \quad P < 10^{-2}\ \text{bar} \\ T &= 1400\ K, \quad \text{for} \quad P > 1\ \text{bar} \\ dT/d\log P &= C, \quad \text{for} \quad 10^{-2} < P < 1\ \text{bar}, \end{aligned}$$

where C is a constant. Determine the constant C and sketch the P-T profile.

(i) State the key chemical processes that could be operating in such an atmosphere as a function of altitude, noting the pressure range where each process could dominate.

(ii) What is the expected chemical composition in the observable atmosphere of this planet assuming solar elemental abundances and chemical equilibrium?

(iii) What are four possible observable signatures of non-equilibrium chemistry in such an atmosphere?

(iv) How would the atmospheric composition of the planet depend on the C/O ratio and metallicity?

(c) Consider the CO-CH_4 conversion in the above atmosphere: $CO + 3H_2 \rightarrow CH_4 + H_2O$. The chemical timescale (τ_{chem}) for the forward reaction at 1 bar pressure is 10^5 s. Derive a constraint on the vertical eddy mixing coefficient (K_{zz}) that would be required to observe CO at 10^{-3} bar. You may assume the planet to have the same mass and radius as Jupiter, and that the atmospheric scale height of Jupiter is 30 km. Make and state any further assumptions required.

4 Atmospheres and Interiors

(a) Discuss briefly, in a few sentences each, three signatures of atmospheric dynamics observed in exoplanets.

(b) Define the geometric albedo (A_g) and bond albedo (A_B) of a planet? Derive the expression for the equilibrium temperature of a planet.

(c) Using a simple one-layer atmosphere model show that the surface temperature (T_s) of a planet due to greenhouse effect can be expressed in the form

$$T_s = \left[\frac{2}{2-\alpha}\right]^{1/4} T_e,$$

where α and T_e need to be defined.

(d) Draw a rough sketch of the M-R curve predicted by detailed theoretical models for solar composition bodies with masses ranging from gas giants to low-mass stars. Identify and explain the main trends in the curve, and the transitions between different object types. State three observational findings outside the solar system that are of relevance to this curve.

(e) Discuss briefly any three important directions of research at the forefront of exoplanetary science.

END OF PAPER