PAPER 315

EXTRASOLAR PLANETS: ATMOSPHERES AND INTERIORS

Attempt no more than THREE questions.

There are FOUR questions in total.

The questions carry equal weight.

STATIONERY REQUIREMENTS

Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS

None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.
1 Spectroscopy of Transiting Exoplanets

(a) Consider a transiting exoplanet with a radius $R_p$ orbiting a star of radius $R_s$ in a circular orbit. $R_p$ corresponds to the nominal surface radius of the planet. It is given that the observable total height of the atmosphere is $H$ above the surface in the spectral range of interest. The planet hosts a cloud deck covering the entire atmosphere horizontally and the top of the cloud deck is at a height $H_c$ from the surface. The optical depth in the cloud-free region of the atmosphere over the total path length in transmission geometry is $\tau_\lambda$ and that in the cloudy region is $\tau_{\lambda,c}$. You may assume cylindrical geometry and that the optical depth does not depend on the pressure-temperature profile. Derive an expression for the transmission spectrum of the planet.

(b) The transmission spectrum of a hot Neptune observed with the Hubble Space Telescope was found to be flat. One hypothesis put forward to explain the spectrum was that the atmosphere hosts a high-altitude cloud deck. Estimate the pressure at the top of the cloud deck if the pressure at the surface of the planet is given as $P_s$.

(c) The transit depth of a transiting exoplanet with a cloud-free atmosphere can be approximated to be of the form $\delta_\lambda = A + B(1 - e^{-\tau_\lambda})$, where A and B are constants. Show that the continuum spectral slope of the transmission spectrum for such an atmosphere in the optical is given by $d\log \delta_\lambda / d\log \lambda \sim m$, where $m$ is a number. For a reasonable assumption, which should be stated, what is a typical value of $m$?
2 Energy transport

(a) The atmospheric temperature profile of a highly irradiated hot Jupiter can be approximated to be of the following form.

\[ T^4 = \frac{3T_{\text{int}}^4}{4} \left( \frac{\tau}{3} + \frac{2}{3} \right) + \frac{3f_c T_{\text{int}}^4}{4} \left[ \frac{\gamma}{\sqrt{3}} + \frac{1}{\gamma\sqrt{3}} + \left( \frac{\gamma}{\gamma\sqrt{3}} - \frac{1}{\gamma\sqrt{3}} \right)e^{-\gamma\tau\sqrt{3}} \right] \]

(i) Define all the quantities in this expression and state the assumptions that need to be made in deriving this expression.

(ii) Derive the condition for which this atmosphere does or does not have a thermal inversion and how can such a condition be met in a hot Jupiter.

(iii) What is a condition in which the atmosphere is stable against convection and why?

(b) The temperature gradient in the deep atmosphere of an irradiated giant planet where it is still dominated by radiative transport can be expressed in the form \( \frac{dT}{dz} = \frac{A\rho}{T^3} \) where \( A \) may be assumed to be constant and \( \rho \) is the density. Derive an expression for the pressure \( P_b \) at the radiative-convective boundary in the deep atmosphere. If \( P_b \) is assumed to be at 0.1 bar for Jupiter in the solar system, estimate the value of \( P_b \) for a typical hot Jupiter orbiting a sun-like star.
3 Atmospheric Chemistry

(a) Describe, in a few sentences for each, four mechanisms of non-equilibrium chemistry in planetary atmospheres. For each mechanism, provide one example in the solar system and one in exoplanets.

(b) The pressure-temperature profile in the atmosphere of a hot Jupiter is given by

\[
T = \begin{cases} 
500 \text{ K, for } P < 10^{-2} \text{ bar} \\
1400 \text{ K, for } P > 1 \text{ bar} \\
\frac{dT}{d \log P} = C, \text{ for } 10^{-2} < P < 1 \text{ bar},
\end{cases}
\]

where \(C\) is a constant. Determine the constant \(C\) and sketch the \(P-T\) profile.

(i) State the key chemical processes that could be operating in such an atmosphere as a function of altitude, noting the pressure range where each process could dominate.

(ii) What is the expected chemical composition in the observable atmosphere of this planet assuming solar elemental abundances and chemical equilibrium?

(iii) What are four possible observable signatures of non-equilibrium chemistry in such an atmosphere?

(iv) How would the atmospheric composition of the planet depend on the \(C/O\) ratio and metallicity?

(c) Consider the CO-CH\(_4\) conversion in the above atmosphere: \(\text{CO} + 3\text{H}_2 \rightarrow \text{CH}_4 + \text{H}_2\text{O}\). The chemical timescale (\(\tau_{\text{chem}}\)) for the forward reaction at 1 bar pressure is \(10^5\) s. Derive a constraint on the vertical eddy mixing coefficient (\(K_{zz}\)) that would be required to observe CO at \(10^{-3}\) bar. You may assume the planet to have the same mass and radius as Jupiter, and that the atmospheric scale height of Jupiter is 30 km. Make and state any further assumptions required.
4 Atmospheres and Interiors

(a) Discuss briefly, in a few sentences each, three signatures of atmospheric dynamics observed in exoplanets.

(b) Define the geometric albedo ($A_g$) and bond albedo ($A_B$) of a planet? Derive the expression for the equilibrium temperature of a planet.

(c) Using a simple one-layer atmosphere model show that the surface temperature ($T_s$) of a planet due to greenhouse effect can be expressed in the form

$$T_s = \left[\frac{2}{2-\alpha}\right]^{1/4} T_e,$$

where $\alpha$ and $T_e$ need to be defined.

(d) Draw a rough sketch of the M-R curve predicted by detailed theoretical models for solar composition bodies with masses ranging from gas giants to low-mass stars. Identify and explain the main trends in the curve, and the transitions between different object types. State three observational findings outside the solar system that are of relevance to this curve.

(e) Discuss briefly any three important directions of research at the forefront of exoplanetary science.