

MATHEMATICAL TRIPOS      Part III

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Tuesday, 12 June, 2018    1:30 pm to 3:30 pm

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PAPER 308

CLASSICAL AND QUANTUM SOLITONS

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

*The questions carry equal weight.*

**STATIONERY REQUIREMENTS**

*Cover sheet*

*Treasury Tag*

*Script paper*

**SPECIAL REQUIREMENTS**

*None*

<p><b>You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.</b></p>
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**1**

Discuss what is meant by a Bogomolny bound and by a Bogomolny equation in the context of solitons in field theory. Derive a Bogomolny bound and a Bogomolny equation in a field theory in one spatial dimension, and find the soliton solution and its energy. Describe an example in *either* two *or* three spatial dimensions.

**2**

Explain what is meant by the collective coordinate dynamics of solitons, and its quantization. Discuss the collective coordinate dynamics in the cases of (i) a  $\phi^4$  kink, (ii) the 2-vortex system in the abelian Higgs model at critical coupling, and (iii) a Skyrmion of baryon number  $B = 1$ . Describe briefly some of the physical properties of quantized states that arise in each of these cases.

**3**

Discuss two examples of how rational maps can be used to exactly or approximately characterize the topological soliton solutions in field theories. [You should explain the role of the Wronskian, and include some remarks on the symmetries of rational maps.]

**END OF PAPER**