UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 12 June, 2018 9:00 am to 11:00 am

PAPER 307

SUPERSYMMETRY

*Attempt all **THREE** questions.*

Questions 1 and 2 carry 30 marks each and Question 3 carries 40 marks

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 [30 marks] Consider the generators of the Poincaré algebra $M_{\mu\nu}, P_\mu$ with indices $\mu, \nu = 0, \cdots, 3$ and add anti-commuting generators $Q_\alpha^A, \bar{Q}_{\dot{\alpha}}^B$ with $\alpha, \dot{\alpha} = 1, 2$ and $A, B = 1, \cdots, \mathcal{N}$ with $\mathcal{N}$ determining the number of supersymmetry generators. Q_α^A and $\bar{Q}_{\dot{\alpha}}^B$ transform in the $(1/2, 0)$ and $(0, 1/2)$ representations of the Lorentz group respectively. Establish the following commutation and anti-commutation relations.

$$\left[Q_\alpha^A, M^{\mu\nu}\right] = (\sigma^{\mu\nu})_\alpha^\beta \, Q_\beta^A\,; \qquad \left\{Q_\alpha^A, \bar{Q}_{\dot{\beta}B}\right\} = 2\,(\sigma^\mu)_{\alpha\dot{\beta}}\, P_\mu \delta_B^A\,; \qquad \left\{Q_\alpha^A, Q_\beta^B\right\} = \epsilon_{\alpha\beta} Z^{AB}$$

and $[Q_\alpha, P^\mu] = \left[\bar{Q}^{\dot{\alpha}}, P^\mu\right] = 0$. Here $Z^{AB} = -Z^{BA}$ are central charges that commute with all generators, $\sigma^{\mu\nu}$ are the $SL(2, C)$ generators and σ^μ correspond to the three Pauli matrices (σ^i) and the two-dimensional identity matrix $(\sigma^0 = \mathbf{1})$.

Using this algebra show that the number of fermions equals the number of bosons in any representation of the algebra.

2 [30 marks] Consider the massless representations of extended supersymmetry (with generators $Q_\alpha^A, \bar{Q}_{\dot{\alpha}}^B$ with $A, B = 1, \cdots, \mathcal{N} \geqslant 1$ and $\alpha, \dot{\alpha} = 1, 2$) in a frame such that the momenta are: $p_\mu = (E, 0, 0, E)_\mu$. Using the supersymmetry algebra, construct the states within a general multiplet and specify the number of states for each helicity.

Establish the following:

- The total number of states in a representation is $2^{\mathcal{N}}$.
- In every multiplet the difference between maximal and minimal helicity is $\mathcal{N}/2$.

It is usually stated that the maximal number of supersymmetries in renormalisable field theories is $\mathcal{N} = 4$, also that the maximal number of supersymmetries in general is $\mathcal{N} = 8$ and that the only supersymmetric theories with a chiral spectrum correspond to $\mathcal{N} = 0, 1$. Provide a short justification for each of these statements.

3 [40 marks] Consider the Wess-Zumino model for a chiral superfield $\Phi(x,\theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + \cdots$ with Kähler potential $K = \Phi^\dagger\Phi$ and superpotential $W = \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3$. Write down the Lagrangian in superspace.

Compute the F dependent part of the Lagrangian [*Hint: Expand the superpotential W in a Taylor series around $\Phi = \varphi$*]. By solving for F determine the scalar potential.

Verify the following:

- The scalar potential is positive semi-definite.
- The mass of the fermion ψ equals the mass of the boson φ.
- The strength of the scalar quartic coupling $|\varphi|^4$ is determined by the strength of the Yukawa coupling $(\varphi\psi\psi)$.

Outline the arguments to establish that the superpotential for this model does not receive perturbative quantum corrections.

END OF PAPER