UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 4 June, 2018 1:30 pm to 4:30 pm

PAPER 135

LOGIC

*Attempt all **FOUR** questions.*

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Let $\{\mathfrak{M}_i : i \in I\}$ be an infinite family of structures all of the same similarity type. Each $\mathfrak{M}_i$ has carrier set M_i.

What is a proper filter on I?

Prove that every filter on I can be extended to a maximal filter.

State and prove Łoś's theorem.

In both cases you will need the Axiom of Choice, and you must make clear where you have used it.

Let each nonprincipal filter F on I be a possible world with carrier set $\prod_{i \in I} M_i$ in a possible world structure which we shall call $\mathfrak{M}_I$. Let the accessibility relation on the set of filters be set inclusion.

Which filter is the designated world?

When do we have $F \models \phi(\vec{x})$ for ϕ atomic?

Supply the recursive clauses for the semantics for complex formulæ in the language of the $\mathfrak{M}_i$.

Prove that $\mathfrak{M}_I \models \phi$ iff, for all nonprincipal ultrafilters $\mathcal{U}$ on I, the ultraproduct $\prod_{i \in I} \mathfrak{M}_i / \mathcal{U} \models \phi$.

What logic does your possible world structure satisfy?

2

State and prove the Omitting Types Theorem.

Explain the connection with standard models of the arithmetic of $\mathbb{N}$.

3

Give two characterisations of the class of partial computable functions, in terms of machines and in terms of syntactic declarations (You need not prove them equivalent). Give a characterisation of this class in terms of λ-definability and show that it is equivalent to the definition in terms of syntactic declarations.

4 What are *many-one reducibility* and *Turing-reducibility*?

Explain the difference.

State and prove the Friedberg-Muchnik theorem.

END OF PAPER