UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 12 June, 2017 1:30 pm to 3:30 pm

PAPER 338

OPTICAL AND INFRARED ASTRONOMICAL TELESCOPES AND INSTRUMENTS

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Telescopes

(a) A star is above the horizon, is west of the meridian and has a declination $\delta < (90^o - \theta)$ where θ is the observer's latitude.

(i) Draw a diagram of the celestial sphere showing the observer's horizon, the zenith Z, the celestial equator and the celestial pole P, assuming the observer is in the northern hemisphere. Show the location of the star on your diagram and label it as X. Show on your diagram the angles that are the observer's latitude θ, and the star's altitude γ and azimuth η. Finally, show on your diagram the angles that are the star's hour angle h and declination δ.

(ii) For the star at X in part (i) show that the coordinates of an alt-az telescope are related to the coordinates of an equatorial telescope by

$$\sin\delta = \sin\gamma\sin\theta + \cos\gamma\cos\theta\cos\eta$$

$$\cos\delta\sin h = \sin\eta\cos\gamma$$

$$\cos\delta\cos h = \sin\gamma\cos\theta - \cos\gamma\sin\theta\cos\eta$$

(iii) How is the hour angle of the star related to its right ascension?

(b) A telescope on the Earth's equator has an alt-az mount. The azimuth axis has a maximum rotation speed of ω RPM (revolutions per minute) and the altitude axis has a total travel of 90^o. Stars that pass through or close to the zenith cannot be tracked properly because the telescope cannot move fast enough causing the telescope to have a so-called *zenith blind spot*.

(i) By considering a star exactly on the celestial equator derive an expression for the angular size of the telescope's zenith blind spot, ϕ, in terms of ω.

(ii) Now consider a similar telescope, not on the equator, with a latitude, θ and a star with a declination, $\delta = \theta$. Modify your answer to part (i) to take into account the telescope's latitude.

(c) A refracting telescope has an objective lens which is a cemented achromatic doublet. This lens has zero axial chromatic aberation for a pair of specific wavelengths, one in the red part of the spectrum (the Fraunhofer C line) and the other in the blue part of the spectrum (the Fraunhofer F line). The focal length, f, of a singlet lens is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_a} - \frac{1}{R_b}\right) \equiv (n-1)\rho$$

where n is the refractive index of the lens and R_a and R_b are the radii of curvature of the two lens surfaces (which define the quantity ρ).

(i) Show that

$$\frac{\rho_1}{\rho_2} = -\frac{n_{2F} - n_{2C}}{n_{1F} - n_{1C}}$$

where the subscripts 1 and 2 refer to the two lenses of the doublet and subscripts C and F refer to the red and blue wavelengths.

(ii) Now consider a third wavelength which is in between the red and blue wavelengths at the Fraunhofer d line. Show that

$$\frac{f_{2d}}{f_{1d}} = -\frac{(n_{2F} - n_{2C})/(n_{2d} - 1)}{(n_{1F} - n_{1C})/(n_{1d} - 1)}.$$

(iii) Using the definition

$$V_d \equiv \frac{n_d - 1}{n_F - n_C}$$

show that

$$f_{1d} = \frac{f_d(V_{1d} - V_{2d})}{V_{1d}} \qquad \text{and} \qquad f_{2d} = \frac{f_d(V_{2d} - V_{1d})}{V_{2d}}$$

where f_d is the focal length of the doublet at the d line wavelength.

(iv) What is the quantity V_d called? Draw a plot area of n_d versus V_d with your axes spanning values that are typical for optical glasses. Indicate the areas where available glasses lie and which areas correspond to crown and flint glasses. What do the V_d values imply for the focal lengths of the two lenses in the doublet?

2 Spectrographs

(a) A telescope has an aperture (i.e. diameter) D, and it has a spectrograph which uses a reflection grating. The resolving power is $R = \lambda/\Delta\lambda$ where λ is the wavelength and $\Delta\lambda$ is the smallest resolvable wavelength separation of two spectral features. The spectrum has a linear dispersion, q, which has units of distance on the detector per unit wavelength. The angular width of the slit on the sky is θ radians.

(i) Draw a sketch of the optical layout of such a system including the telescope, the slit, the collimator, the grating and the camera. Also draw a sketch of the intensity of light versus position on the detector in the dispersion direction for monochromatic light (assuming the collimator and camera have no aberations and that diffraction at the slit is negligible). Use this sketch to explain how R, q, the physical width, p, of the monochromatic slit image on the detector and the slit's apparent wavelength extent are related.

(ii) Define a quantity L by

$$L = R\theta D.$$

Show that

$$L = \frac{RpA_{cam}}{f_{cam}}$$

where A_{cam} is the aperture of the beam entering the camera (in the dispersion direction) and f_{cam} is the focal length of the camera.

(iii) Show that

$$L = \frac{\lambda q A_{cam}}{f_{cam}}.$$

(iv) Show that

$$L = \frac{Wm\lambda}{d}$$

where m is the spectral order and d is the groove spacing of the grating.

(v) Finally, show that

$$L = W(\sin\alpha + \sin\beta)$$

where W is the illuminated length on the surface of the grating, α is the angle of incidence on the grating and β is the angle of diffraction from the grating with α and β both measured from the grating normal. What is the physical significance of the quantity L?

(b) An integral field device feeds the light from a square patch of sky which is β arcsec on a side, to a set of spectrographs. N spectrographs are needed to accomodate the combination of the étendue ($A\Omega$) of the field area and the number of spectral resolution elements (SREs) in the wavelength direction. The spaxels are square and are θ arcsec on a side. Consider a scenario where β is kept constant but θ is allowed to vary and this drives the physical size of a spectrograph. You should also assume that a single spectrograph always has the same type of detector (which is a fixed size and therefore does not scale in size as θ varies) and that the width of an SRE in pixels is a constant independent of θ.

(i) By considering the volumes of the collimator, the collimated space and the camera separately (V_{coll}, V_{grat} and V_{cam}) derive scaling laws that show how the total combined volume of all N spectrographs scales with θ and the number of spaxels, M.

(ii) What is the limiting factor as θ gets smaller?

(c) For a single spectral order of a cross-dispersed echelle grating there is a range of wavelengths, S, that arrive at the detector closer to the y-axis than they do in any of the other orders [The y-axis is a line through the camera's optical axis and orthogonal to the main dispersion direction of the echelle grating]. This is called the free spectral range and the detector width must be at least equal to its physical length for all spectral orders to ensure that there are no spectral coverage gaps in the echellogram.

(i) Draw a diagram showing two spectral orders with order m and $(m+1)$ which are separated on the detector by a cross disperser. Indicate on your diagram the directions of increasing wavelength for both the main (echelle grating) dispersion and the cross dispersion.

(ii) Show that

$$S = \frac{K}{m(m+1)}$$

where K is a constant. What is the physical meaning of the constant K?

3 High contrast imagers and detectors

(a) A high contrast imaging system has a deformable mirror (DM) with $N \times N$ actuators in a square format. The telescope has a circular primary mirror of diameter D which is imaged on to the DM such that N actuators span the diameter of the telescope. The total wavefront error can be thought of as the sum of many sinusoidal ripples in the wavefront each of which can be characterised by its amplitude, phase and wavelength, Λ (which should not be confused with the wavelength of the light).

(i) What is the shortest spatial scale of the wavefront error (measured at the primary mirror) that the DM can correct?

(ii) Let a single wavefront error mode be one for which $D = j\Lambda/2$ where j is an integer. Show that the number of modes that can be corrected, M, for the full telescope aperture is given by

$$M = \frac{\pi}{4}\left[\frac{N}{2}\right]^2.$$

(iii) The system is used to observe a star. The intensity of a speckle of starlight produced by a single mode of the residual wavefront error (i.e. after the DM) is given by

$$C \equiv \frac{I_{spec}}{I_{star}} = (\pi h_0/\lambda)^2$$

where λ is the wavelength of the light, h_0 ($\ll \lambda$) is the amplitude of the mode that gives rise to the speckle and I_{star} is the total intensity of the light from the star. This equation also defines the speckle contrast C. Show that

$$h_{rms} = \frac{N\lambda\sqrt{C}}{4\sqrt{\pi}}$$

where h_{rms} is the rms amplitude of the total wavefront error due to all the modes combined.

(iv) Show that the DM can correct wavefront errors and therefore reduce the speckle contrast within an angle θ from the star (measured in the direction of a row or column of actuators) given by

$$\theta = \frac{N\lambda}{2D}.$$

Comment on the shape of the corrected region and its size in terms of spatially resolved elements.

(b) Light from a star falls on a patch of n pixels during an observation which has a total exposure time T. Some unwanted background light also falls on the same pixel patch as the star light. Let Q be the total number of photons collected from the star, B be the total number of background photons collected in the patch and r be the rms detector readnoise in electrons per read per pixel. During the exposure another patch of n pixels collects photons from the background only and the photon arrival rate of the background is the same for both patches.

(i) Assuming that Q is estimated by subtracting one patch measurement from the other, derive an expression for the signal-to-noise ratio Z.

(ii) Derive the following expression for the exposure time T of the observation

$$T = \frac{Z^2(R_Q + 2R_B) + \sqrt{Z^4(R_Q + 2R_B)^2 + 8nR_Q^2 Z^2 r^2}}{2R_Q^2}$$

where R_Q is the average photon arrival rate from the object in the patch and R_B is the average photon arrival rate from the background in the patch.

(c) A detector measures the intensity of light in units of counts which are also sometimes called data-numbers or analogue to digital units (ADUs). The gain g for a detector is defined as the number of detected photons per count so if a pixel value in the data file is N counts then the number of detected photons is Ng. To measure the gain the detector is illuminated uniformly (both spatially and temporally) and images are taken such that the counts in the data files are slightly less than the saturation level. The same illumination level is used for all m images (m is an even integer). Half of the images are added together to make an image called A and the other half are added together to make an image called B. Finally an image called C is made by dividing image A by image B.

(i) If E is the standard deviation of the data numbers in image C and the average of the data numbers in image A (or B) is N counts then show that the gain is given by

$$g = \frac{2}{NE^2}.$$

(ii) Discuss how flatfield noise (sensitivity variations from pixel to pixel) affects the accuracy of the value of g determined by this method.

(iii) Discuss how readout noise affects the accuracy of the value of g determined by this method.

(iv) What effect would a spatially non-uniform illumination have and what can be done with the data to minimise the error in the determination of g?

(v) Similarly, discuss the situation where the illumination varies with time.

END OF PAPER