UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 12 June, 2017 1:30 pm to 3:30 pm

PAPER 328

BOUNDARY VALUE PROBLEMS FOR LINEAR PDES

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Let $q(x,t)$ satisfy the Stokes equation

$$q_t + q_x + q_{xxx} = 0, \quad 0 < x < +\infty, \; 0 < t < T, \tag{1}$$

and the initial and boundary conditions

$$q(x,0) = q_0(x), \quad 0 < x < +\infty, \tag{2}$$

$$q_x(0,t) = g_1(t), \quad 0 < t < T, \tag{3}$$

where $0 < T < +\infty$, the given functions q_0 and g_1 have sufficient smoothness, q_0 has sufficient decay as $x \to +\infty$, and $\dot{q}_0(0) = g_1(0)$.

Find an integral representation for the solution $q(x,t)$ in terms of $q_0(x)$ and $g_1(t)$.

2 Let $q(x,t)$ satisfy the initial-boundary value problem

$$q_t = q_{xx} + \beta q_x, \quad 0 < x < L, \; 0 < t < T, \tag{1}$$

$$q(x,0) = q_0(x), \quad 0 < x < L, \tag{2}$$

$$q_x(0,t) = g_1(t), \quad 0 < t < T, \tag{3}$$

$$q_x(L,t) = h_1(t), \quad 0 < t < T, \tag{4}$$

where L, T, and β are given positive constants, the given functions q_0, g_1, and h_1 have sufficient smoothness, $\dot{q}_0(0) = g_1(0)$, and $\dot{q}_0(L) = h_1(0)$.

Obtain an integral representation for the solution $q(x,t)$ in terms of $q_0(x)$, $g_1(t)$, and $h_1(t)$.

3

Let $q(x, y)$ satisfy the Laplace equation

$$q_{xx} + q_{yy} = 0, \quad 0 < x + \infty, \;\; 0 < y < +\infty, \tag{1}$$

and the initial and boundary conditions

$$q(0, y) = g_1(y), \quad 0 < y < +\infty, \tag{2}$$

$$q(x, 0) = g_2(x), \quad 0 < x < +\infty, \tag{3}$$

where the given functions g_1 and g_2 have sufficient smoothness, $g_1(y)$ has sufficient decay as $y \to +\infty$, $g_2(x)$ has sufficient decay as $x \to +\infty$, and $g_1(0) = g_2(0)$.

Find an integral representation for the derivative q_z, $z = x + iy$, in terms of $g_1(y)$ and $g_2(x)$.

END OF PAPER