UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 2 June, 2017 1:30 pm to 4:30 pm

PAPER 317

STRUCTURE AND EVOLUTION OF STARS

Attempt no more than ***THREE*** *questions.*

There are ***FOUR*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Discuss the surface boundary conditions for a star. Assume planar geometry and write down the differential equation of radiation transfer that determines the specific intensity I_ν in terms of the emission coefficient j_ν, the opacity κ_ν and the density of the stellar atmosphere material ϱ. Consider the case of a grey atmosphere and use the Eddington's first approximation to obtain his closure approximation

$$cP_{\text{rad}}(\tau) = \tfrac{4}{3}\pi J(\tau)$$

where $P_{\text{rad}}(\tau)$ is radiation pressure and $J(\tau)$ is the mean intensity, τ is the optical depth and c is the speed of light.

The equation of radiative transfer for isotropic scattering in a plane parallel grey atmosphere is

$$\mu\,\tfrac{\mathrm{d}I(\mu,\tau)}{\mathrm{d}\tau} = I(\mu,\tau) - J(\tau),$$

where $\mu = \cos\theta$ and θ is the angle measured from axis z perpendicular to the plane-parallel atmosphere.

Assume that this equation has a solution of the form:

$$I(\mu,\tau) = \alpha(\beta + \tau + \mu + \tfrac{\gamma}{1+\epsilon\mu}e^{-\delta\tau}),$$

where α, β, γ, ϵ and δ are constants. Find the mean intensity $J(\tau)$, the flux $F(\tau)$ and radiation pressure $P_{\text{rad}}(\tau)$.

Show that the condition of radiative equilibrium in the atmosphere, $F(\tau) = F =$ constant, implies that $\alpha = \frac{3}{4\pi}F$ and ϵ is very small but non-zero. Then, in the limit of large τ, obtain that $cP_{\text{rad}}(\tau) = \frac{4\pi}{3}J(\tau)$. From the equation of radiative transfer deduce that that $\delta = \epsilon$.

The exact solution of the grey atmosphere problem can be written in terms of the Hopf function $q(\tau)$ as:

$$J(\tau) = \tfrac{3}{4}F(\tau + q(\tau)).$$

Assume that $q(0) = 0.57$, $q(\infty) = 0.71$ and find β, γ. How much brighter would the centre of the stellar disk ($\mu = 1$) appear compared to the limb ($\mu = 0$) of such star?

2

Write down the equations of stellar structure for a spherically symmetric star which does not suffer any mass loss. Describe clearly the criterion for the use of an appropriate equation of energy transfer using the concepts of radiative and convective gradients.

Assume that the core of a star is isothermal, with $T = 10^7 K$ and a mass $M_{\rm core} \simeq 1 M_\odot$. Just outside the core, there is a very thin layer which produces essentially all of star's luminosity. Outside the energy producing layer there is the star's radiative envelope. Assuming P, ϱ and $M_{\rm r}$ behave as power laws of radius through the envelope, find a solution to the equations of hydrostatic equilibrium, the ideal gas law and radiative transfer with Kramer's opacity and calculate the ratio of the stellar radius to the core radius for a photospheric temperature of 4000 K. What kind of star is described by the above simple assumptions?

3

Obtain the equation for the gradient of the velocity with radius for a steady, spherically symmetric stellar wind, assuming that the temperature gradient is determined by the equation of radiative transfer. Discuss the properties of the solutions.

Let the pressure P be given by $P = P_{\rm gas} + P_{\rm rad}$ and let $\beta = \frac{P_{\rm gas}}{P}$, where gas pressure $P_{\rm gas}$ is given by the ideal gas law and $P_{\rm rad}$ is radiation pressure for a gas of photons. The critical luminosity $L_{\rm crit}$ is given by $L_{\rm crit} = \frac{4\pi c G M_{\rm r}}{\kappa}$, where $M_{\rm r}$ is the mass contained in a sphere of radius r and κ is the opacity. Assume now that $\frac{L_{\rm r}}{L{\rm crit}} = 1$, where $L_{\rm r}$ is the luminosity at radius r. Explain in terms of β why such wind cannot be accelerated to achieve supersonic velocities.

4

Write an essay on the equation of state for stellar matter. Starting with the momentum distribution function $f(\mathbf{p})$ obtain relations between pressure and the part of the internal energy of particles that corresponds to their kinetic energy for both the non-relativistic and relativistic cases. Discuss the Maxwellian gas and a gas of photons and derive the relevant equations of state. What is the equation of state for a degenerate gas in the limit of high density. On a log-log graph of temperature versus density explore the various regimes including the approximate divisions between the extreme relativistic and non-relativistic cases as well as the extreme degenerate and non-degenerate cases. Quantitatively explain the location of these boundaries. Sketch also an approximate location of the curve where pairs of electrons and positrons are important.

END OF PAPER