UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 8 June, 2017 9:00 am to 12:00 pm

PAPER 315

EXTRASOLAR PLANETS: ATMOSPHERES AND INTERIORS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Planetary Atmospheres

(a) The emission spectrum of the dayside of a transiting hot Jupiter observed in the near-infrared with the Hubble Space Telescope was used to retrieve its atmospheric properties. The average pressure-temperature (P-T) profile in a region of the dayside atmosphere probed by the spectrum was found to satisfy the following relation

$$P = P_0 e^{\alpha\sqrt{T-T_0}}, \tag{1}$$

where T is the temperature in K, P is the pressure in bar, and P_0, T_0, and α are free parameters.

(i) Derive the condition(s) for which this P-T profile does or does not have a thermal inversion.

(ii) Derive the condition(s) for which this region of the atmosphere can be unstable against convection.

(iii) Sketch the P-T profile in (1) for a hot Jupiter. Make and state any assumptions required.

(iv) Explain whether or not the P-T profile in (1) can satisfy any region of the Earth's atmosphere and under what conditions.

(b) Derive the expression for the transmission spectrum of a transiting exoplanet of radius R_p orbiting a star of radius R_s. You may assume a plane parallel planetary atmosphere of height H and cylindrical geometry, such that the wavelength-dependant optical depth τ is independent of the impact parameter of a ray of star light traversing the atmosphere. The transmission spectrum must be expressed as the transit depth as a function of R_p, R_s, H, and τ. Make and state any other assumptions necessary.

(c) The emergent spectrum of an exoplanet was found to be of a double-peaked structure, with the two peaks occurring at wavelengths of 0.7 μm and 1.5 μm respectively. What could be causing the two peaks? The planet-star flux ratio of the spectrum in the far infrared was found to be 0.1% and the star was found to be half as large as the Sun with a similar age. What is the approximate radius of the planet? Make and state any assumptions required. Which detection method is most likely to characterize the atmosphere of such a planet and why?

2 Planetary Atmospheres

(a) The atmosphere of a Neptune-sized transiting exoplanet was observed during transit in the ultraviolet using the Hubble Space Telescope. The observations revealed a large exosphere of atomic species surrounding the planet and a comet-like tail.

(i) Derive an approximate condition under which hydrostatic thermal escape can cause the atomic species to escape the planet. Estimate a typical temperature at the base of the exosphere, the exobase, required for escape of atomic hydrogen from this planet. What is the typical pressure at the exobase? You may assume the following values in SI units: Gravitational constant $\sim 7 \times 10^{-11}$, proton mass $\sim 10^{-27}$, Boltzmann's constant $\sim 10^{-23}$, mass of Neptune $\sim 10^{26}$, and radius of Neptune $\sim 2 \times 10^{7}$.

(ii) Describe three mechanisms which can cause atmospheric escape in planetary atmospheres.

(b) (i) Starting with the First Law of thermodynamics derive the condition for thermochemical equilibrium in a planetary atmosphere at a given temperature and pressure.

(ii) Derive an expression for the Gibbs Free Energy of a closed system as a function of its temperature T, pressure P, and the volume mixing ratios of chemical species. Make and state any assumptions required.

(c) The thermal emission spectrum of the dayside atmosphere of a transiting hot Jupiter revealed copious amounts of CO as inferred from a strong CO emission feature in the spectrum. However, a transmission spectrum of the same planet in the CO band showed a low-amplitude feature equivalent to 3 scale heights of the night-side atmosphere, with the continuum of the spectrum probing a pressure of 0.1 bar.

(i) It is argued that clouds at the day-night terminator may explain the transmission spectrum. Estimate the pressure at the top of the cloud deck required to explain the findings. Make and state any assumptions required.

(ii) What could be other explanations to the findings?

(iii) Explain whether or not strong convection can influence the observable composition on either the dayside or night side atmosphere of the planet?

3

Planetary Atmospheres

(a) (i) Show how the luminosity of a newly formed object depends on the energy generation rate from gravitational contraction.

(ii) The intrinsic luminosity of a planet of a given mass evolves with time as $L \propto t^{-\alpha}$, where $\alpha = 3/2$. Consider a Jupiter-mass exoplanet at an orbital separation of 5 AU and an age of 50 Myr. What is the ratio of its intrinsic luminosity to the intrinsic luminosity of present-day Jupiter ($L_{\mathrm{int,J}}$)? Make and state any assumptions required. What is the best method to find such exoplanets and what are two important factors to consider while making such observations?

(iii) Now consider that the planet above migrated after 50 Myr and reached a stable circular orbit at a distance 0.05 AU from its star. It stayed in this orbit thereafter until its host star evolved to be like the present-day Sun. At that point, what is the ratio of its intrinsic luminosity to the incident irradiation it receives from its star? You may express your answer in terms of the irradiation received by present-day Jupiter ($L_{\mathrm{irr,J}}$) and $L_{\mathrm{int,J}}$ defined above. List two observational signatures that suggest whether or not such a migration process occurs in nature. Make and state any assumptions necessary

(b) Using the structure equations and a polytropic model derive the Lane-Emden equation which is of the form

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2\frac{d\theta}{d\xi}\right] = -\theta^n. \tag{1}$$

Make and state any assumptions required and define the quantities involved.

4

Short Questions (Answer all questions):

(a) State three methods to observe exoplanetary atmospheres, the merits and limitations of each method, and the applicable spectral range. Name one major telescope that is suitable for each method.

(b) Derive the flux received by an observer on Earth from an isotropic isothermal planet of radius R at a distance d. Make and state any assumptions required.

(c) Show that the specific intensity is invariant of distance.

(d) Show using a simple one-layer atmosphere how emergent spectral features depend on the temperature gradient in an atmosphere.

(e) State three key parameters that affect the chemical compositions of hydrogen-rich planetary atmospheres in thermochemical equilibrium. Discuss three chemical observables expected in hot Jupiter atmospheres.

(f) State three non-equilibrium chemical processes that can occur in planetary atmospheres and state one example for each process in exoplanets and solar system planets.

(g) State two observational approaches used to infer clouds/hazes in exoplanets. Discuss three observable signatures of clouds/hazes in each approach along with the relevant wavelength range.

(h) Discuss the "Inflated hot Jupiters" problem. What are the two broad classes of solutions considered to solve the problem? List 2 specific solutions in each class.

(i) Discuss three factors that can influence the observable radii of super-Earths.

(j) Discuss three emerging directions of research in exoplanetary science.

END OF PAPER