UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 13 June, 2017 9:00 am to 11:00 am

PAPER 307

SUPERSYMMETRY

Attempt all ***THREE*** *questions.*

There are ***THREE*** *questions in total.*

Questions 1 and 2 carry 35 marks each, Question 3 carries 30 marks.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Consider $\mathcal{N} = 2$ supersymmetry with central charges Z^{AB}, $A, B = 1, 2$. Write the algebra for the supersymmetry generators $Q^A_\alpha, \bar{Q}^A_{\dot{\alpha}}$ with $\alpha, \dot{\alpha}, A = 1, 2$ and define the associated creation and annihilation operators. For $Z^{AB} = 0$ construct and enumerate the number of the states in a massive representation of the algebra starting from a spin 0 state $|\Omega\rangle$. For $Z^{AB} \neq 0$ prove the BPS condition relating mass and charge. Describe the properties of the BPS states saturating the condition. Generalise this result for any even $\mathcal{N}$.

2

Using the definition of the supercovariant derivatives

$$\mathcal{D}_\alpha \equiv \partial_\alpha + i(\sigma^\mu)_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \qquad \bar{\mathcal{D}}_{\dot{\alpha}} \equiv \bar{\partial}_{\dot{\alpha}} + i\theta^\beta(\sigma^\mu)_{\beta\dot{\alpha}}\partial_\mu$$

define a chiral superfield. Write a general expression for a chiral superfield $\Phi(y, \theta)$ with $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ in terms of its components. Use this to write the full expression of $\Phi(x, \theta, \bar{\theta})$. Consider now a chiral superfield X with the extra nilpotency constraint $X^2 = 0$. Solve for each of the components of X. Write the most general superpotential and Kähler potential for the field X. Compute the corresponding contribution to the scalar potential. What can be said about supersymmetry breaking in this case. If an extra chiral superfield Y is introduced satisfying $XY = 0$ solve for the components of Y. Write the most general superpotential and Kähler potential as a function of X and Y.

3

Consider $\mathcal{N}=1$ supergravity with three chiral superfields S, T, and C. In Planck units, the Kähler potential and superpotential are given by

$$\begin{aligned} K &= -\log\left(S+S^*\right) \;-\; 3\,\log\left(T+T^*-CC^*\right) \\ W &= C^3 \;+\; a\,e^{-\alpha S} \;+\; b\;, \end{aligned}$$

where a, b are arbitrary complex numbers and $\alpha > 0$. Compute the scalar potential. Find the auxiliary field for S, T, C and verify that supersymmetry is broken. Assuming that C denotes a matter field with vanishing vev, find a minimum of the potential. Are there flat directions? A typical Kähler potential derived from string compactifications includes the term

$$K = -3\log\Gamma(\tau_i)$$

where Γ is a homogeneous function of degree one of moduli fields τ_i. Using the homogeneity equations $\tau_i\Gamma_i = \Gamma$ and $\tau_i\Gamma_{ij} = 0$ (where $\Gamma_i = \partial\Gamma/\partial\tau_i$, etc.) show that

$$\tau_i K_{ij} = 3\Gamma_j/\Gamma \qquad \Gamma_i K^{-1}_{ij}\Gamma_j/\Gamma^2 = 1/3$$

and deduce from this that if the superpotential does not depend on the τ_i fields then the corresponding contribution to the $N=1$ supergravity scalar potential V vanishes.

END OF PAPER