UNIVERSITY OF
CAMBRIDGE

MATHEMATICAL TRIPOS   Part III

Wednesday, 7 June, 2017   9:00 am to 12:00 pm

PAPER 135

LOGIC

Answer **QUESTION 1** and **ANY THREE** other questions

.

*There are **SIX** questions in total.*

The questions carry equal weight.

**You may not start to read the questions
printed on the subsequent pages until
instructed to do so by the Invigilator.**

1

In this question you may use excluded middle but not AC.

A set is *Dedekind* if it is infinite but has no countably infinite subset.

(1) Show that if X is Dedekind then so too is the set of finite repetition-free sequences from X.

Define D-trees inductively as follows. A D-tree has a root d which is a member of D; its children form a repetition-free finite sequence of $(D \setminus \{d\})$-trees.

(2) Prove that if D is Dedekind so is the set of D-trees.

2

What are *many-one reducibility* and *Turing reducibility*?

State and prove the Friedberg-Muchnik theorem.

3

State and prove the Omitting Types theorem for a first-order language.

What is a universal sentence? Prove that any theory the class of whose models is closed under substructure has an axiomatisation consisting entirely of universal sentences.

4

State and prove the Ehrenfeucht-Mostowski theorem. Your proof should use ultraproducts not Ramsey's theorem.

5

What is the negative interpretation for first-order logic?

Prove that the negative interpretation of a classical thesis ϕ is always a constructive thesis, and is classically equivalent to ϕ.

6

Show that every partial computable function can be represented by a λ-term acting on Church numerals.

END OF PAPER