UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 31 May, 2016 9:00 am to 12:00 pm

PAPER 317

STRUCTURE AND EVOLUTION OF STARS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Consider a white dwarf star of mass M, radius R, and luminosity L. A typical white dwarf may be approximated by an isothermal electron-degenerate core, which comprises most of the star's mass, surrounded by an outer layer composed of gas, mean molecular weight μ, in radiative equilibrium with negligible radiation pressure and opacity given by Kramer's law $\kappa = \kappa_0 \varrho T^{-3.5}$, at density ϱ and temperature T, with $\kappa_0 = const.$ Assume that the gas in the outer layer obeys the perfect gas equation of state. In the core, the degenerate electron gas pressure is given by $P = K_1(\frac{\varrho}{\mu_e})^{5/3}$, ϱ is the density and μ_e is the mean molecular weight of the electrons and K_1 is a constant.

(a) Find the pressure P as a function of temperature T in the outer layers of this white dwarf. Then find L/M in terms of the core temperature T_c, where $T_c \gg T_{\rm eff}$, the effective surface temperature. Assume that the density ϱ is continuous between the core and the outer layers.

(b) Show that the temperature profile throughout the outer layers is given by

$$T(r) = \frac{4}{17} \frac{\mu m_{\rm H}}{k} GM \left(\frac{1}{r} - \frac{1}{R} \right) ,$$

where $m_{\rm H}$ is the atomic mass unit and k is Boltzman's constant.

(c) Show that the outer layer's thickness $s = R - r_b \ll R$, where r_b is the inner radius of the outer layer.

(d) By what ratio s_1/s_2 would the thickness of the outer layer have to change to produce a drop in luminosity from L_1 to L_2.

2

Homologous radiative stars are composed of material of uniform composition that behaves as an ideal gas. Its opacity is given by $\kappa \propto \varrho^\lambda \, T^\nu$ and the energy generation is given by $\epsilon \propto \varrho \, T^\eta$

(a) Obtain the relation between stellar radius R and mass M in the form $R \propto M^X$, where $X = (\eta + \nu + \lambda - 1)/(3 + \eta + \nu + 3\lambda)$

(b) When the opacity is given by the Kramer's law with $\lambda = 1$ and $\nu = -3.5$ and the *pp* chain dominates energy generation with $\eta = 4$, obtain relations between stellar radius R, luminosity L and mass M in the form $R \propto M^X$, $L \propto M^Y$, giving X and Y explicitly. Find the slope on which these stars are in the H-R diagram.

(c) When the opacity is given by electron scattering and the CNO cycle dominates energy generation with $\eta = 16$, obtain relations $R \propto M^X$, $L \propto M^Y$, giving X and Y explicitly. Find the slope on which these stars are in the H-R diagram.

Which of the cases (b) and (c) can be used to model greater and lower stellar masses?

3

Define the adiabatic exponents Γ_1 and Γ_2 and derive them for a mixture of a perfect gas and radiation in terms of $\beta = P_{\rm gas}/P$ where P is the total pressure. Calculate Γ_1 and Γ_2 for $\beta = 0, \frac{1}{2}, 1$.

Write down the Schwarzschild criterion for convective instability in terms of Γ_2 and the temperature gradient.

A star of mass M has uniform opacity κ and uniform β. It has a convective core of mass $M_{\rm c}$, there is no nuclear (or otherwise) energy generation outside the core and energy transport is radiative outside the core. Show that the mass fraction of such core $\frac{M_{\rm c}}{M}$ is given by $\Gamma_2/4(\Gamma_2 - 1)$.

4

Write an essay on energy generation in stars. Start with the energy balance equation and then proceed through various contributions to energy generation and losses. While discussing nuclear fusion reactions describe in detail the *pp* chain, *CNO* cycle and the *triple* $-\,\alpha$ reaction and then comment on possible energy generation in further burning.

END OF PAPER