UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 27 May, 2016 9:00 am to 12:00 pm

PAPER 315

EXTRASOLAR PLANETS: ATMOSPHERES AND INTERIORS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Energy Transport and Temperature Profiles

a. A transiting Jupiter-sized exoplanet orbiting a sun-like star at 0.05 AU was observed in thermal emission with an infrared space telescope in a 1-μm-wide photometric bandpass centered at 20 μm. Assume that there is no significant line absorption in this band due to any prominent molecules expected in hot Jupiter atmospheres, besides H_2. The observations measured the secondary eclipse depth of the planet to be 0.5 %.

- Estimate the brightness temperature (T_1) of the planet in this bandpass. Make and state any assumptions required.
- Estimate a nominal pressure this temperature would correspond to in the planet's atmosphere. How would the temperature change for pressures larger than this pressure?
- Another observation made in another photometric bandpass centered at 4.5 μm found a brightness temperature (T_2) of 1500 K in that bandpass. Give two reasons why $T_1 \neq T_2$. Provide an illustrative sketch of the pressure-temperature profile of the atmosphere.
- Another observation was made in another photometric bandpass in the J band (centered at 1.2 μm) which also has very little molecular line opacity. How would the temperature in this bandpass (T_3) compare with T_1 and T_2?

b. Consider three categories of planets: (1) Earth and larger planets in the solar system, (2) irradiated hot Jupiters (HJs), and (3) directly-imaged giant exoplanets on distant orbits.

- Draw qualitative sketches of atmospheric pressure-temperature (P-T) profiles expected for the three categories of planets. Mark which parts of the atmospheres are dominated by radiative versus convective temperature gradients, and at what pressures are the typical radiative-convective boundaries in each case.
- State two differences between thermal inversions in hot Jupiters and those in solar system planets.
- State two differences between P-T profiles of HJs and directly-imaged giant planets currently known.

c. State three modes of energy transport in planetary atmospheres and interiors. Show that an atmosphere in hydrostatic equilibrium will be unstable against convection for $dT/dz \leqslant -g/C_p$, where T is the atmospheric temperature, z is the radial distance, g is the gravity, and C_p is the specific heat capacity at constant pressure.

2

Atmospheric Chemistry

a. State and briefly explain the dominant chemical processes in planetary atmospheres as a function of altitude or pressure. Provide a sketch and identify nominal pressures corresponding to the transitions between the different processes. If convenient you may assume a particular planet type, e.g. a hot Jupiter.

b. A transiting hot-Neptune, GJ 436b, orbiting an M Dwarf star was observed using the Spitzer infrared space telescope to characterize its atmospheric properties. Thermal emission measurements revealed that the average dayside temperature of the atmosphere was below 500 K for pressures (P) below 0.1 bar and over 1300 K for pressures above 1 bar.

- What are the four dominant molecules expected in this atmosphere in each of the two pressure regimes mentioned above?
- Consider a reaction A1 + A2 $\rightarrow$ B1 + B2. In chemical equilibrium, it is known that the forward reaction is favored at low temperatures and the reverse reaction is favored at high temperatures. However, observations have found copious amounts of A1 in the upper atmosphere at P$\lesssim$0.1 bar, implying that some form of mixing is taking place in the atmosphere. Estimate the strength of the mixing coefficient required for this to occur. The reaction timescale for the forward reaction at $P{\sim}1$ bar is 10^5s. Make and state any assumptions required. For reference, a representative scale height of Jupiter's atmosphere is 30 km and gravity of Neptune is $\sim 0.4\times$ that of Jupiter.

c. At typical temperatures prevalent in hot Jupiter atmospheres ($\sim$1000-3000 K) the H_2O mixing ratio is expected to be $\sim 5\times 10^{-4}$, assuming a solar elemental abundance and chemical equilibrium.

- A thermal emission spectrum of the dayside of a hot Jupiter revealed a minimum dayside temperature of 1800 K and a H_2O abundance of 10^{-6}. What could be possible reasons for such a low H_2O abundance?
- A transmission spectrum of the same planet revealed an average terminator temperature of 1000 K and an H_2O abundance of 10^{-4}. What could explain this observed abundance at the terminator?
- How would you reconcile the different H_2O abundances between the dayside and terminator of the same planet?

3

Planetary Interiors and Surfaces

a. What are the two categories of giant planets which can have inflated radii, i.e. radii too large for their masses? How is the radius of the planet inferred in each case? What are the two broad classes of solutions that have been proposed to explain inflated radii in giant exoplanets? State two specific solutions in each class.

b. What is the maximum radius an isolated sub-stellar mass object could have in steady state? Sketch a theoretical mass-radius curve for astronomical objects ranging from ice-giant planets to low-mass stars, showing the approximate power-law exponents where applicable. Identify and explain the transitions between the different object types. Briefly comment on the source(s) of energy and the mechanism of energy transport in the interiors of each class of objects.

c. Starting with the appropriate structure equation(s) show that a newly formed planet starts hot and cools during its evolutionary lifetime, assuming no significant external source of energy. How does this phenomenon help in the detection of exoplanets? What properties of such exoplanets can be observed?

d. Derive the equilibrium temperature (T_e) of a planet with a Bond albedo A_B orbiting a star with temperature $T_\star$ and radius $R_\star$ at an orbital separation a. Using a simple one-layer atmosphere model derive the surface temperature of a rocky planet due to the green house effect from its atmosphere. Assume that α is the fraction of infrared radiation trapped by the atmosphere.

4

Short Questions (Answer all questions):

a. Discuss briefly three methods of observing exoplanetary atmospheres and the pros and cons of each method.

b. Discuss briefly four mechanisms of atmospheric escape in highly irradiated exoplanetary atmospheres.

c. Discuss briefly three different approaches to model exoplanetary atmospheres.

d. State what wavelengths and telescopes would you observe in if you want to detect the following in exoplanetary atmospheres: (a) atomic species, (b) molecular species, and (c) atmospheric escape.

e. Discuss briefly three observational signatures of atmospheric dynamics in hot Jupiters.

f. Discuss briefly three observable signatures of clouds or hazes in exoplanetary atmospheres. State the type of observation and the approximate wavelength range in which each signature can be observed.

g. What are super-Earths? What are the various observables that can help in constraining the interior compositions of super-Earths?

h. What are primary and secondary atmospheres? Which planets in the solar system have primary atmospheres and which have secondary atmospheres?

i. Discuss key developments in the theory of thermal inversions in hot Jupiters.

j. Discuss three effects of non-equilibrium atmospheric chemistry in planets in the solar system and three effects in exoplanets.

END OF PAPER