UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 3 June, 2016 9:00 am to 11:00 am

PAPER 307

SUPERSYMMETRY AND EXTRA DIMENSIONS

*Attempt **QUESTION 1** and no more than **TWO** of Questions 2-4*

*There are **FOUR** questions in total.*

Question 1 is worth 60 marks.
Questions 2-4 are worth 20 marks each.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Provide a concise answer to the following questions:

(i) Construct the massless representations of $\mathcal{N} \geqslant 1$ supersymmetry with states labelled by momenta p^μ and helicity λ. Find the largest values of $\mathcal{N}$ to have a renormalisable field theory ($|\lambda| \leqslant 1$) and to have one single graviton ($|\lambda| = 2$). Explain how chirality puts strong constraints on the number of supersymmetries. [15 marks]

(ii) Illustrate using massive and massless $\mathcal{N} = 1$ supersymmetric multiplets how the standard Higgs mechanism can be implemented in supersymmetric theories. How does this differ from the super Higgs effect? [10 marks]

(iii) Explain how in $\mathcal{N} = 1$ theories for chiral superfields the strength of the quartic coupling of scalar fields $g^2|\varphi|^4$ and that of the Yukawa couplings $g\varphi\psi\psi$ are determined by the same parameter g. You may use the Wess Zumino model to illustrate this. Explain in a few words how this may be relevant to address the hierarchy problem for the Higgs mass. [10 marks]

(iv) Using the $\mathcal{N} = 1$ supersymmetry algebra show that the number of bosons equals the number of fermions in any multiplet. Prove also that the energy is not negative. What can we conclude from this result regarding the value of the energy when supersymmetry is broken? [15 marks]

(v) Define the supersymmetry transformation of a general scalar superfield and prove that the product of two superfields is a superfield. Is $S(x, \theta, \bar{\theta}) = \phi(x)$ a superfield? Explain. [10 marks]

2

Show that the supercovariant derivatives defined as:

$$\mathcal{D}_\alpha \equiv \partial_\alpha + i(\sigma^\mu)_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \qquad \bar{\mathcal{D}}_{\dot{\alpha}} \equiv \bar{\partial}_{\dot{\alpha}} + i\theta^\beta(\sigma^\mu)_{\beta\dot{\alpha}}\partial_\mu$$

satisfy the algebra

$$\left\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\right\} = 2i(\sigma^\mu)_{\alpha\dot{\beta}}\partial_\mu \qquad \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \left\{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\right\} = 0$$

Show that if $\Phi = \Phi(y, \theta, \bar{\theta})$ with $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ then a chiral superfield can be written as $\Phi(y, \theta) = \varphi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$.

3

Starting with a general Lagrangian for chiral and vector superfields use arguments of symmetry and holomorphy to prove that the superpotential of $\mathcal{N} = 1$ supersymmetric theories does not receive quantum corrections in perturbation theory. You may introduce spurion superfields to keep better control of the couplings and the symmetries.

4

Consider a renormalisable $\mathcal{N} = 1$ supersymmetric theory for chiral superfields with F term supersymmetry breaking. Prove that

$$StrM^2 = \sum_j (-1)^{2j+1}(2j+1)m_j^2 = 0$$

where j represents the total spin of the particles and m_j their mass. What implications can this result have for the MSSM?

Consider the O'Raifertaigh model with three chiral superfields X, Y, Z and superpotential

$$W = \lambda X(Z^2 - \mu^2) + MYZ \qquad M >> \mu$$

find the mass spectrum of the corresponding scalar and fermion fields around the minimum field configuration $x = y = z = 0$ where x, y, z are the scalar components of X, Y, Z respectively and verify that the supertrace formula above is satisfied.

END OF PAPER