UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 26 May, 2016 9:00 am to 12:00 pm *Draft 4 July, 2016*

PAPER 301

QUANTUM FIELD THEORY

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Write an essay explaining the Feynman rules for photons interacting with electrons and positrons. Your essay should start from a description of the path integral for free photons, electrons and positrons and why it is equivalent to the operator formalism. You should then explain how the electromagnetic interaction arises and how it is consistent with gauge invariance. Then you should explain how the Feynman diagrams arise in the computation of the expectation values of operators, how to remove vacuum diagrams and finally what the rules are.

2

Define Dirac gamma matrices.

How do the gamma matrices behave under the transpose operation and under Hermitian conjugation?

Write down the Dirac equation and explain how it describes spin 1/2 particles and antiparticles.

The wavefunction for particles of spin s and momentum p^a is $\Psi = e^{ip\cdot x}u_s(p)$. Derive the equation satisified by $u_s(p)$.

Find the equation satisfied by $\bar{u}_s(p)$.

Let $\gamma^{ab} = \frac{1}{2}[\gamma^a, \gamma^b]$, and show that

$$\bar{u}_{s'}(p')\gamma^{ab}(p'_b - p_b)u_s(p) + \bar{u}_{s'}(p')(2m\gamma^a + p^a + p'^a)u_s(p) = 0.$$

3

Starting from the action for the free real Klein-Gordon field, explain the process of canonical quantization and illustrate how to derive the commutation rules for the creation and annhilation operators for this case.

Consider the expression

$$\langle 0|T\phi(x_1)\phi(x_2)|0\rangle = G_F(x_1 - x_2).$$

Explain the physical meaning of the left hand-side of the above expression. Starting from the field operator for the scalar field, derive an expression for the Fourier transform of $G_F(x_1 - x_2)$ and explain what is meant by the "$i\epsilon$ prescription ".

Evalaute

$$\langle 0|T\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)|0\rangle$$

in terms of G_F.

4

One way of quantizing the electromagnetic field is to consider a different starting point from the usual Maxwell action. Suppose the action is given by

$$\int d^4x \ \left(-\frac{1}{4} F_{ab} F^{ab} + \frac{1}{2\xi} (\partial_a A^a)^2 \right)$$

where ξ is a new scalar field. Is this action invariant under gauge transformations?

What are the equations of motion for A_a and ξ.

Assuming that ξ is constant, find the A_a equation of motion in momentum space.

Find the Green's function $G^{ab}(p)$ for the field A^a in momentum space.

Show that $p^a G_{ab} \neq 0$ for all $\xi \neq 0$.

END OF PAPER