UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 26 May, 2016 1:30 pm to 4:30 pm

PAPER 123

ALGEBRAIC NUMBER THEORY

*Attempt **ALL** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

State a version of Hensel's lemma, and use it to show that $\mathbb{Q}_3^\times/(\mathbb{Q}_3^\times)^3 \cong \mathbb{Z}/3\mathbb{Z}\times\mathbb{Z}/3\mathbb{Z}$.

2

(a) Let $K/\mathbb{Q}_p$ be a finite extension, and let L/K be a finite Galois extension. Define the upper and lower ramification groups of $G = \mathrm{Gal}(L/K)$.

(b) Calculate the upper and lower ramification groups in case $K = \mathbb{Q}_3$ and L is the splitting field of the polynomial $f(X) = X^3 - 3$.

3

(a) Let $K/\mathbb{Q}_p$ be a finite extension, and let $f(X) \in K[X]$ be a monic polynomial with $f(0) \neq 0$. Define the Newton polygon $N_K(f)$ of $f(X)$.

(b) Let L/K be a Galois, totally ramified extension of degree n, let $\pi \in \mathcal{O}_L$ be a uniformizer, and let $f(X) \in \mathcal{O}_K[X]$ denote the minimal polynomial of π. Let $g(X) = f(\pi X + \pi)/\pi^n X \in \mathcal{O}_L[X]$. Show that an integer m is a slope of $N_L(g)$ if and only if $G_m \neq G_{m+1}$.

4

Calculate, with justification, the Hilbert class field of $K = \mathbb{Q}(\sqrt{-31})$. *[You may use the fact that the discriminant of the polynomial $f(X) = X^3 + aX + b$ equals $-4a^3 - 27b^2$.]*

END OF PAPER