UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 8 June, 2015 9:00 am to 11:00 am

PAPER 65

BINARY STARS

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

A binary system comprises two stars of masses M_1 and M_2 in a circular orbit with period P. Show that the total orbital angular momentum J is given by

$$J = \frac{G^{2/3} P^{1/3} M_1 M_2}{(2\pi)^{1/3} (M_1 + M_2)^{1/3}},$$

where G is Newton's constant.

The star of mass M_1 is transferring mass to the star of mass M_2 while also losing mass isotropically in a stellar wind. A fraction f of the total mass lost by star 1 is accreted by star 2. Explain why the total angular momentum lost by the system is

$$\dot{J} = (1 - f)\dot{M}_1 a_1^2 \Omega,$$

where a_1 is the distance of star 1 from the centre of mass of the system and $\Omega = 2\pi/P$.

By considering the conservation of angular momentum, or otherwise, deduce that

$$P \propto M_1^{-3f} M_2^{-3} (M_1 + M_2)^{-2}.$$

The effective Roche lobe radius of star 1 can be approximated by

$$R_{\mathrm{L}} = 0.46a \left(\frac{M_1}{M_1 + M_2} \right)^{\frac{1}{3}}.$$

Show that

$$\frac{\dot{R}_{\mathrm{L}}}{R_{\mathrm{L}}} = \frac{\dot{M}_1}{M_1} \left\{ f \left[2q + \frac{4q}{3(1+q)} - 2 \right] + \frac{1}{3} - \frac{4q}{3(1+q)} \right\},$$

where $q = M_1/M_2$.

The radius R_1 of star 1 is such that $R_1 \propto M_1^{-n}$. If star 1 remains just filling its Roche lobe show that

$$f \left[2q + \frac{4q}{3(1+q)} - 2 \right] = \frac{4q}{3(1+q)} - n - \frac{1}{3}$$

and comment on what happens when q and n are such that this cannot be satisfied if $0 \leqslant f \leqslant 1$.

2

A binary system of total mass M consists of two stars of masses M_1 and M_2 in an elliptical orbit and sufficiently separated that they behave as point mass objects. Instantaneously, the vector from star 2 to star 1 is $\boldsymbol{r}$ and the velocity of star 1 relative to star 2 is $\boldsymbol{v} = \dot{\boldsymbol{r}}$. Show that the energy of the orbit

$$E = \mu \left(\frac{1}{2} v^2 - \frac{GM}{r} \right),$$

where $\mu = M_1 M_2 / M$ is the reduced mass, $r = |\boldsymbol{r}|$ and $v = |\boldsymbol{v}|$ are the instantaneous separation and relative speed of the stars and G is Newton's constant. Show further that the orbit can be parametrised by

$$r = \frac{l}{1 + e \cos\theta},$$

with $0 < \theta < 2\pi$ and that

$$E = -\frac{GM\mu}{2a} = \text{const},$$

where $l = a(1 - e^2)$ and a is the semi-major axis.

In a system with a circular orbit star 1 of mass M_1 undergoes a supernova explosion to leave a neutron star of mass M_1' in a time much shorter than the orbital period. The mass lost to the gravitational binding energy of the neutron star may be neglected. Matter is ejected in an asymmetric manner such that the neutron star experiences a kick of speed $u = \alpha v$, with $\alpha < 1$, at an angle ψ to $\boldsymbol{v}$. Show that the new total mass M' and semi-major axis a' are related by

$$\frac{M'}{a'} = \frac{2M'}{a} - (1 + 2\alpha \cos\psi + \alpha^2) \frac{M}{a}$$

and from there that the system can unbind if

$$M' < \frac{1}{2}(1 + \alpha^2) M$$

but could remain bound if

$$M' > \frac{1}{2}(1 - \alpha)^2 M.$$

If the system becomes unbound show that the stars eventually recede from one another at a speed

$$V = \left\{ (1 + 2\alpha \cos\psi + \alpha^2) - \frac{2M'}{M} \right\}^{\frac{1}{2}} v.$$

3

Write an essay on Cataclysmic Variables. Include a diagram of the standard features of non-magnetic systems and discussions of the nova and dwarf nova phenomena, their orbital period distribution, evolutionary driving mechanisms and formation.

END OF PAPER