UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday, 29 May, 2015 1:30 pm to 3:30 pm

PAPER 61

THE ORIGIN AND EVOLUTION OF GALAXIES

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

i) Use the shape and evolution of the power spectrum of the spatial distribution of matter in the late-time Universe (redshift $z < 100$) to explain what is meant with hierarchical galaxy formation. Explain when and how this shape was established, and how it relates to the shape of the power spectrum of spatial fluctuations of the energy density in the early Universe. Explain the difference between warm and cold dark matter, and how this affects galaxy formation.

ii) The volume cooling rate of gas with temperature T can be written as $\mathcal{C}(T) = \Lambda(T)n_{\mathrm{H}}^2$, where $\Lambda(T)$ is the cooling function and n_H is the number density of hydrogen. Sketch a diagram of the cooling function of metal-free primordial gas for $10^4K < T < 10^8K$. Discuss the main characteristic features of the cooling curve and the processes responsible. How does the presence of metals affect the cooling curve.

iii) Sketch the curve in the number density-temperature plane, at which the cooling time of a uniform cloud of metal-free primordial gas with temperature equal to the virial temperature equals its free-fall time. Discuss how the cooling diagram has been used to explain the upper mass limit of galaxies.

[You may find the following helpful for labelling your diagrams. For metal-free primordial gas: $\log(\Lambda(T = 15000\mathrm{K})/(\mathrm{ergcm}^{-3})) \approx -21.8, \log(\Lambda(T = 10^7\mathrm{K})/(\mathrm{ergcm}^{-3}) \approx -23.0$. The number density of hydrogen at redshift $z = 0$ is $n_{\mathrm{H}} \approx 2 \times 10^{-7}\mathrm{cm}^{-3}$. The Boltzmann constant can be approximated as $k_{\mathrm{B}} \approx 1.38 \times 10^{-16}\mathrm{ergK}^{-1}$ and the gravitational constant as $G \approx 6.67 \times 10^{-8}\mathrm{cm}^3\mathrm{s}^{-2}\mathrm{g}^{-1}$.]

2

An observed spectrum of a QSO at $z = 3$ shows several Lyα absorption features with average separation of 304 $\AA$. Calculate the proper length that corresponds to the average separation of the absorbers.

Assume that the absorption features are caused by neutral hydrogen in a population of randomly orientated thin disc galaxies with radius $R_{\rm d}$ and comoving space density $n_{\rm gal} = 2.8$ Mpc^{-3}. Calculate $R_{\rm d}$.

Assume that the discs have formed from a quarter of the baryons in dark matter haloes with virial velocity $v_{\rm vir}$ settling into centrifugal support with the specific angular momentum of the baryons at the edge of the disc equal to the specific angular momentum of the halo at the virial radius. Assume further that the ratio of rotational velocity at the virial radius to virial velocity of the haloes is, $v_{\rm rot}^{\rm h} = 0.1 v_{\rm vir}$, and that the dark matter does not contribute to the gravitational potential at $R \leqslant R_{\rm d}$. Calculate the virial radius and the virial velocity of the haloes.

State and explain any additional assumptions you make.

[Assume the Universe to be flat with matter density and cosmological parameters $\Omega_{\rm mat} = 1 - \lambda = 0.25$, baryonic density parameter $\Omega_{\rm bar} = 0.05$ and Hubble constant H_0= 70 km s^{-1} Mpc^{-1} . The rest frame wave length of Lyα is $\approx 1216\AA$.]

3

i) Assume that the (comoving) number density of galaxies at $z = 3$ with stellar mass M_* is given by,

$$n(M_*, z = 3)\,\mathrm{d}M_* = \left(\frac{M_*}{M_{*,\mathrm{ch}}}\right)^{-7/4} \frac{\phi_{\mathrm{ch}}}{M_{*,\mathrm{ch}}} \exp\left[-(M_*/M_{*,\mathrm{ch}})^{1/2}\right]\mathrm{d}M_*,$$

with characteristic stellar mass $M_{*,\mathrm{ch}} = 10^{10} M_\odot$ and and characteristic number density $\phi_{\mathrm{ch}} = 10^{-2}\,\mathrm{Mpc}^{-3}$. Compare the mass density of stars in these galaxies with the critical density of the Universe today.

ii) With the Press-Schechter ansatz, the mass fraction of the matter density in the Universe in collapsed objects with mass greater than M is,

$$f(> M, t) = \mathrm{erfc}\left(\frac{\delta_\mathrm{c}(t)}{\sqrt{2}\sigma(M, z = 0)}\right) \quad \text{with} \quad \mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp\left(-y^2\right)\mathrm{d}y.$$

Explain the meaning of $\delta_\mathrm{c}(t)$ and $\sigma(M, z)$. Assume that the fraction of baryons in collapsed dark matter haloes forming stars is $f_* = \mathrm{const.}$, independent of mass. Show how the stellar mass function in (i) can be derived with the Press-Schechter Ansatz if the power spectrum describing the Gaussian density fluctuations is assumed to be a power law $P(k) \propto k^n$ on the relevant scales.

iii) Calculate n and f_*. Why is the value of n different from $n = 1$, the power law index expected for primordial density fluctuations in the early Universe?

iv) Calculate the value of $\sigma(M = 10^{14} M_\odot, z = 3)$. How realistic is the assumption $f_* = \mathrm{const.}$ in light of the stellar mass fraction of dark matter haloes inferred from observations?

[Assume the Universe to be flat with matter density and cosmological parameters $\Omega_{\mathrm{mat}} = 1 - \lambda = 0.25$ and baryonic density parameter $\Omega_{\mathrm{bar}} = 0.05$. Assume that the mean matter density at $z = 0$ is $3.5 \times 10^{10} M_\odot\,\mathrm{Mpc}^{-3}$. You may find the approximation $\int_0^\infty x^{-1/2} \exp(-x)\,dx \approx 1.77$ helpful.]

END OF PAPER