UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 28 May, 2015 1:30 pm to 4:30 pm

PAPER 58

STRUCTURE AND EVOLUTION OF STARS

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Write down the equations of stellar structure, assuming that energy transport is by radiative diffusion in a perfect gas with uniform mean molecular weight μ and neglecting all microscopic changes in heat content of an element of matter with density ρ. Radiation pressure can be neglected.

Assume that the density distribution $\rho(r)$ in the star is linear and is given by

$\rho = \rho_{\rm c}(1 - \frac{r}{R})$, where ρ_c is the central density of the star r is the distance from the centre of the star and R is the radius of the star.

(a) Derive expressions for mass within radius r, $m_{\rm r} = m(r)$, pressure $P_{\rm r} = P(r)$ and temperature $T_{\rm r} = T(r)$, and give expressions for the central pressure $P_{\rm c}$ and the central temperature $T_{\rm c}$ in terms of the radius R and the total mass of the star M. You can assume that $\frac{r}{R} \ll 1$.

(b) Find the ratio of the radiation pressure to the gas pressure at the centre of this star in terms of M. Find the value of M for which the radiation pressure would be equal to the perfect gas pressure.

(c) Assume now that the rate of nuclear energy generation is $\epsilon = \epsilon_0 \ \rho \ (\frac{T}{T_0})^\nu$. Derive an expression for luminosity $L_{\rm r}$ in terms of M, R, and the integral $I_\nu = \int_0^1 f(x)dx$, with $x = \frac{r}{R}$ and $f(x) = x^2(1-x)^{2+\nu} \ (1 + 2x - \frac{9}{5}x^2)^\nu$.

2

A star with mass M and radius R is made of a perfect gas and has constant density ρ and mean molecular weight μ.

(a) Find the central pressure $P_{\rm c}$ and the central temperature $T_{\rm c}$ of such a star in terms of its total mass and density.

(b) What is the gravitational potential energy, Ω, of such a star in terms of M and R?

(c) Assume that the star shrinks in radius and supports its luminosity from its gravitational energy reservoir, thus converting its gravitational energy into luminosity. Quote the virial theorem for spherical star built of a perfect gas. Assume virial equilibrium and find an equation for evolution of luminosity.

(d) Assume now that the star is emitting at the Solar luminosity rate and has the mass and radius of the Sun. Find the relevant timescale for this star's evolution and determine if such a process can be ruled out as the source of the Sun's power.

(e) Relate a model of a constant density star to the theory of polytropes. Derive the Lane-Emden equation and solve it for the appropriate polytrope.

3

Suppose that you have a mixture of simple perfect gas and black body radiation, Assume that the as pressure $P_{\rm g} = \frac{k}{\mu H}\rho T$ and the radiation pressure $P_{\rm rad} = \frac{1}{3}aT^4$.

(a) Calculate specific heat at $c_{\rm P}$ in terms of $\beta = \frac{P_{\rm g}}{P}$

(b) Define adiabatic exponents Γ_i, $i = 1, 2, 3$ and calculate the ratio of specific heats γ in terms of Γ_i. What are the relations between γ and Γ_i for a perfect gas, a mixture of gas and radiation and pure black body radiation.

(c) Derive the Ledoux criterion for dynamical stability in gas pressure dominated stellar matter with a composition gradient.

4

Derive the equations of linear adiabatic stellar oscillations and discuss what determines their frequencies. Discuss the acoustic modes.

END OF PAPER