UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday, 8 June, 2015 1:30 pm to 4:30 pm

PAPER 48

SUPERSYMMETRY AND EXTRA DIMENSIONS

Attempt no more than ***THREE*** *questions.*

There are ***FOUR*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Using the Jacobi identity, calculate $[Q_\alpha,\ P^\mu]$. What does this imply for the masses of different states in a supermultiplet (provide a brief argument)?

The O'Raifertaigh model has three chiral superfields $\Phi_1,\ \Phi_2,\ \Phi_3$ with Kähler potential $K = \Phi_i^\dagger \Phi_i$ and superpotential

$$W = g\Phi_1\left(\Phi_3^2 - m^2\right) + M\Phi_2\Phi_3.$$

Assuming that $M \gg m$ and $m^2 < M^2/(2g^2)$, analyse the mass spectrum of the theory, commenting on any supersymmetry breaking and on the supertrace operator.

Why is this not a good model for direct supersymmetry breaking in the MSSM?

2

Cover the following topics on the $N = 1$ chiral superfield Lagrangian:

(a) The definition of a chiral superfield.

(b) Which functions of chiral superfields are chiral superfields.

(c) An expansion of the chiral superfield in terms of $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$.

(d) Write the Lagrangian in terms of superspace integrals.

(e) Renormalisability.

(f) A derivation of the $F-$dependent terms.

(g) Eliminate F.

[*Hint: You may find the covariant derivative* $\bar{\mathcal{D}}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^\beta(\sigma^\mu)_{\beta\dot{\alpha}}\partial_\mu$ *useful.*]

3

Demonstrate that the volume of a $N-1$ sphere of radius r is

$$V_{N-1} = \frac{2\pi^{N/2}}{\Gamma(N/2)}\, r^{N-1} \tag{1}$$

[*Hint: It may help to consider the integral* $I_N = \int d^N x e^{-\rho^2}$ *with* $\rho^2 = \sum_{i=1}^N x_i^2$.] Use this result to derive an expression for the electric (and gravitational) potential in D dimensions.

Write an explicit expression for the four-dimensional Planck scale M_{planck} in terms of the $(n+4)$-dimensional Planck scale M_* if the extra-dimensional space is a n-dimensional sphere. Estimate the values of the radius if $M_* = 1$ TeV for $n = 1, 2, 6$.

Show that the potential due to a point particle in five dimensions reduces to the 4-dimensional potential at distances much larger than the size of the fifth dimension.

4

Using the Dirac algebra in $D = 2k$ dimensions, where k is a positive integer, find the dimensionality of the spinorial representation of the Lorentz group $SO(1, D-1)$. Is the representation irreducible? What is the difference with $D = 2k+1$?

Write the field content of IIA and IIB supergravities in $D = 10$. Count the number of degrees of freedom of each field, explaining in detail each derivation.

Perform the dimensional reduction to $D = 9$ and count the number of degrees of freedom for each multiplet. Is the spectrum chiral? Explain. Perform directly the reduction from $D = 11$ to $D = 9$ and compare. Perform dimensional reduction of IIA supergravity in $D = 10$ to $D = 4$ and compare the number of degrees of freedom.

If some antisymmetric tensors appear in the final spectrum show that they are either non-physical or that they are equivalent to scalar fields in $D = 4$. Derive this equivalence in detail keeping track of the formal dependence on the coupling constant in the effective Lagrangian.

END OF PAPER