UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Wednesday, 3 June, 2015 1:30 pm to 3:30 pm

PAPER 42

CONTEST THEORY

Attempt no more than ***THREE*** *questions.*

There are ***FIVE*** *questions in total.*

The questions carry equal weight.

This is ***OPEN BOOK*** *examination*
Candidates may only bring into the examination handwritten
or personally typed lecture notes and handouts from this course.
No other material, or copies thereof, are allowed.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Consider a principal who uses an auction mechanism to collect money from $n \geqslant 1$ individuals (players) for a public project (each individual receives a unit allocation if any allocation is made to the individuals at all) and whose goal is to maximize revenue. The public project may or may not be realized depending on the bids put in by the individuals. Consider this as a game with incomplete information where individuals hold private valuations according to regular prior distributions $F_1, F_2, \ldots, F_n$ with respective density functions $f_1, f_2, \ldots, f_n$ and supports $[\underline{v}_1, \bar{v}_1]$, $[\underline{v}_2, \bar{v}_2]$, …, $[\underline{v}_n, \bar{v}_n]$.

What is the optimum auction mechanism with respect to maximizing the expected revenue in a Bayes-Nash equilibrium?

Show an explicit characterization of the optimal auction mechanism for the case of uniform prior distributions on $[0, 1]$.

2

Consider a game G_1 that models the contest with rank-order allocation of prizes among $n > 2$ players with private valuations that are independently and identically distributed according to a prior distribution function F, and where players incur unit marginal costs of production. In this contest, there are two prizes: a first place prize of value 2 and a second place prize of value 1.

Consider another game G_2 under the same assumptions except that the contest consists of two subcontests in which all players make simultaneous effort investments. In the first subcontest, there is only a first place prize of value 1, and in the second subcontest there is a first place prize and a second place prize of value 1 each.

Compare the expected total efforts in the symmetric Bayes-Nash equilibria of games G_1 and G_2.

Compare the expected total efforts in the symmetric Bayes-Nash equilibria of games G_1 and G_2 in the case when in game G_2, the second subcontest offers only a first place prize but of value 2 instead of 1.

3

Consider a system of three simultaneous contests that offer prizes of values $w_1 \geqslant w_2 \geqslant w_3 > 0$, and $n \geqslant 2$ players with identical valuations and linear production costs with unit marginal costs of production. Each player is assumed to participate in exactly two contests that are strategically selected by the player. The payoff functions of the players are assumed to be quasi-linear in the expected reward and the production cost.

Show that there exists a unique symmetric mixed-strategy Nash equilibrium and characterize it.

4

Consider a two-player contest with valuations $v_1 \geqslant v_2 > 0$ where a unit prize is allocated according to the proportional allocation mechanism, and players incur quadratic production costs. The payoff functions of the players are given by

$$s_1(b_1, b_2) = v_1 x_1(b_1, b_2) - b_1^2 \text{ and } s_2(b_1, b_2) = v_2 x_2(b_1, b_2) - b_2^2,$$

where

$$x_1(b_1, b_2) = \begin{cases} \frac{b_1}{b_1+b_2}, & \text{if } b_1 + b_2 > 0 \\ \frac{1}{2}, & \text{if } b_1 + b_2 = 0 \end{cases} \quad \text{and } x_1(b_1, b_2) + x_2(b_1, b_2) = 1.$$

Show that there exists a unique pure-strategy Nash equilibrium, and provide an explicit characterization of it.

5

Outcomes of pair comparisons are given for a set of three players 1, 2, and 3: player 1 won once against player 2, player 1 lost once against player 3, and player 2 won $k \geqslant 1$ times against player 3. Suppose that the outcomes of pair comparisons are generated according to the Bradley-Terry model with parameters $\theta_1, \theta_2, \theta_3$. Sort the players in decreasing order of the maximum likelihood estimates of the parameters $\theta_1, \theta_2, \theta_3$.

END OF PAPER