UNIVERSITY OF CAMBRIDGE

# MATHEMATICAL TRIPOS Part III

Friday, 6 June, 2014 1:30 pm to 3:30 pm

## PAPER 78

## FLUID DYNAMICS OF ENERGY SYSTEMS

*Attempt no more than **FOUR** questions.*

*There are **SIX** questions in total.*

*The questions carry equal weight.*

***STATIONERY REQUIREMENTS***

*Cover sheet*
*Treasury Tag*
*Script paper*

***SPECIAL REQUIREMENTS***

*None*

**You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.**

**1**

Calculate the effective permeability in the $x$ direction of a two dimensional porous rock of thickness $0 < y < H$ and length $0 < x < L(>> H)$ which is composed of two layers of rock, of thickess $y = Hx/L$ and permeability $k_1$ and $y = H(1 - x/L)$ and permeability $k_2$, and which is bounded above and below, at $y = 0$ and $y = H$ by impermeable rock.

Find the travel time $\tau(y)$ of a parcel of fluid travelling from $x = 0$ to $x = L$ released from the point $(0, y)$ at $t = 0$ and show that the maximum difference in travel time between two parcels of fluid released from $x = 0$ is

$$\Delta\tau = \frac{\phi HL}{2Q}\left[\frac{k_1^2 - k_2^2}{k_1 k_2}\right].$$

[You may assume there is a steady flow through the formation with total (two dimensional) flux $Q$ in the $x$-direction, and that the formation has uniform porosity $\phi$.]

Comment on the implications of this model for the recovery of oil by injection of water from a layered permeable rock in which different layers have permeabilities ranging by a factor of 10.

**2**

Find the velocity potential and the streamfunction for a flow in a two dimensional porous medium produced by a uniform flow $U$ in the x-direction combined with a flow from a source of strength $Q$ at the origin. Find the maximum distance upstream (i.e. in the region $x < 0$) travelled by the fluid from the source, and show that far downstream the source fluid is confined within the region $-b < y < b$ where $b = Q/2U$.

Show that fluid issuing from the source at time $t$ first reaches the point $x(> 0)$ at a time $\tau$ given by

$$\tau = \frac{b}{\pi U}\ln\left(\frac{\pi x}{b} + 1\right)$$

earlier than fluid far from the source, $y >> b$, which passes the line $x = 0$ at time $t = 0$.

**3**

Fluid migrates in the $x$ direction through a porous layer which lies in the region $0 < y < H$, $0 < x < L$, $L >> H$, and which has permeability $k = k_o y/H$. If a pressure gradient is applied in the $x$ direction which produces a mean flow speed $U$, find the equation which describes the evolution of average concentration of contaminant

$$\overline{c} = \frac{1}{H}\int_0^H c dy$$

in a pulse of contaminated fluid as it propagates along the layer, in the limit that $P = H^2U/D_T L << 1$. [You may assume the pore scale (transverse and longitudinal) dispersion in the porous medium is given by the constant values $D_T$ and $D_L$.] What is the physical interpretation of the condition $P$?

Find an expression for the average concentration in the porous layer, $\overline{c}$ in the case that the contaminant is supplied from a source at $x = 0$ with concentration $c = 1$ for $t > 0$, while the region $x > 0$ is initially uncontaminated.

**4**

A wetting fluid of saturation $s$ migrates through a porous layer initially with a non-wetting fluid of saturation $1 - s_o$, and viscosity $\mu_{nw}$ and a residual saturation $s_o$ of the wetting fluid of viscosity $\mu_w$. If the relative permeabilities of the wetting and non-wetting fluids are $k_w$ and $k_{nw}$ and the capillary pressure is $p_c(s)$, find an equation for the evolution of the saturation of the wetting fluid, $s(x,t)$, in the case that the wetting fluid is injected into the porous medium at $x = 0$ with constant flux $Q$ per unit area in the $x$ direction.

Explain why in general the saturation is expected to develop a shock front when the capillary pressure is small, and show the shock speed is given by

$$\frac{dF}{ds} = \frac{F(s_s) - F(s_o)}{s_s - s_o}$$

where $s_s$ is the saturation upstream of the shock and $F$ is the fractional flow of the wetting phase. Explain what effect the capillary pressure has on the shock.

In the case $F = 2 - 1/s$ for $s > s_0 > 0$, explain how the saturation evolves in time as wetting fluid is injected to displace the non-wetting fluid.

**5**

Relatively cold $CO_2$ is injected into a porous layer filled with water, and which has a sloping upper boundary at an angle $\theta$ to the horizontal. The $CO_2$ is driven by the density difference between the $CO_2$ and water, and is modified by the temperature difference between the $CO_2$ and the reservoir. The $CO_2$ has viscosity

$$\mu = \mu_o - \lambda(T - T_r)$$

and density

$$\rho = \rho_w - \Delta\rho - \alpha(T - T_r)$$

where $T_r$ is the reservoir temperature. If the $CO_2$ is injected with a mass flux $Q$ per unit length of a line well arranged across the slope, and has initial temperature $T_r - \Delta T$, such that $\Delta\rho > \alpha\Delta T$, find the depth of the steady current as it propagates upslope both near source, where the current has temperature $T_r - \Delta T$, and in the distal region where the current has adjusted to the reservoir temperature. [You may assume that the thermal front migrates as a local region of adjustment with speed $\Gamma u$ where $u$ is the Darcy speed of the cold injected current. You may neglect cross-current conduction of heat.]

If the current is able to leak through the overlying cap rock when the overpressure of the current is $\Delta p$ find the critical values of the injection rate so that (i) there is leakage from the cold region of the current only; (ii) there is leakage from all the current.

Describe in qualitative terms how the cross-current conduction of heat changes these dynamics.

**6**

A semi-infinite porous medium of porosity $\phi$ and permeability $k$ is brought in contact with a reservoir of water through an inlet of radius $R_s$. The water wets the porous medium and hence capillary forces drive imbibition into the porous matrix. Neglecting the early time inertia of the fluid, consider the axisymmetric spreading of the hemispherical front of radius $R(t)$ assuming a fixed capillary pressure at the advancing front $p_c$ and a constant inlet pressure $p(R_s) = p_0$.

Using mass conservation and Darcy's law within the porous medium, find the radial extent of the front $R(t)$ and hence the flux $Q(t)$ as implicit functions of time. Compare the asymptotic behaviour for $R(t)$ and $Q(t)$ to one-dimensional (i.e. purely vertical) imbibition.

Finally, find the steady-state radius in the case where a constant evaporative loss per unit area of the interface $F_e$ balances the imbibition flux through the porous medium.

## END OF PAPER