UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday, 3 June, 2014 1:30 pm to 4:30 pm

PAPER 71

BIOLOGICAL PHYSICS

Attempt ***ALL*** *questions.*

There are ***THREE*** *questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

A microsphere of radius a and drag coefficient ζ is constrained to move along the x-axis, and is acted on by an optical trap which is moving in the positive x-direction at velocity v_T. When the trap is located at a point x_0 it exerts a force $F(x - x_0)$, so the overdamped dynamics of the particle is

$$\zeta \dot{x} = F(x - v_T t) \ .$$

Suppose that the trap has compact support, so that $F(x) = 0$ for $x < -X_L$ and for $x > X_R$. If the trap starts to the left of the particle, find the particle's net displacement Δx after the trap has passed it by, and the time Δt spent by the particle interacting with the trap. What is the condition that assures that the particle does not remain trapped as $t \to \infty$? Assuming this is the case, show that whatever the form of $F(x)$ the net displacement is always in the direction of the trap motion, and suggest a heuristic explanation for this result. Find the asymptotic behaviour of Δx for large trap velocities.

The trap is now moved around a circle of radius $R \gg a$. Derive the particle's net rotational frequency f_p as a function of the trap angular frequency $f_T = v_T/(2\pi R)$, the displacement Δx in each kick, the interaction time Δt and the potential width $2X_0 = X_R - X_L$. Confirm that in the regime of suitably large trap velocity, which you should define precisely, one obtains the intuitive result $f_p \simeq (\Delta x/2\pi R) f_T$. Specializing to the case of a triangular trapping potential, with $F(x) = F$ for $-X_0 < x < 0$ and $F(x) = -F$ for $0 < x < X_0$, obtain an explicit expression for f_p/f_c as a function of the two quantities $\alpha = X_0/(\pi R)$ and $\beta = f_T/f_c$, where $2\pi R f_c = F/\zeta$.

2

A long cylindrical vesicle of radius R_0, aligned along the z-axis, is subject to a tension $\sigma \gg \kappa/R_0^2$, where κ is the bending modulus. Thus, its energy is well-approximated by $\sigma\mathcal{S}$, where $\mathcal{S}$ is the total surface area of the vesicle. Assuming that fluctuations in the radius preserve axisymmetry, so the fluctuating radius $R(z)$ does not depend on the cylindrical polar angle, find the spectrum of thermal fluctuations as a function of the longitudinal wavevector q, at fixed enclosed volume of fluid. You may take $R(z) = \rho_0 + u_q \sin qz$, where ρ_0 is to be determined by volume conservation. Explain the significance of your result for $qR_0 < 1$.

A circular inclusion of radius R_0 in a lipid membrane consists of a distinct phase from the surrounding lipids, so there is a line tension γ between the two. Find the spectrum of thermal fluctuations in the radius, at fixed enclosed area, as above. Explain the significance of the result for the mode with $qR_0 = 1$.

3

Two parallel charged, planar, laterally-infinite membranes are located at $z = \pm d/2$. The upper one has charge density $\sigma_+ = \alpha \cos(kx)$, while the lower has $\sigma_- = \alpha \cos(kx + \theta)$, where θ is a constant phase shift. Within Debye–Hückel theory, find the electrostatic energy as a function of θ by computing the electrostatic potential ϕ in the region between the sheets. Find the value of θ that minimizes the energy, averaged over one wavelength of the charge modulation, and explain the physical content of this result.

END OF PAPER