UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 29 May, 2014 1:30 pm to 4:30 pm

PAPER 56

THE ORIGIN AND EVOLUTION OF GALAXIES

*Attempt no more than **THREE** questions.*

*There are **FOUR** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Give a brief account of how galaxies form within the ΛCDM paradigm for structure formation. Explain what is meant by hierarchical galaxy formation by referring to the ΛCDM power spectrum and discuss the role of galaxy mergers in shaping galaxies. Take care to discriminate between linear and non-linear evolution of the evolution of dark matter and baryons and to discuss the role of cooling and angular momentum. Discuss how the cooling properties of baryons determine the characteristic masses of galaxies. Discuss briefly how the shape of the mass function of dark matter haloes and the luminosity function of galaxies are related.

2

Thin disc galaxies with radius $R_{\rm d}$ have formed at $z = 3$ within dark matter haloes with virial velocity $v_{\rm vir} = 280\,{\rm km\,s^{-1}}$. Assume that after virialization of the haloes hosting the disc galaxies half of all baryons in each halo have settled into centrifugal support with the specific angular momentum of the baryons at the edge of the disc equal to the specific angular momentum of the halo at the virial radius. Assume further that the ratio of rotational velocity at the virial radius to virial velocity of the halo is given by $v_{\rm rot}^{\rm h}(r_{\rm vir}) = 0.1\,v_{\rm vir}$, and that the dark matter does not contribute to the gravitational potential at $R \leqslant R_{\rm d}$.

Calculate the radius $R_{\rm d}$ and the rotational velocity $v_{\rm rot}^{\rm d}$ of the disc.

The neutral hydrogen in the galactic discs causes Lyα absorption features in the spectra of background QSOs. The mean separation between the Lyα absorption features in a spectrum is $608\AA$. Calculate the comoving space density of the galactic discs.

State and explain any additional assumptions you make.

[Assume the Universe to be flat with matter density and cosmological constant parameters, $\Omega_{\rm mat} = 1 - \lambda = 0.25$, baryonic density parameter $\Omega_{\rm bar} = 0.05$ and assume the Hubble constant to evolve with redshift as, $H(z) = 70\,(0.25\,(1+z)^3 + 0.75)^{1/2}$ km s^{-1} Mpc^{-1}. The rest-frame wavelength of Lyα is $\approx 1216\AA$.]

3

A supernova at $z = 9$ leaves behind a black hole with mass $M_0 = 10 M_\odot$. Assume that the black hole grows by continuous spherical accretion of gas at the Eddington accretion rate.

Starting from the balance between gravitational and radiative forces on the accreting gas show that the black hole mass will grow as

$$M_{\rm bh} = M_0 \exp[t/t_{\rm e-fold}], \qquad \text{with} \qquad t_{\rm e-fold} \approx \epsilon_{\rm r} \frac{c\,\sigma_{\rm T}}{4\pi G\, m_p} \approx 4.5\, \epsilon_{\rm r}\, 10^8 {\rm yr},$$

where $\epsilon_{\rm r}$ is the radiative efficiency of accretion, $\sigma_{\rm T}$ is the cross-section for Thomson scattering and $m_{\rm p}$ is the proton mass.

Assume that the black hole is located in a region of the Universe where hydrogen is not yet reionized and emits 30% of its accretion luminosity in the form of photons above the hydrogen ionization threshold with spectral shape $L_\nu \propto \nu^{-2}$ and $\epsilon_{\rm r} = 0.1$. Estimate the time it takes for the hydrogen ionization front to reach a dark matter halo located at a distance of 1 Mpc from the black hole if you neglect recombinations. You can assume that all baryons are in the form of hydrogen and that there are no other sources of ionizing photons.

The dark matter halo has a mass of $2 \times 10^5 M_\odot$ and a virial temperature of $\approx 500K$. Assume the baryons in the dark matter halo to be in hydrostatic equilibrium with a temperature close to the virial temperature. What will happen to the baryons in the halo when the ionization front overtakes the halo? How long will this take? Assume that the halo has collapsed at $z = 9$.

State and explain any additional assumptions you make.

[Assume the Universe to be flat with matter density and cosmological constant parameters, $\Omega_{\rm mat} = 1 - \lambda = 0.25$, baryonic density parameter $\Omega_{\rm bar} = 0.05$. The critical density $\rho_{\rm crit} = 3H_0^2/8\pi G$ at $z = 0$ is $\approx 2.8 h^2 \times 10^{11} M_\odot {\rm Mpc}^{-3}$ with $h = 0.7$. The rest mass energy of a proton is $m_{\rm p} c^2 \approx 938$ MeV and the ionization threshold for hydrogen is ≈ 13.6eV.]

4

With the Press–Schechter ansatz, the mass fraction of the matter density in the Universe in collapsed objects with mass greater than M is

$$f(>M,t) = \mathrm{erfc}\left(\frac{\delta_c(t)}{\sqrt{2}\sigma(M, z=0)}\right) \quad \text{with} \quad \mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp\left(-y^2\right) \mathrm{d}y.$$

Explain the meaning of $\delta_\mathrm{c}(t)$ and $\sigma(M,z)$ and discuss how $\sigma(M,z)$ is related to the power spectrum of the matter density.

Assume $\sigma(M, z=3) \approx 0.85\,(v_\mathrm{vir}/250\,\mathrm{km\,s^{-1}})^{-1/2}$. Use the Press–Schechter formalism to calculate the characteristic time $t_\mathrm{d} = (\partial \ln(n)/\partial t)^{-1}$ for the comoving space density n of dark matter haloes with virial velocity $v_\mathrm{vir} = 10\,\mathrm{km\,s^{-1}}$ at $z = 19$ to increase.

Discuss and explain qualitatively how this time changes with

(i) increasing mass,

(ii) decreasing redshift.

[Assume the Universe to be flat with matter density and cosmological constant parameters, $\Omega_\mathrm{mat} = 1 - \lambda = 0.25$ and assume the Hubble constant to evolve with redshift as $H(z) = H_0\,(0.25\,(1+z)^3 + 0.75)^{1/2}$. You may also find it helpful to assume that $H_0^{-1} \approx 10^{10} h^{-1}$ yr with $h = 0.7$.]

END OF PAPER