UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Wednesday, 4 June, 2014 1:30 pm to 3:30 pm

PAPER 43

SUPERSYMMETRY

*Attempt no more than **TWO** questions.*

*There are **THREE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

List the superfield content and representations under SU(3)×SU(2)×U(1)$_Y$ of the MSSM fields. Briefly discuss the hypercharge gauge boson anomaly. Show explicitly that the contribution from each Standard Model family is zero, then argue for the addition of an extra Higgs doublet in the MSSM from anomaly arguments.

Discuss the other reason for introducing a second Higgs doublet into the MSSM.

2

(a) Find the integers A, B, C, D, E where

$$\begin{aligned} \partial^2(\theta\theta) &= A, & \bar\partial^2(\bar\theta\bar\theta) &= B, \\ \theta^\alpha\theta^\beta &= \frac{1}{C}\epsilon^{\alpha\beta}(\theta\theta), & \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta} &= \frac{1}{D}\epsilon^{\dot\alpha\dot\beta}(\bar\theta\bar\theta), \\ (\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta) &= \frac{1}{E}(\theta\theta)(\bar\theta\bar\theta)\eta^{\mu\nu} \end{aligned}$$

(b) Give the condition that restricts a general $N = 1$ superfield to be a vector superfield V.

(c) The covariant derivatives are defined in x^μ coordinates as

$$D_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu, \qquad \bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} - i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu.$$

Show that

$$\begin{aligned} D_\alpha V(y^\mu,\ \theta^\beta,\ \bar\theta^{\dot\alpha}) &= [\partial_\alpha + 2i(\sigma^\mu\bar\theta)_\alpha\partial_\mu]V(y^\mu,\ \theta^\beta,\ \bar\theta^{\dot\alpha}), \\ \bar D_{\dot\alpha} V(y^\mu,\ \theta^\alpha,\ \bar\theta^{\dot\beta}) &= -\bar\partial_{\dot\alpha} V(y^\mu,\ \theta^\alpha,\ \bar\theta^{\dot\beta}). \end{aligned}$$

(d) Consider the vector superfield in Wess-Zumino gauge

$$V(x^\mu,\ \theta^\alpha,\ \bar\theta^{\dot\alpha}) = (\theta\sigma^\mu\bar\theta)V_\mu(x^\mu) + (\theta\theta)(\bar\theta\bar\lambda(x^\mu)) + (\bar\theta\bar\theta)\theta\lambda(x^\mu) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x^\mu).$$

Show that the field strength superfield $W_\alpha = -\frac{1}{4}\bar D^2 D_\alpha V$ can be written

$$W_\alpha(y^\mu,\ \theta^\beta,\ \bar\theta^{\dot\alpha}) = \lambda_\alpha(y^\mu) + \theta_\alpha D(y^\mu) + \ldots$$

where '$\ldots$' denotes terms involving $\bar\lambda(y^\mu)$ and $V(y^\mu)$, which you need not find.

(e) What kind of superfield is W_α? Give a reason for your answer.

[Note that $\mathrm{Tr}(\sigma^\mu\bar\sigma^\nu) = 2\eta^{\mu\nu}$. Use the following conventions from the lectures: $\epsilon^{12} = -\epsilon_{12} = \epsilon^{\dot1\dot2} = -\epsilon_{\dot1\dot2} = +1$, $\partial_\alpha\theta^\beta = \delta^\beta_\alpha$, $\bar\partial_{\dot\alpha}\bar\theta^{\dot\beta} = \delta^{\dot\beta}_{\dot\alpha}$, $\theta\theta = \theta^\alpha\theta_\alpha$, $\bar\theta\bar\theta = \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}$.]

3

The Polonyi model of $N = 1$ supergravity has a single chiral superfield z, a superpotential

$$W = m^2(z + \beta),$$

where $\beta > 0$ is a real constant, and Kähler potential

$$K = |z|^2.$$

(a) Calculate whether the model breaks or preserves supersymmetry, giving any necessary conditions involving the parameters m, β and $\langle z \rangle$.
(b) Calculate the scalar potential of z, $V(z)$.
(c) Imposing the (approximate) observational constraint of a zero cosmological constant $V(\langle z \rangle) = 0$, where $\langle z \rangle$ is the vacuum expectation value of the scalar superfield, show that

$$\beta = -\sqrt{A} + B,$$

where A and B are integers that you should find.
(d) Show that

$$\langle z \rangle = \sqrt{C} + D,$$

in Planck units, where C and D are integers which you should find.

[In $M_{Pl} = 1$ units, the general $N = 1$ supergravity scalar potential is given by

$$V = e^K \left(K^{-1}_{ij} D_i W (D_j W)^* - 3|W|^2 \right),$$

where $D_i W = \partial_i W + W \partial_i K$.]

END OF PAPER