UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday, 5 June, 2014 1:30 pm to 4:30 pm

PAPER 21

COMPUTABILITY AND LOGIC

*Attempt no more than **FOUR** questions.*

*There are **FIVE** questions in total.*

The questions carry equal weight.

STATIONERY REQUIREMENTS
Cover sheet
Treasury Tag
Script paper

SPECIAL REQUIREMENTS
None

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1

Prove Tennenbaum's theorem that there is no recursive nonstandard model of Peano Arithmetic.

2

(i) Prove that, for every regular language L, there is a unique minimal FDA that recognises it.

(ii) An **interleaving** of two words w_1 and w_2 is a word obtained by inserting the characters from w_1 into w_2 in the order in which they appear in w_1. Thus, for example, both the strings $b0a1c$ and $ba01c$ are interleavings of the two strings bac and 01.

Now let L_1 and L_2 be regular languages over alphabets Σ_1 and Σ_2 respectively. Let the *interleaving* $L_1 \oplus L_2$ of two languages L_1 and L_2 be the set of words that can be obtained by interleaving words from L_1 with words from L_2.

If L_1 and L_2 are both regular must $L_1 \oplus L_2$ be regular?

3

Prove Scott's Isomorphism Theorem for $\mathcal{L}_{\omega_1,\omega}$.

4

(i) Let $<_A$ and $<_B$ be recursive (*i.e.* decidable sets of ordered pairs) dense linear orderings of $\mathbb{N}$ without endpoints. Must there exist a recursive isomorphism between them?

(ii) Find a natural deduction proof for $((((A \to B) \to A) \to A) \to B) \to B$ using only the two rules for $\to$ and the "identity rule": $\dfrac{A \quad B}{A}$

Then decorate your proof appropriately with λ-terms.

5

(i) State and prove Kruskal's theorem on the wellquasiordering of trees over a wellquasiorder.

(ii) If $\langle X, \leqslant_X \rangle$ is a wellquasiorder, must the relation $X' \leqslant X''$ iff $(\forall x' \in X')(\exists x'' \in X')(x' \leqslant_X x'')$ be a wellquasiorder?

END OF PAPER